Adult Basic Education

Mathematics

Mathematics 2105C

Geometry

Curriculum Guide

Prerequisites: Mathematics 2105A, 2105B

Credit Value: 1

Mathematics Courses **[General College Profile]**

Mathematics 2105A
Mathematics 2105B
Mathematics 2105C
Mathematics 3107A
Mathematics 3107B
Mathematics 3107C
Mathematics 3109A
Mathematics 3109B
Mathematics 3109C

Table of Contents

To the Instructor

I. Introduction to Mathematics 2105C

One of the purposes of this course is to teach students to measure lengths and estimate measurement of objects using both metric and imperial units. Students will work with and create scale drawings as well as sketch 3-D designs using isometric dot paper. They will apply ratio and proportion to solve problems using similar triangles. Students will use the primary trigonometric ratios sine, cosine, and tangent in solving problems involving right triangles.

The study of trigonometry, starting with similar triangles, enables students to solve many ratio, proportion, and distance problems as well as problems that require determining the lengths of sides of triangles and measures of unknown angles. These skills are particularly useful for construction trades such as carpentry.

II. Prerequisites

Students may need some review in perimeter, area, surface area and volume. Students should be familiar with substituting values into formulas. In addition, students should know the basic properties of a triangle.

III. Textbook

Essentials of Mathematics 10 is designed to emphasize the skills needed in adult life as well as in the workplace. Students should appreciate that mathematics is practical and useful for accomplishing real-world activities. With this in mind, this resource has been developed with contents that are real and relevant to the lives of students.

Each chapter begins with an introduction which presents the key mathematical ideas that will be encountered. The following categories are in each chapter:

Chapter Goals: Located on the bottom of each introductory page, this section lists the major concepts to be learned.

Chapter Project and Project Activity: Each chapter contains a guided project. This type of group work is not well suited for the Adult Basic Education environment. Therefore, **these sections have been omitted from the course**. However, if there are several students working on the same chapter, instructors may use their discretion in assigning the **Chapter Project**, or some modification of it, for an assessment.

To the Instructor

<u>Exploration</u>: Most of the concepts are introduced, developed and explained in these lessons. In this section, **Examples** and **Solutions** for typical problems are provided. The instructor should ensure that students carefully study and understand each **Example** before proceeding.

<u>Class Discussion, Small Group Discussion and Pairs Activities</u>: As the titles imply, these activities are provided to give students an opportunity to work collaboratively. Some of these sections have been assigned in the Study Guide, especially if they can be completed by a student working alone.

<u>Mental Math</u>: The questions contained in these sections are often calculations that are similar to those required in the **Solutions** to the **Examples**. Although called **Mental Math**, students should <u>not</u> be required to complete these activities without pencil and paper. If students have difficulty with these problems, the instructor should provide practice worksheets. The solutions to **Mental Math** are found in the *Teacher Resource Book 10*.

<u>Notebook Assignment</u>: This section provides a series of problems similar to those in the **Exploration**. Students should attempt these problems only after the **Exploration** problems have been understood and all assigned **Mental Math** and practice worksheets have been completed. The textbook contains only answers to **Notebook Assignment**, but the *Teacher Resource Book 10* has solutions with workings and some explanations.

<u>Chapter Review</u>: This section contains a series of questions that review the chapter outcomes. Answers are in the textbook as well as the *Teacher Resource Book 10*.

<u>Case Study</u>: This part requires students to express their understanding of the skills they have learned. Answers are in the textbook as well as the *Teacher Resource Book 10*.

IV. <u>Technology</u>

The use of technology in our society is increasing and technological skills are becoming mandatory in the workplace. It is assumed that all students have a scientific calculator and its manual for their individual use. Ensure that the calculator used has “scientific” on it as there are calculators designed for business and statistics which would not have the functions needed for this course. Although students will sometimes use a calculator, they should first complete most problems using pencil and paper.

V. Curriculum Guides

Each new ABE Mathematics course has a Curriculum Guide for the instructor and a Study Guide for the student. The Curriculum Guide includes the specific curriculum outcomes for the course. Suggestions for teaching, learning, and assessment are provided to support student achievement of the outcomes. Each course is divided into units. Each unit comprises a **two-page layout of four columns** as illustrated in the figure below. In some cases the four-column spread continues to the next two-page layout.

Curriculum Guide Organization:
The Two-Page, Four-Column Spread

Unit Number - Unit Title

Outcomes	**Notes for Teaching and Learning**
Specific curriculum outcomes for the unit.	Suggested activities, elaboration of outcomes, and background information.

Unit Number - Unit Title

Suggestions for Assessment	**Resources**
Suggestions for assessing students' achievement of outcomes.	Authorized and recommended resources that address outcomes.

VI. Study Guides

The Study Guide provides the student with the name of the text(s) required for the course and specifies the sections and pages that the student will need to refer to in order to complete the required work for the course. It guides the student through the course by assigning relevant reading and providing questions and/or assigning questions from the text or some other resource. Sometimes it also provides important points for students to note. (See the *To the Student* section of the Study Guide for a more detailed explanation of the use of the Study Guides.) The Study Guides are designed to give students some degree of independence in their work. Instructors should note, however, that there is much material in the Curriculum Guides in the *Notes for Teaching and Learning* and *Suggestions for Assessment* columns that is not included in the Study Guide and instructors will need to review this information and decide how to include it.

