Adult Basic Education
Mathematics

Mathematics 3109C

Personal Finance
Statistics

Curriculum Guide

Prerequisites: Mathematics 2105A, 2105B, 2105C
                       Mathematics 3109A, 3109B

Credit Value: 1

<u>Mathematics Courses</u> **[General College Profile]**

Mathematics 2105A
Mathematics 2105B
Mathematics 2105C
Mathematics 3107A
Mathematics 3107B
Mathematics 3107C
Mathematics 3109A
Mathematics 3109B
Mathematics 3109C

Table of Contents

To the Instructor

I. Introduction to Mathematics 3109C

In the first unit, students will become familiar with different types of life and property insurance. They will learn how to decide how much life insurance is needed and then use Life Insurance tables in the textbook to calculate the monthly cost or premium. Students will also learn about mortgages and different payment options and how to calculate the interest that will be paid on a given mortgage. The instructor should ensure that students have access to and know how to use an on-line mortgage calculator.

In the second unit, students will analyze data. They will learn about percentile rank, calculate standard deviation, as well as draw and label normal curves.

II. Prerequisites

Students should be able to work with percents and use the order of operations. They should also be able use a calculator to find the square root.

III. Textbook

Essentials of Mathematics 12 is designed to emphasize the skills needed in adult life as well as in the workplace. Students should appreciate that mathematics is practical and useful for accomplishing real-world activities. With this in mind, this resource has been developed with contents that are real and relevant to the lives of students.

Each chapter begins with an introduction which presents the key mathematical ideas that will be encountered. The following categories are in each chapter:

Chapter Goals: Located on the bottom of each introductory page, this section lists the major concepts to be learned.

Chapter Project and Project Activity: Each chapter contains a guided project. This type of group work is not well suited for the Adult Basic Education environment. Therefore, **these sections have been omitted from the course**. However, if there are several students working on the same chapter, instructors may use their discretion in assigning the **Chapter Project**, or some modification of it, for an assessment.

Exploration: Most of the concepts are introduced, developed and explained in these lessons. In this section, **Examples** and **Solutions** for typical problems are provided. The instructor should ensure that students carefully study and understand each **Example** before proceeding.

To the Instructor

<u>Class Discussion, Small Group Discussion and Pairs Activities</u>: As the titles imply, these activities are provided to give students an opportunity to work collaboratively. Some of these sections have been assigned in the Study Guide, especially if they can be completed by a student working alone.

<u>Mental Math</u>: The questions contained in these sections are often calculations that are similar to those required in the **Solutions** to the **Examples**. Although called **Mental Math**, students should <u>not</u> be required to complete these activities without pencil and paper. If students have difficulty with these problems, the instructor should provide practice worksheets. The solutions to **Mental Math** are found in the *Teacher Resource Book 12.*

<u>Notebook Assignment</u>: This section provides a series of problems similar to those in the **Exploration**. Students should attempt these problems only after the **Exploration** problems have been understood and all assigned **Mental Math** and practice worksheets have been completed. The textbook contains only answers to **Notebook Assignment**, but the *Teacher Resource Book 12* has solutions with workings and some explanations.

<u>Chapter Review</u>: This section contains a series of questions that review the chapter outcomes. Answers are in the textbook as well as the *Teacher Resource Book 12*.

<u>Case Study</u>: This part requires students to express their understanding of the skills they have learned. Answers are in the textbook as well as the *Teacher Resource Book 12*.

IV.   <u>**Technology**</u>

The use of technology in our society is increasing and technological skills are becoming mandatory in the workplace. It is assumed that all students have a scientific calculator and its manual for their individual use. Ensure that the calculator used has "scientific" on it as there are calculators designed for business and statistics which would not have the functions needed for this course. Although students will sometimes use a calculator, they should first complete most problems using pencil and paper.

To the Instructor

V.   Curriculum Guides

Each new ABE Mathematics course has a Curriculum Guide for the instructor and a Study Guide for the student. The Curriculum Guide includes the specific curriculum outcomes for the course. Suggestions for teaching, learning, and assessment are provided to support student achievement of the outcomes. Each course is divided into units. Each unit comprises a **two-page layout of four columns** as illustrated in the figure below. In some cases the four-column spread continues to the next two-page layout.

Curriculum Guide Organization:
The Two-Page, Four-Column Spread

Unit Number - Unit Title	
Outcomes Specific curriculum outcomes for the unit.	**Notes for Teaching and Learning** Suggested activities, elaboration of outcomes, and background information.

Unit Number - Unit Title	
Suggestions for Assessment Suggestions for assessing students' achievement of outcomes.	**Resources** Authorized and recommended resources that address outcomes.

VI.   Study Guides

The Study Guide provides the student with the name of the text(s) required for the course and specifies the sections and pages that the student will need to refer to in order to complete the required work for the course. It guides the student through the course by assigning relevant reading and providing questions and/or assigning questions from the text or some other resource. Sometimes it also provides important points for students to note. (See the *To the Student* section of the Study Guide for a more detailed explanation of the use of the Study Guides.) The Study Guides are designed to give students some degree of independence in their work. Instructors should note, however, that there is much material in the Curriculum Guides in the *Notes for Teaching and Learning* and *Suggestions for Assessment* columns that is not included in the Study Guide and instructors will need to review this information and decide how to include it.

VII.   <u>Resources</u>

Essential Resources

Essentials of Mathematics 12, ISBN: 0-7726-4997-9

Essentials of Mathematics 12, