Adult Basic Education
Mathematics

Mathematics 3107C

Variation and Formulas

Study Guide

Prerequisites: Mathematics 2105A, 2105B, 2105C
Mathematics 3107A, 3107B

Credit Value: 1

Text: *Essentials of Mathematics 12*, Baron, Celia; Pacific Educational Press, 2003.

Mathematics Courses [General College Profile]

Mathematics 2105A
Mathematics 2105B
Mathematics 2105C
Mathematics 3107A
Mathematics 3107B
Mathematics 3107C
Mathematics 3109A
Mathematics 3109B
Mathematics 3109C

Table of Contents

I. Introduction to Mathematics 3107C

The main goal of Variation and Formulas is to investigate four different types of variations. You will learn how to recognize and establish a basic formula for each type of variation when given a graph, a table of values, or a description of the variation. You will also learn how to interpret graphs of various types of relations as well as manipulate and evaluate formulas.

To be successful in this course, you should know how to apply the order of operations correctly. You should also know how to correctly plot a graph and be familiar with the various terms associated with a relation, such as *independent* and *dependent variables*. These topics were discussed in Mathematics 3107B. Knowing how to change a percent to its decimal equivalent is important in this unit as well as knowing how to divide by a fraction. At various points throughout this guide you will be asked to see your instructor for review worksheets on these topics.

II. Resources

You will require the following:

- *Essentials of Mathematics 12*
- scientific calculator
- graph paper

Notes concerning the textbook:

Glossary: Knowledge of mathematical terms is essential to understand concepts and correctly interpret questions. Written explanations will be part of the work you submit for evaluation, and appropriate use of vocabulary will be required.

Your text for this course includes a Glossary where definitions for mathematical terms are found. Be sure you understand such definitions and can explain them in your own words. Where appropriate, you should include examples or sketches to support your definitions.

Examples: You should study the **Examples** in each section carefully and see your instructor if you have any questions. These **Examples** have full solutions to problems that will be a great help when answering assigned questions from **Notebook Assignment**.

Chapter Project: Unless your instructor directs you differently, you should omit all **Chapter Projects** and **Project Activity**.

To the Student

Notes concerning technology:

You should have a scientific calculator (the word "scientific" should be written on it) and the instruction booklet that belongs with it. Scientific calculators are fairly inexpensive. Even though your calculator will be a useful tool, you should be able to solve most exercises by using paper and pencil.

III.   Study Guide

This Study Guide is required at all times. It will lead you through the course and you should take care to complete each unit of study in the order given in this Guide.

To be successful, you should read the **References and Notes** first and then, when indicated by the ▶▶ symbols, complete the **Work to Submit** problems. Many times you will be directed to see your instructor, and this is vital, especially in a Mathematics course. If you have only a hazy idea about what you just completed, nothing will be gained by continuing on to the next set of problems.

To the Student

The Study Guide has the following format:

Reading for this Unit: In this box, you will find the name of the text, and the chapters, sections and pages used to cover the material for this unit. As a preliminary step, skim the referenced section, looking at the name of the section, and noting each category. Once you have completed this overview, you are ready to begin.

References and Notes	Work to Submit
This left hand column guides you through the material to read from the text. It will also refer to specific **Examples** found in each section. You are directed to study these **Examples** carefully and see your instructor if you have any questions. The **Examples** are important in that they not only explain and demonstrate a concept, but also provide techniques or strategies that can be used in the assigned questions. You should read and understand the **Hints** and **New Terms** that are at the bottom of selected pages in the textbook. The symbols ▶▶ direct you to the column on the right which contains the work to complete and submit to your instructor. You will be evaluated on this material. This column will also contain general notes which are intended to give extra information and are not usually specific to any one question.	There are two basic categories included in this column that correspond to the same categories in the sections of the text. They are **Mental Math** and **Notebook Assignment**. **Mental Math:** These problems should be completed using pencil and paper. If you have difficulty, you should see your instructor for extra practice problems. Usually the skills that are applied in **Mental Math** are those required to successfully complete **Notebook Assignment**. Your instructor will provide the answers to **Mental Math**. **Notebook Assignment:** This section provides a series of problems similar to those in the **Exploration**. You should attempt these problems only after the **Exploration** problems have been understood and all assigned **Mental Math** and practice worksheets have been completed. The textbook contains answers to **Notebook Assignment**. Your instructor will provide more detailed solutions with workings and some explanations. This column will also contain **Notes** which give information about specific questions.