VII. Resources

Essential Resources

Essentials of Mathematics 10, ISBN: 0-7726-4675-9

Essentials of Mathematics 10, Teacher Resource Book 10, ISBN: 0-7726-4808-5

Resources

http://mathforum.org
http://edHelper.com
http://www.purplemath.com/index.htm
http://www.educationindex.com/math/
http://www.learner.org/exhibits/dailymath/resources.html
http://www.onlineconversioncom

VIII. Recommended Evaluation

Written Notes	10%
Assignments	10%
Test(s)	30%
Final Exam *(entire course)*	50%
	100%

Geometry

Geometry

Outcomes

1.1 Measure the length of objects using the SI system.

1.1.1 Use ratios to convert between metric units of length.

1.2 Solve problems involving length, area, volume and surface area in the SI system.

1.2.1 Recognize and name the following figures: square, rectangle, parallelogram, trapezoid, triangle and circle.

1.2.2 Recognize and name the following objects: rectangular solid, sphere, cone, cylinder and pyramid.

1.2.3 Substitute values into formulas for area, surface area and volume.

Notes for Teaching and Learning

Again, be reminded that the **Chapter Project** is omitted.

The intent of this course is to teach the concepts of measurement, spatial geometry and trigonometry. Note that the calculations were done using the π key on a scientific calculator. Students may have slightly different answers if they use 3.14 for π.

The instructor should point out the π button on the scientific calculator.

When completing **Exploration 2**, students may need some review on the metric prefixes. The instructor should show students how to use a ratio which equals 1 to change from one metric unit to another. Encourage students to use this method when converting measurements. When understood and used properly, this method works better than the method of moving decimal points right or left.

Area, volume and surface area formulas may need some review. The instructor will need to work through some of the formulas and show students how to substitute values to find solutions. Students will need extra practice exercises.

The formulas on pages 258 and 259 of *Essentials of Mathematics 10* don't have to be memorized, but students should know how and when to use them.

Students should be able to identify the figures and objects on these two pages.

The instructor should demonstrate to students the reason why area units are squared and volume units are cubed.

Suggestions for Assessment

Study Guide questions 1.1 to 1.3 will meet the objectives of Outcomes 1.1 and 1.2.

Practice Exercise 1, *Area and Perimeter* has been assigned in the Study Guide. It could be used for an assessment. A copy of this worksheet is in the Appendix of this Curriculum Guide.

Resources

Essentials of Mathematics 10, Measurement in the Metric System, pages 247 - 264

Teacher Resource Book 10, pages 161 - 171

Blackline Master 11, Ruler

Appendix, Practice Exercise 1, *Area and Perimeter*

Geometry

Outcomes

1.3 Measure lengths using the imperial system.

1.3.1 Change from inches to feet and vice versa.

1.4 Solve problems involving length, area, volume and surface area in the imperial system.

Notes for Teaching and Learning

It is important that students learn the imperial system because measurement in construction is done using this system.

Students may be unfamiliar with imperial measurements which involve fractions. The instructor should use a ruler to show students the one-sixteenth increments in an inch.

Students can then count the increments and see, visually, how two-sixteenths equals one-eighth. Alternatively, the instructor could demonstrate that $\frac{1}{16} + \frac{1}{16} = \frac{2}{16} = \frac{1}{8}$ (reduced to lowest terms).

If fractions are involved when finding area or volume, students should be able to use pencil and paper, but they can also use the fraction button on their calculator or change the fraction to a decimal.

Suggestions for Assessment

Study Guide questions 1.4 and 1.5 will meet the objectives of Outcomes 1.3 and 1.4.

Practice Exercise 2, *Area, Perimeter and Volume* has been assigned in the Study Guide. It could be used for an assessment. A copy of this worksheet is in the Appendix of this Curriculum Guide.

Resources

Essentials of Mathematics 10, Imperial Measurement, pages 265 - 273

Teacher Resource Book 10, pages 172 - 178

Blackline Master 11, Ruler

Appendix, Practice Exercise 2, *Area, Perimeter and Volume*

Geometry

Outcomes

1.5 Estimate length, area and volume of objects in metric and imperial systems.

Notes for Teaching and Learning

As students are estimating in imperial and metric measurement, the instructor should advise students to check to see that their answers are reasonable. Students should have some ideas about measurement which should help in estimating measured quantities. For example, students should know that a metre is a little more than 3 feet and that a kilometre is about 0.6 miles.

Students shouldn't be expected to convert between the imperial and metric systems. The following site, www.onlineconversioncom gives the conversion factors for almost any conversion required.

Suggestions for Assessment

Study Guide questions 1.6 and 1.7 will meet the objectives of Outcome 1.5.

Resources

Essentials of Mathematics 10, Metric and Imperial Estimation, pages 276 - 280

Teacher Resource Book 10, pages 179 - 181

www.onlineconversion.com

Outcomes

1.6 Enlarge or reduce a dimensioned object according to a specific scale.

1.6.1 Determine the dimensions of a given object from a scale drawing.

1.6.2 Determine the dimensions of a scale drawing when the dimensions of the object and the scale are known.

Notes for Teaching and Learning

If possible, the instructor could provide scale models of cars or other items to give students a sense of scale.

The instructor should help students understand the effects that changing scale has on area and volume by asking them to predict and then calculate what happens to the area of a table when its dimensions are doubled and what happens to a box volume when the dimensions are doubled.

The instructor should remind students that a scale factor of 1:4 means 1 unit on the model represents 4 units of the same units on the actual object. If the scale factor is written 1 cm represents 4 m, then the 4 m should be changed to cm (or 1 cm change to m) before the scale factor can be written as a fraction or ratio.

