

## Calculation of the Langelier Index

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The Langelier index (LI) is an approximate measure of the degree of saturation of calcium carbonate in water. It is calculated using the **pH**, **alkalinity** (reported as  $CaCO_3$  in mg/litre), **calcium concentration** (reported as  $Ca^{2+}$  in mg/litre), **total dissolved solids** (reported as  $TDS$  in mg/litre), and **water temperature** (reported as  $t$  in degrees Celsius, °C) of a water sample collected at the tap. These parameters are highlighted in **red** in the calculations listed below.

The Langelier Index (LI) is calculated as follows:

$$LI = pH - pH_s$$

where **pH** = measured pH of the tap water  
 $pH_s$  = calculated saturation pH of the tap water

$$pH_s = pK'_2 + pCa^{2+} - pK'_s - \log(2[Alk]) - \log \gamma_m$$

In order to calculate  $pH_s$ ,  $pK'_2$ ,  $pK'_s$  and  $pCa^{2+}$  must first be computed. Explanations of how to do these calculations are given below. In addition, concentrations of  $Ca^{2+}$  and  $Alk$  must also be converted to moles per litre. These steps are described below.

**1.** To compute  $pK'_2$ , which is the negative log of the activity constant  $K'_2$  ( $-\log K'_2$ ):

**a.** Determine the Ionic Strength (I), in moles per litre (M), of the water:

$$I(M) = (2.5 \times 10^{-5} \text{ moles / mg}) \times [TDS(\text{mg / litre})]$$

where **TDS = Total Dissolved Solids (mg/litre)**

**b.** Determine  $\gamma_m$ , the activity coefficient of monovalent ions (ions that are able to form only one covalent or ionic bond – having only one valence) using the Davies relationship:

$$\text{If } I < 0.5M, \quad \log \gamma_m = -AZ^2 \left( \frac{\sqrt{I}}{1 + \sqrt{I}} - 0.2I \right)$$

$$\text{If } 0.5M < I < 1.0M, \quad \log \gamma_m = -AZ^2 \left( \frac{\sqrt{I}}{1 + \sqrt{I}} \right)$$

where  $M$  = molarity (moles per litre)

$$A = 1.82 \times 10^6 (DT)^{-\frac{3}{2}}$$

$D = 78.3$ , the dielectric constant for water and  $T$  = temperature in Kelvins (K). To convert **temperature ( $t$ )** in degrees centigrade ( $^{\circ}\text{C}$ ) to Kelvins (K):

$$[T = t(^{\circ}\text{C}) + 273]$$

and  $Z$  = the oxidation number of the chemical species in question, which for monovalent ions = 1

$$\text{If } 0.5M < I < 1.0M, \log \gamma_m = -AZ^2 \left( \frac{\sqrt{I}}{1 + \sqrt{I}} \right)$$

c. Calculate  $pK_2$  :

$$pK_2 = \frac{2902.39}{T} + 0.02379(T) - 6.498$$

From which we calculate  $K_2$  :

$$K_2 = 10^{-pK_2}$$

d. Calculate  $\gamma_D$ , the activity coefficient of divalent ions (ions having two valences):

$$\log \gamma_D = -AZ^2 \left[ \frac{\sqrt{I}}{1 + \sqrt{I}} \right]$$

where as defined earlier;

$$A = 1.82 \times 10^6 (DT)^{-\frac{3}{2}}$$

$D = 78.3$ , the dielectric constant for water, and  $T$  = temperature in Kelvins (K). To convert **temperature ( $t$ )** in degrees centigrade ( $^{\circ}\text{C}$ ) to Kelvins (K):

$$[T = t(^{\circ}\text{C}) + 273]$$

and  $Z$  = the oxidation number of the chemical species in question, which for divalent ions = 2

Then calculate  $\gamma_D$  :

$$\gamma_D = 10^{\log \gamma_D}$$

e. Calculate  $K'_2$  :

$$K'_2 = \frac{K_2}{\gamma_D}$$

f. We can then calculate  $pK'_2$ :

$$pK'_2 = \log \frac{1}{K'_2}$$

2. To compute  $pK'_s$  which is the negative log of the activity constant  $K'_s$  ( $-\log K'_s$ ):

a. First compute  $pK_s$ :

$$pK_s = 0.01183t + 8.03$$

where  $t = \text{temperature in degrees Centigrade } (^{\circ}\text{C})$

b. Convert the  $pK_s$  value to  $K_s$ :

$$K_s = 10^{-pK_s}$$

c. Using the value of  $\gamma_D$  calculated earlier, calculate  $K'_s$ :

$$K'_s = \frac{K_s}{(\gamma_D)^2}$$

d. We can then calculate  $pK'_s$ :

$$pK'_s = \log \frac{1}{K'_s}$$

3. To compute  $pCa^{2+}$ :

a. Convert the concentration of  $Ca^{2+}$  (mg/litre) to moles per liter:

$$Ca^{2+}(\text{moles / litre}) = \frac{[Ca^{2+}(\text{mg / litre})] \times 10^{-3}}{40}$$

c. Calculate  $pCa^{2+}$ :

$$pCa^{2+} = \log \frac{1}{[Ca^{2+}(\text{moles / litre})]}$$

4. **Alkalinity (Alk)** is reported as mg/litre  $CaCO_3$ . It is necessary to convert the given **alkalinity** concentration to moles/litre:

$$[Alk(\text{moles / litre})] = \frac{[CaCO_3(\text{mg / litre})] \times 10^{-3}}{100}$$

**Reference:**

Benefield, L., Judkins, J. & Weand, B. 1982. *Process Chemistry for Water and Wastewater Treatment*. Prentice-Hall, Inc. Englewood Cliffs, New Jersey.