IV.   Recommended Evaluation

Written Notes	10%
Assignments	10%
Test(s)	30%
Final Exam *(entire course)*	<u>50%</u>
	100%

Variation and Formulas

To meet the objectives of this unit, students should complete the following:

Reading for this unit:	*Essentials of Mathematics 12*		
	Chapter 6:	Exploration 1:	pages 281, 284 - 295
		Exploration 2:	pages 296 - 303
		Exploration 3:	pages 305 - 311
		Exploration 4:	pages 314 - 324
		Exploration 5:	pages 325 - 333
		Exploration 6:	pages 334 - 344
		Case Study:	pages 351 - 353

References and Notes	**Work to Submit**
Omit **Chapter Project** and all references to **Project Activity**. Read **Exploration 1.** Study and work through the calculations given in **Examples 1** to **3** on pages 285 - 289. **Recall:** In **Example 1** on page 285, the independent variable is graphed on the horizontal axis and the dependent variable is graphed on the vertical axis. To find the formula for a direct variation use the general form from Mathematics 3107B: *dependent variable = (slope)(independent variable).*	

Variation and Formulas

References and Notes	Work to Submit
Recall: In **Example 2b** on page 286, to complete the table of values, use the formula from 2a and substitute in the given values of p. **Example:** $A = 80p$ $A = 80(2) = 160\, sq.\, feet$	
Note: Direct variations have straight line graphs that pass through the point (0, 0). The formula for a direct variation always has the form $y = kx$, where y is the dependent variable, x is the independent variable and k is the constant of variation.	
Note: In **Example 3f** and **3g** on page 289, the formula is used to interpolate and extrapolate the data. The graph from 3d can also be used.	
Note: On the bottom of page 289 "example 2" should be replaced with "example 1".	
Read **Hints** and **New Terms** on the bottom of the pages.	
Answer the following questions. ▶▶	1.1 Briefly describe, in your own words, *direct variation*. Include a sketch in your description.

Variation and Formulas

References and Notes	Work to Submit
See your instructor for solutions to **Mental Math.**	1.2 **Mental Math**, page 290 Answer questions 1 - 3.

References and Notes

See your instructor for solutions to **Mental Math.**

Note: You should complete the **Small Group Activity** individually. Since you are doing this activity on your own, you will have to create two tables. Using the points from these tables, graph two lines on the same set of axes and then answer the questions.

Work to Submit

1.2   **Mental Math**, page 290
Answer questions 1 - 3.

1.3   **Small Group Activity**, page 284
(*See note below on part c.*)

Part c: Recall that to calculate slope, use
$$slope = \frac{rise}{run} \, .$$

1.4   **Notebook Assignment**, pages 292 - 295
Answer questions 1 - 4.
(*See notes below on questions 1b, 2a, 3a, and 4d.*)

Answer questions 5 - 8.
(*See note below on question 8.*)

Answer questions 9 and 10.
(*See note below on question 10a.*)

Question 1b: The graph on page 292 is missing the following points: (1, 1.25), (2, 2.50), (3, 3.75), and (4, 5.00).

Question 2a: Make sure that your graph has a title and the axes are labelled and contain units of measure.

Question 3a: To determine the constant of variation, write the general form of a direct variation using *C* for cost and *a* for area. Substitute in the given values for cost and area to find *k*.

Variation and Formulas

References and Notes	Work to Submit
	Question 4d: The time you calculate in this problem will be in hours. You must convert this time from hours to minutes to find time to the nearest minute.
Answers to **Notebook Assignment** are in the back of your textbook. Your instructor will have detailed solutions to these problems.	**Question 8:** First find the constant of variation, k, then use this value and the value of the sales to calculate her commission.
	Question 10a: Since k was found using minutes, you must convert 5 hours to minutes to find the amount of grass the student would cut.
Read **Exploration 2.** Study and work through the calculations given in **Examples 1** and **2** on pages 298 - 300.	
Note: Partial variations are similar to direct variations. Partial variations have straight line graphs with a slope equal to the constant of variation, k. The graph does **not** pass through (0, 0). The formula for a partial variation always has the form $y = kx + F$, where y is the dependent variable, x is the independent variable, k is the constant of variation, and F is the fixed value.	
Answer the following questions. ▶▶	1.5 Briefly describe, in your own words, *partial variation*. Include a sketch in your description.