So, $4\,\cancel{\text{m}} \times \frac{100\,cm}{1\,\cancel{m}} = 400$ cm. Therefore 1 cm represents 400 cm and this scale can then be written 1:400 or $\frac{1}{400}$.

Geometry

Suggestions for Assessment

Study Guide questions 1.8 to 1.11 will meet the objectives of Outcome 1.6.

Practice Exercise 3, *Word Problems* and Practice Exercise 4, *Scale Drawings* have been assigned in the Study Guide. Both worksheets are in the Appendix of this Curriculum Guide.

Resources

Essentials of Mathematics 10, Working to Scale, pages 281 - 288

Teacher Resource Book 10, pages 182 - 186

Appendix, Practice Exercise 3, *Word Problems* Practice Exercise 4, *Scale Drawings*

Outcomes

1.7 Draw three-dimensional objects using isometric dot paper.

Notes for Teaching and Learning

The instructor should provide students with isometric dot paper.

Students should be made aware of the fact that, on isometric dot paper, all distances between the dots are the same.

Suggestions for Assessment

Study Guide questions 1.12 to 1.14 will meet the objectives of Outcome 1.7.

Resources

Essentials of Mathematics 10, Isometric Dot Paper, pages 297 - 302

Teacher Resource Book 10, pages 189 and 190

Teacher Resource Book 10, Blackline Master 14 (Isometric Dot Paper)

Outcomes

1.8 Apply ratio and proportion to solve for missing sides or angles of similar triangles.

1.8.1 State the properties of similar triangles.

Notes for Teaching and Learning

Students will require a short review in the basic properties of a triangle. The bottom of page 304 of *Essentials of Mathematics 10* shows the proper way to label sides and vertices of triangles.

Students may need to be reminded of the definition of similar triangles. Similar triangles have the same shape. They will have the same shape if their corresponding angles are equal. Instructors should point out that when $\triangle ABC$ is similar to $\triangle DEF$, this can be written $\triangle ABC \sim \triangle DEF$. This means that:

$\angle A = \angle D$, therefore, $\angle A$ and $\angle D$ are corresponding angles;

$\angle B = \angle E$, therefore $\angle B$ and $\angle E$ are corresponding angles; and

$\angle C = \angle F$, therefore, $\angle C$ and $\angle F$ are corresponding angles.

Note: When using the notation $\triangle ABC \sim \triangle DEF$, the pairs of corresponding angles must be written in the same position.

Corresponding

$$\triangle ABC \sim \triangle DEF$$

Corresponding

Instructors should ensure that, before students move on to the next **Exploration**, they understand the ratios that are equivalent in similar triangles.

Students should be advised that when they are drawing similar triangles they should avoid using a $30°$- $60°$- $90°$ triangle and draw the base horizontally.

Suggestions for Assessment

Study Guide questions 1.15 and 1.16 will meet the objectives of Outcome 1.8.

Resources

Essentials of Mathematics 10, Using Similar Triangles, pages 303 - 310

Teacher Resource Book 10, pages 191 - 195
Blackline Master 15 (Protractor)

Geometry

Outcomes

1.9 Use the trigonometric ratios, sine, cosine, and tangent in solving right triangles.

1.9.1 Use a scientific calculator to determine trigonometric ratios.

Notes for Teaching and Learning

Although the Pythagorean Theorem is not mentioned in the textbook, the instructor should teach it as an extra topic. Often, in the application of problems requiring the use of Pythagorean relationship, buildings, poles and towers are used. Students should be advised that, unless otherwise stated, these structures are assumed to be at right angles to the ground.

Since some students may find trigonometry difficult at first, the instructor may need to provide more guidance and extra practice problems.

The instructor should ensure that students can quickly and correctly identify the sides of a right triangle as opposite, adjacent and hypotenuse.

When solving a problem, and finding the correct trigonometric function, students will need to be guided in the algebraic manipulations required to isolate the variable.

The instructor should ensure that students can correctly use their scientific calculators when solving problems which involve trigonometric ratios.

Geometry

Suggestions for Assessment

Study Guide questions 1.17 and 1.18 will meet the objectives of Outcome 1.9.

Resources

Essentials of Mathematics 10, Using Trigonometry, pages 311 - 316

Teacher Resource Book 10, pages 196 - 199

Outcomes

1.10 Use trigonometry to find hard-to-measure distances.

1.10.1 Solve angles of elevation problems.

1.10.2 Solve angles of depression problems.

1.10.3 Solve problems involving trigonometric ratios with and without drawings provided.

Notes for Teaching and Learning

Instructors must ensure that students have a good understanding of the topics which were developed in **Exploration 9** before they start this topic, Indirect Measurement.

This **Exploration** should really emphasize the power of trigonometry when it is not possible to make a measurement of a hard-to-reach area.

This **Exploration** may take a little more time than others. Omit the activity “Build a Clinometer” on pages 318 and 319.

Geometry

Suggestions for Assessment

Study Guide questions 1.19 to 1.21 will meet the objectives of Outcome 1.10.

Practice Exercise 5, *Trigonometry*, is suitable for a homework assignment.

The activity, **Packing the Packages**, could be used as a review of the chapter. If possible, students could work in pairs to complete this activity. This activity has NOT been assigned in the Study Guide.

The following or similar questions could be used for an assessment:

Chapter Review, questions 4, 5, 6, 8, 10, 11, 14, 16 - 31.