Variation and Formulas

References and Notes	Work to Submit
You should understand the terms **fixed cost** (a cost that remains constant) and **variable cost** (a cost that changes depending upon the amount of goods purchased).	1.6 **Notebook Assignment**, pages 302 and 303 Answer questions 1 and 2. (*See notes below on questions 1a and 1b.*) Answer questions 4 - 6. (*See notes below on questions 4c and 5a.*) **Question 1a:** To write the formula, use *C* to represent the cost to rent the car, and *d* to represent the distance travelled in kilometres. **Question 1b:** Create a table of values to graph the formula from question 1a. Choose your own values for the number of kilometres travelled. **Question 4c:** To determine the total charge for a repair, convert 30 minutes to hours. **Question 5a:** To write the formula for this variation, convert 6% to its decimal equivalent.
Read **Exploration 3.** Study and work through the calculations given in **Examples 1** and **2** on pages 306 - 308.	

Variation and Formulas

| References and Notes	Work to Submit
Note: In a direct squared variation, the independent variable is squared. The graph of this variation is curved and passes through (0, 0). The formula for a direct squared variation always has the form $y = kx^2$ where y is the dependent variable, x is the independent variable and k is the constant of variation.	
Answer the following questions. ▶▶	1.7 Briefly describe, in your own words, *direct squared variation*. Include a sketch in your description.
	1.8 **Mental Math**, page 306 Answer questions 1 and 2. (*See note below on question 2.*)
	Question 2: Be sure to use the correct order of operations when doing this calculation. Square the number represented by x first and then multiply by 5.
Note: You should complete the **Small Group Activity** individually.	1.9 **Small Group Activity**, page 306 (*See note below on* **Small Group Activity**)
	Note: Recall that the area of a square is given by: $$A = l^2.$$
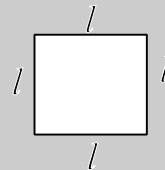
Variation and Formulas

References and Notes	Work to Submit
Remember: Make sure you include units (**examples:** m, km, h, m/s) in your answers when necessary.	1.10 **Notebook Assignment**, pages 309 - 311 Answer questions 1 and 2. (*See note below on question 2a*.) Answer questions 3 - 7. (*See note below on question 6b*.)
	Question 2a: To answer this question ask yourself, which graph shows direct variation? Partial variation? Direct squared variation?
	Question 6b: You do not need to do a comparison of the stopping distances of the vehicles. Simply do two separate calculations using the speeds given in the question to find the stopping distances.
You are not required to complete **Problem Analysis** and **Games** on pages 312 and 313. Read **Exploration 4.** Study and work through the calculations given in **Examples 1** to **4** on pages 316 - 321. **Note:** The graph of an inverse variation decreases as you move to the right along the horizontal axis. The graph is curved and does **not** pass through the point (0, 0). The formula for an inverse variation always has the form $y = \dfrac{k}{x}$, where *y* is the dependent variable, *x* is the independent variable and *k* is the constant of variation.	

Variation and Formulas

References and Notes	Work to Submit

Note: In **Example 2** on page 318, the term "inversely proportional" means that as the independent variable *increases* the dependent variable *decreases*. Since the interest rate is given as a percent, then to calculate *k* use $r = 6$.
 	
Note: Example 2d on page 319 mentions *The Rule of 72*. You do not need to know this rule. However, if you would like to know more about this rule you can see *Essentials of Mathematics 11*, page 37.
 	
Answer the following questions. ▶▶
 	
If necessary, see your instructor for review on dividing by a fraction.
 	
Note: You should complete the **Small Group Activity** individually. Since you are doing this activity on your own, you will need to construct three tables, one for each equation given. Use these tables to create graphs of the equations.
	
	1.11 Briefly describe, in your own words, *inverse variation*. Include a sketch in your description.
	
	1.12 **Mental Math**, page 316
	        Answer questions 1 and 2.
	
	1.13 **Small Group Activity**, page 315 (omit part e)
	        (*See notes below on parts c and d*.)
	
	Part c: To answer this question substitute $x = \dfrac{1}{100}$ into the three equations given.
	
	Part d: To answer this question substitute $x = 100$ into the three equations given.

Variation and Formulas

References and Notes	Work to Submit
	1.14 **Notebook Assignment**, pages 322 - 324 Answer questions 1 - 3c, 4 and 5. (*See note below on question 5*.) Answer questions 6 - 8. (*See notes below on questions 6 and 7*.)
	Question 5: Hertz is the unit of measure of frequency.
	Questions 6: Litres are the units of measure of the volume of a gas and atmospheres are the units of measure of pressure.
	Question 7: To answer this question, use the formula: $$I = \frac{k}{r}.$$
Read **Exploration 5.** Study and work through the calculations given in **Examples 1** to **3** on pages 328 and 329. **Note:** In **Example 1** on page 328, what kind of variation is represented in each question? Direct variation, partial variation, direct squared variation, or inverse variation? **Note:** In **Example 2** on page 328, the golfer takes her first shot at the point (0, 0) on the graph. Answer the following questions. ▶▶	
	1.15 **Mental Math**, page 331 (*See note below on part c*.)
	Part c: In this question, "blood glucose level" refers to the amount of sugar in a person's blood.

Variation and Formulas

References and Notes	Work to Submit
	1.16 **Notebook Assignment**, pages 330 - 333 Answer question 1. (*See note below on question 1c*.) Answer questions 2 - 4. (*See note below on question 4*.) Answer questions 5 - 8. (*See notes below on questions 5 and 7c*.)
	Question 1c: To answer this question see **Example 4** on page 321.
	Question 4: Take the point where the rocket takes off from the ground to be at a height of 0. That is, $H = 0$ when the rocket takes off.
	Question 5: Carla's house is at the origin, (0, 0), on the graph.
	Question 7c: Sketch your graph so that at $t = 0$ minutes, you start to fill up the tub. As time goes on, you wash your dog, then empty the tub so that all of the water has drained out at $t = 20$ minutes.
Read **Exploration 6.** Study and work through the calculations given in **Examples 1** to **4** on pages 336 - 340. There is a π button on your calculator. Be sure you know how to use it. You should note that if you use the π button when solving formulas you will obtain a slightly different answer than if you use $\pi = 3.14$.	

Variation and Formulas

References and Notes	Work to Submit
Note: In **Example 2** on page 338, the diameter of a circle is equal to half the radius.	
Note: In **Example 3** on page 339, the interest rate of 5% must be changed to its decimal equivalent to answer this question.	
If necessary, see your instructor for review on order of operations and changing a percent to its decimal equivalent.	
Answer the following questions. ▶▶	**1.17** **Mental Math**, page 335 Answer questions 1 and 2.
Note: You should complete the **Small Group Activity** individually.	**1.18** **Small Group Activity**, page 335
See your instructor for **Practice Exercise 1**, *Solving Simple Equations*.	**1.19** **Practice Exercise 1**, *Solving Simple Equations*
Recall: When solving formulas, isolate the desired variable first before substituting in the given values. This topic was discussed in Mathematics 3107B. **Do not forget** to include units in your final answer.	**1.20** **Notebook Assignment**, pages 341 - 344 Answer questions 1 - 4. (*See notes below on questions 1b, 1e, and 4b*.) Answer questions 5 - 7. (*See notes below on questions 5 and 7*.)

Variation and Formulas

References and Notes

Note: Refer to the list of formulas on pages 336 and 337 to answer the **Notebook Assignment** questions. You do not need to memorize these formulas, but you do need to know how to solve them.

Work to Submit

Answer questions 8 - 11.
(*See notes below on questions 9 and 10.*)

Questions 1b and 5: Change the percents to their decimal equivalents to answer these questions.

Question 1e: To solve for the hypotenuse, *h*, take the square root of the value obtained for h^2.
$$h = \sqrt{h^2}$$

Question 4b: See your instructor if you need help when trying to rearrange the formula to solve for *e*.

Question 7: Recall that the shape of a tin can is a cylinder. To find the amount of metal required to manufacture 24 cans, find the surface area of 1 tin can using the given information and multiply that surface area by 24.

Question 9: To answer 9a use the formula for the volume of a sphere to find the radius of the balloon. To answer 9b use the formula for the surface area of a sphere to find the amount of material needed to make the balloon.

Question 10: In this question, let *r* = 2.5 ohms, *s* = 4 ohms, and *t* = 5.5 ohms.

1.21   **Case Study,** pages 351 - 353
Answer questions 1 and 2.
(*See notes below on questions 1a, 1c, and 2.*)

Answer questions 3 and 4.
(*See note below on question 3c.*)

Variation and Formulas

References and Notes	Work to Submit
	Question 1a: To find how far the skydiver has fallen, subtract the value obtained for D from 2000 m.
	Question 1c: To answer this question, the value of D is $D = 3000 - 200 = 2800$ m.
	Question 2: In this question, the term "directly proportional" means direct variation.
	Question 3c: To determine the value of the constant of variation, k, first write the general formula for the appropriate variation and use a value of t and D from the given table of values. For example, use $t = 2$ and $D = 161$.