Resources

Essentials of Mathematics 10, Indirect Measurement, pages 317, 320 - 325

Chapter Review, pages 327 - 332

Teacher Resource Book 10, pages 200 and 211

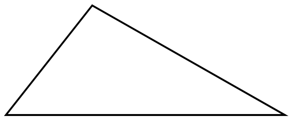
Appendix, Practice Exercise 5, *Trigonometry*

Activity, **Packing the Packages**

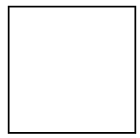
Appendix

Practice Exercise 1: Area and Perimeter

Name each of the figures.

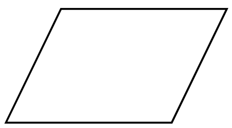


1) ____________________

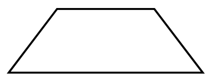


2) ____________________

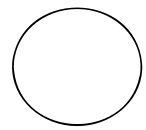
3) ____________________



4) ____________________



5) ____________________



6) ____________________

Match each formula with the area it represents.

7.__________ $\frac{1}{2}bh$ — a. area of a trapezoid

8. __________ πr^2 — b. area of a rectangle

9. __________ bh — c. area of a square

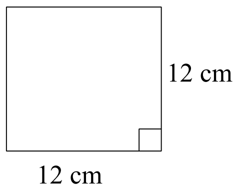
10. __________ lw — d. area of a triangle

11. __________ $\frac{1}{2}(a + b)h$ — e. area of a parallelogram

12__________ s^2 — f. area of a circle

Find the perimeter and area of each figure.

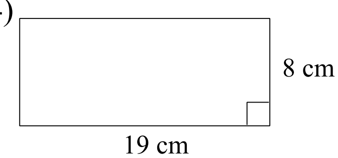
13)



P ______________________

A ______________________

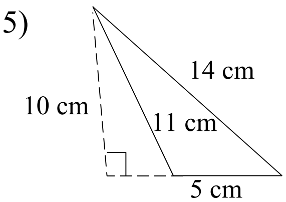
14)



P ______________________

A ______________________

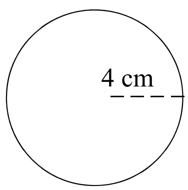
15)



P ______________________

A ______________________

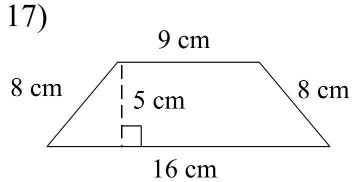
16)



P ______________________

A ______________________

17)



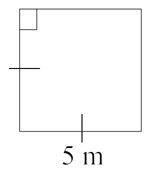
P ______________________

A ______________________

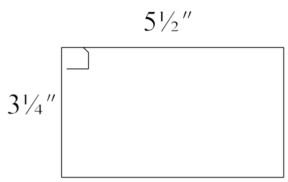
Practice Exercise 2: **Area, Perimeter and Volume**

1. Find the perimeter of each of the following:

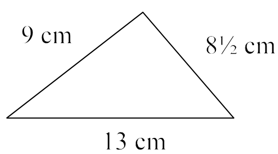
a)



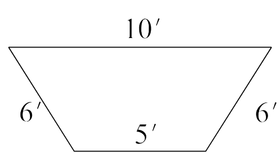
b)



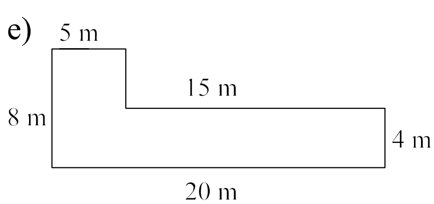
c)



d)

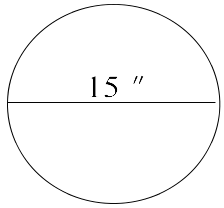


e)



2. Find the circumference of the following:

a)



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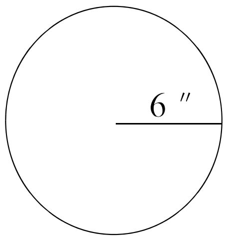
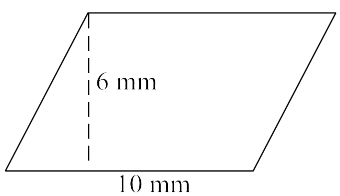
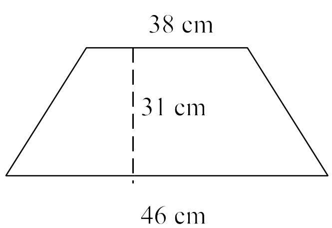


3. Find the area of the following figures:

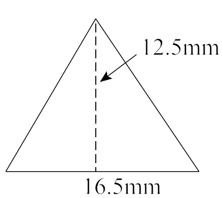
a)



b)



c)



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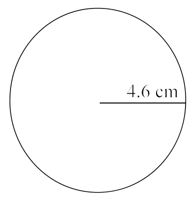
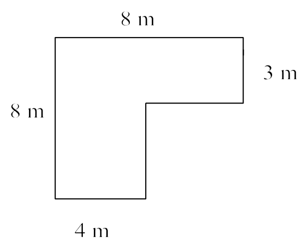
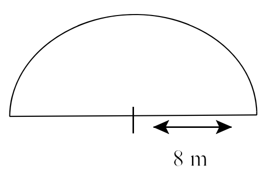


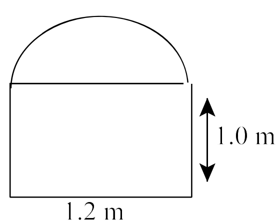
e)



f)

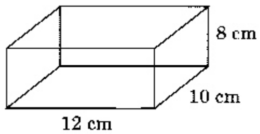


g)

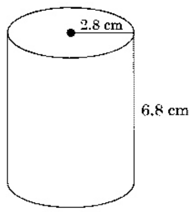


4. Determine the volume of the following shapes:

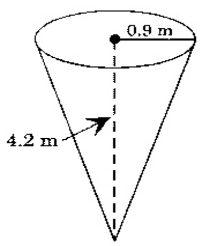
a)



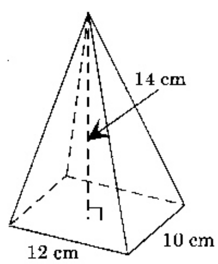
b)



c)

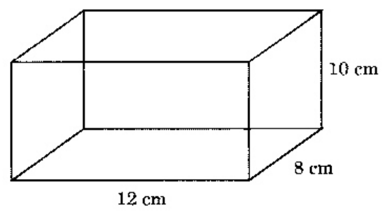


d)



Practice Exercise 3: **Word Problems**

1. Determine the surface area of the following:



2. One side of a parallelogram measures 50 mm, while the second side measures 45 mm. Find the perimeter of the parallelogram.

3. A cylindrical tank has a diameter of 5 metres and a height of 3.5 metres. Find the surface area of the tank.

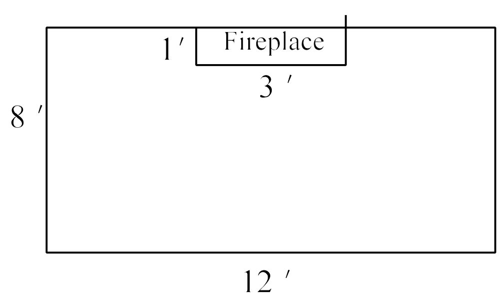
4. The diameter of a pizza is 26.2 cm. Find the area of the pizza.

5. Find the volume of a sphere with a diameter of 10 metres.

6. Find the volume of a cone that is 5 inches tall and has a diameter of 3.5 inches.

7. If a circular fish pond is 9.5 metres across, what is the area of the top surface and what is the distance around the pond?

8. The following diagram is an illustration of a great room.



a) If the entire great room except for the fireplace is to be covered with carpet, what area is to be carpeted?

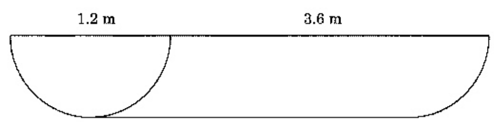
b) If the carpet costs $6.99 per square foot, how much would it cost, including tax, to purchase the carpet?

9. The perimeter of a square rug is 48 cm. What is the area?

10. Determine the quantity of wheat that can be stored in a cylindrical bin with a cone-shaped top. The diameter of the base of the bin is 7 m and its height is 5 m. The cone top has a height of 2 m.

11. Form two cylinders from a rectangular piece of paper, one by joining the long sides, one by joining the short sides. Which of these cylinders will have greater volume, or will they hold the same amount?

12. A trough has the following shape. What volume of water can this tank hold to the nearest cubic metre?



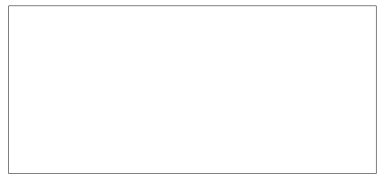
13. A storage box measures 1.3 by 1.2 by 1.0 yards. How much plywood would it take to build the storage box? A sheet of plywood measures 4′ × 8′ and costs $22.00. What is the cost to build the box?

14. A juice box measure 10 cm × 6 cm × 3 cm. What is its volume?

15. Have students determine the volume of four cubes with side lengths of 1 cm, 2 cm, 3 cm and 4 cm. Have students examine the relationship between the side lengths and the volumes.

Practice Exercise 4: **Scale Drawings**

1. The following diagram represents a coffee table. If the scale is 1:30, determine the actual dimensions of the table.



2. A recreation centre has dimensions of 35 m in length and 27 m in width. Represent the recreation centre with a scale drawing where 1 cm represents 5 m.

3. Complete the following chart:

Drawing Length (cm)	**Actual Length (cm)**	**Scale**
5.6	560	
0.6	600	
3.3	1650	
	200	1:20
	5000	1:1000

4. Using the scale $\frac{1}{4}$ inch = 1 foot, find the actual length in feet represented by the following lengths on a drawing:

a) 3″

b) $2\frac{1}{4}$ ″

c) $4\frac{3}{4}$ "

5. Using the scale $\frac{1}{8}$ inch = 1 foot, how long should a segment be drawn to represent an object whose actual length is:

a) 32 feet

b) 5 yards

c) 12 feet

d) 4 feet

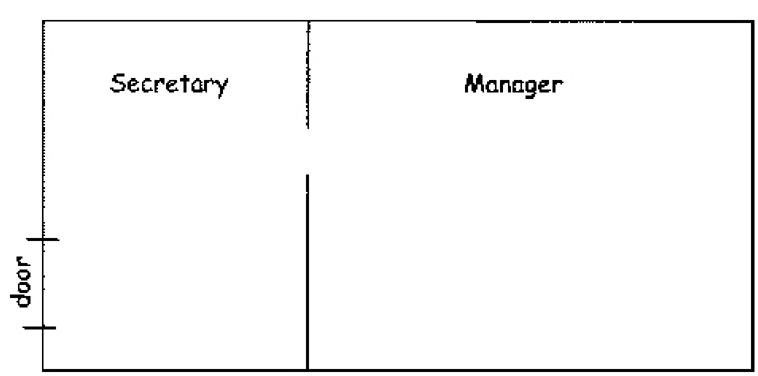
6. A scale drawing of a family room has dimensions of 4.5 cm by 3.25 cm. In the scale drawing, 1 cm represents 3 m. What are the dimensions of the actual family room?

7. An apartment building is 150 m tall. If the height of the building in the scale drawing is 25 cm, what scale is being used in the drawing?

8. Represent a circular window with a diameter of 50 cm. Use a scale of 1:10.

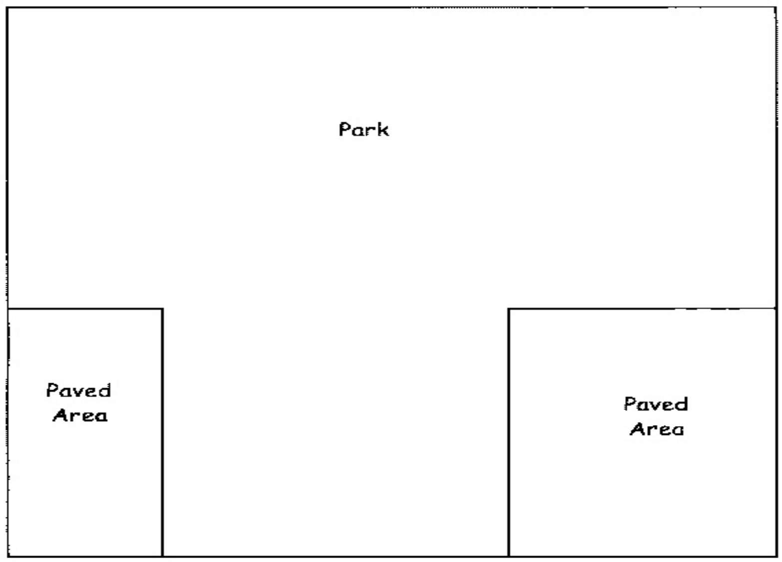
9. Represent a rectangular room measuring 12 feet by 24 feet with a scale drawing. Use a scale of 1 inch representing 2 feet.

10. The following is a drawing of an office, drawn to a scale of 1 cm = 1 m. How much carpet would be needed for the secretary's office? Manager's office?



11. The park below is drawn to a scale of 1 cm = 20 m. A groundskeeper must apply grass seed to the unpaved areas, taking care not to get any on the paved areas. The directions on the grass seed recommend 3.75 kg/100 m^2.

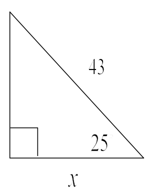
Calculate how much grass seed the groundskeeper will need to apply.



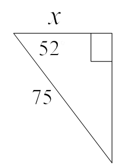
Practice Exercise 5: **Trigonometry**

1 Use the sine, cosine or the tangent ratio to find *x* for each figure.

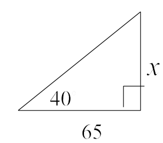
a)



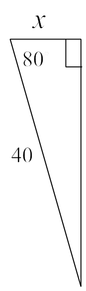
b)



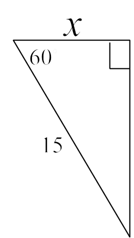
c)

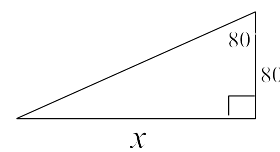


d)



e)





2 If a grasshopper jumped a vertical distance of 1.5 cm and a horizontal distance of 8.0 cm, at what angle did it rise from the ground?

3 A 6 m ladder is leaning against the wall of a house. The angle between the ladder and the ground is 68°. How far away is the ladder from the wall? How high up the house does the ladder reach? Give your answers accurate to two decimal places.

4 An airplane is flying at an altitude of 10 km. You spot the airplane at an angle of 55°. How far away is the airplane directly from you? Give your answer to the nearest kilometre.

Data Table for #1 of the **Notebook Assignment** on pages 306 and 307 of Student Text			
$\angle A$		$\frac{a}{d}$	
$\angle B$		$\frac{b}{e}$	
$\angle C$		$\frac{c}{f}$	
$\angle D$		$\frac{a}{b}$	
$\angle E$		$\frac{d}{e}$	
$\angle F$		$\frac{a}{c}$	
$\frac{\angle A}{\angle D}$		$\frac{d}{f}$	
$\frac{\angle B}{\angle E}$		$\frac{b}{c}$	
$\frac{\angle C}{\angle F}$		$\frac{e}{f}$	
length of side a		length of AM (cm)	
length of side b		length of DN (cm)	
length of side c		area of ΔABC	
length of side d		area of ΔDEF	
length of side e		$\frac{area\ of\ \Delta\ ABC}{area\ of\ \Delta\ DEF}$	
length of side f			

Answer Key for Practice Exercise 2: Area, Perimeter and Volume

1a) $p = 4a$
$p = 4(5)$
$p = 20\text{m}$

1b) $p = 2l + 2w$
$p = 2\left(5\tfrac{1}{2}\right) + 2\left(3\tfrac{1}{4}\right)$
$p = 11 + 6\tfrac{1}{2}$
$p = 17\tfrac{1}{2}$ "

1c) $p = 9 + 8\tfrac{1}{2} + 13$
$p = 30\tfrac{1}{2}$ cm

1d) $p = 10 + 6 + 5 + 6$
$p = 27'$

1e) $p = 5 + 15 + 4 + 20 + 8$
$p = 52$ m

2a) $C = \pi d$
$C = 15\pi$
$C = 47.1''$

2a) $C = \pi d$
$C = 15\pi$
$C = 47.1''$

2b) $C = 2\pi r$
$C = 2\pi\ (6)$
$C = 12\pi$
$C = 37.7''$

3a) $A = bh$
$A = 6 \times 10$
$\text{A} = 60\ \text{mm}^2$

3b) $A = \tfrac{1}{2}(a + b)h$
$A = \tfrac{1}{2}(38 + 46)(31)$
$A = \tfrac{1}{2}(84)(31)$
$A = 1302\ \text{cm}^2$

3c) $A = \tfrac{1}{2}bh$
$A = \tfrac{1}{2}(16.5)(12.5)$
$A = 103.1\ \text{mm}^2$

3d) $A = \pi r^2$
$A = \pi\ (4.6)^2$
$A = 21.6\pi$
$A = 66.5\ \text{cm}^2$

3e) $A_1 = (8)(3)$
$A_1 = 24$
$A_2 = (5)(4)$
$A_2 = 20$
$A_T = 24 + 20$
$A_T = 44\ \text{m}^2$

3f) $A = \tfrac{1}{2}\pi r^2$
$A = \tfrac{1}{2}(\pi)(8)^2$
$A = \tfrac{1}{2}(\pi)(64)$
$A = 32\pi$
$A = 100.5\ \text{m}^2$

3g) rectangle: $A = lw$

$A = (1)(1.2)$

$A = 1.2 \text{ m}^2$

semicircle: $A = \frac{1}{2}\pi r^2$

$A = \frac{1}{2}\pi(0.6)^2$

$A = \frac{1}{2}(0.36)\pi$

$A = 0.18\pi$

$A = 0.6 \text{ m}^2$

total area:

$A = 1.2 \text{ m}^2 + 0.6 \text{ m}^2$

$A = 1.8 \text{ m}^2$

4a) $V = lwh$

$V = (8)(10)(12)$

$V = 960 \text{ cm}^3$

4b) $V = \pi r^2 h$

$V = \pi(2.8)^2(6.8)$

$V = 53.3\pi$

$V = 167.5 \text{ cm}^3$

4c) $V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}(\pi)(0.9)^2(4.2)$

$V = 3.6 \text{ m}^3$

4d) $V = \frac{1}{3}lwh$

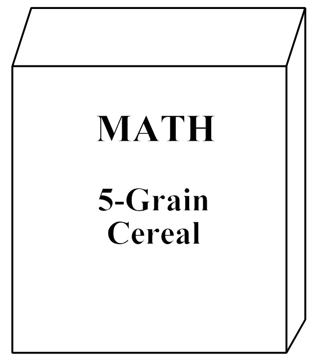
$V = \frac{1}{3}(12)(10)(14)$

$V = \frac{1}{3}(1680)$

$V = 560 \text{ cm}^3$

Packing the Packages

Did you ever notice that cereal comes in tall, thin boxes and that laundry soap comes in short, wide boxes? Is the way a product is packaged important? What box shape holds the most and uses the least amount of material to make? Let's explore the amount of packaging material needed to wrap a product.



1. Most cereal boxes are right rectangular prisms. That is, they have two parallel rectangular regions called *bases*, which are connected by four other rectangular regions called *lateral surfaces*. Suppose that your favorite cereal comes in a box that is 25 cm high, 22 cm long, and 7 cm wide. Sketch a picture of the box on a separate sheet of paper, and label it with the dimensions.

2. a) What are the dimensions of the bottom of the box? ______________________________

   b) What are the dimensions of the front of the box? ________________________________

   c) What are the dimensions of the side of the box? _________________________________

3. What part of the cereal box has the same dimensions as the bottom panel? _____________

4. Finding the amount of cardboard needed to make the cereal box is sometimes easier if you use a flat pattern of the box. This flat pattern is called a *net*. On a separate sheet of paper, draw a sketch showing the cereal box if you cut it apart and flattened it.

   Take an empty cereal box, and cut along as many edges are necessary to lay the box completely flat and keep it in one piece. Did your sketch match the box?

   ____

   To calculate the amount of material needed to make the box, you must find the surface area of the box.

5. How many rectangular regions make up the net? __________________________________

6. On your sketch, label the front, back, top, bottom and the right and left sides of the box. Also, label the dimensions of the box.

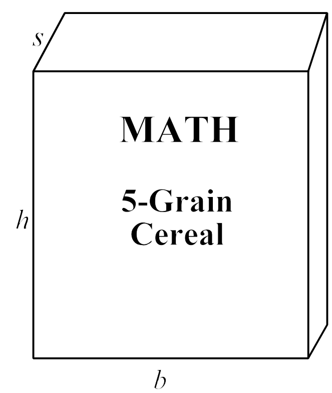
7. a) Find the area of the front panel of the cereal box. ______________________________

b) Find the area of the top panel of the cereal box. _______________________________

c) Find the area of one of the side panels of the cereal box. ________________________

8. Find the total surface area of the box. ____

9. You can develop a formula for the surface area, *SA*, of all boxes that are rectangular prisms by representing the different edges of a box with letters or variables. Assume that a box is sitting on its bottom surface with its front panel facing you, as pictured at right. Let *b* represent the bottom front edge of the box. Let *s* represent the top edge of the left side of the box. Finally, let *h* represent the left edge of the front panel of the box.



Write an equation that represents the surface area (*SA*) of the box

____

10. The amount of space inside the box is called its *volume*. You can find the volume of the box by calculating the number of one-unit cubes needed to fill the box. The one-unit cubes used here are cubic centimetres or one centimetre cubes. A dice is an example of a cube; all of the faces are the same size and shape.

a) If you begin to fill the box with one-centimetre cubes, how many one-centimetre cubes would complete the first layer in the bottom of the cereal box?

____

b) How many layers of one-centimetre cubes are needed to fill the whole box?

____

c) Find the number of one-centimetre cubes needed to fill the box?

____

d) Write an equation that related the dimensions of the box to the total number of one-centimetre cubes that would fill the cereal box.

____

11. Using the same variables that you used in problem 9, write an equation that represents the volume, *V*, of the cereal box. ____

12. Notice that the number of one-centimetre cubes on the bottom layer of the box is the same as the number of square centimetres in the area of the bottom panel. Will you always obtain this result? Explain your reasoning.

The NL.MATH Corporation manufacturers manipulatives for mathematics classrooms. One popular manipulative is a set of 100 blocks that are one centimetre cubes. NL.MATH Corporation is trying to find ways to decrease its packaging costs. The company has decided to arrange each set of blocks in a box that is shaped like a rectangular prism.

13. Complete the chart to find all of the possible box arrangements for 100 blocks. Turning a box or standing it upright does not constitute a new or different box shape. Find the surface area and the cost to manufacture the box if boxes can be made for 0.8 ¢, or $0.008, per square centimetre. Since the box must hold exactly 100 centimetre blocks, the volume of each box will be the same.

Base Front	Side	Height	Volume	Surface Area (cm^2)	Cost: ($0.008 per cm^2)
100	1	1	100		
50			100	304	
			100		
			100		
			100		
	10	1	100		$1.92
			100		
			100		

14. a) What are the dimensions of the box that costs the most to manufacture?

____

b) What does this box look like? ____

c) Why this shape is the most expensive one for the box to have? _____________

____

15. a) What are the dimensions of the box that costs the least to manufacture?
____

b) How does its shape differ from that of the most expensive box? ____________________
____

c) Why would you expect this box to cost less? ________________________________
____

d) If the box did not have to hold the cubes but still had a volume of 100 cubic centimetres, what would be the shape of the least expensive box? ____________________
____

16. An employee suggested that 125 one-centimetre blocks could be packaged for the same cost as 100 blocks.

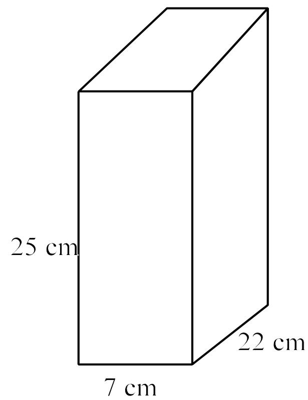
a) What are the dimensions of the box that would most economically hold 125 one-centimetre cubes? ____

b) Find the surface area and the cost of this package. ______________________________

c) Is the employee correct? Explain. ____

Answer Key for Packing the Packages

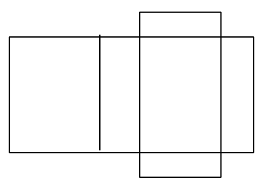
1. Sample box



2. a) 7 cm × 22 cm
   b) 25 cm × 22 cm
   c) 7 cm × 25 cm

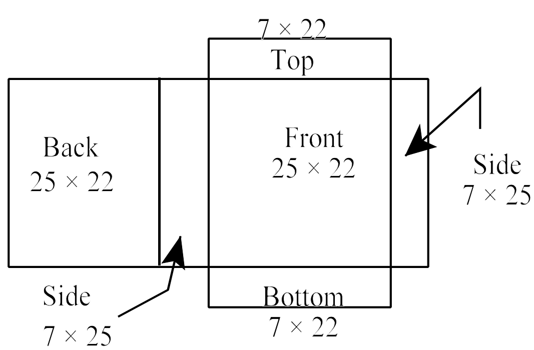
3. The top panel

4. Sample net



5. Six rectangular regions

6. Sample net



7. a) 550 cm^2
b) 154 cm^2
c) 175 cm^2

8. 1758 cm^2

9. $SA = 2bs + 2sh + 2bh$

10. a) 154 cubes
b) 25 layers
c) 25 × 154 = 3850 cubes
d) Total # of cubes = 7 × 22 × 25 = 3850

11. $V = bsh$

12. Yes, the faces of the cubes that are touching the bottom of the box exactly cover the same area as the bottom panel.

13. Sample chart

Base Front	Side	Height	Volume	Surface Area (cm^2)	Cost (\$0.008 per cm^2)
100	1	1	100	402	\$3.22
50	2	1	100	304	\$2.43
25	2	2	100	208	\$1.66
25	4	1	100	258	\$2.06
20	5	1	100	250	\$2.00
10	10	1	100	240	\$1.92
10	5	2	100	160	\$1.28
5	5	4	100	130	\$1.04

14. a) 100 cm × 1 cm × 1 cm
b) The box would be long and skinny.
c) It has the greatest surface area.

15. a) 5 cm × 5 cm × 4 cm
    b) It is more compact and its shape is closer to that of a cube.
    c) It has less surface area.
    d) A cube

16. a) 5 cm × 5 cm × 5 cm
    b) The box would require 150 square centimetres of material and would cost $1.20 to produce.
    c) No, 100 cubes can be packaged for less, since the 5 cm × 5cm × 4cm box has a surface area of 130 square centimetres and costs $1.04.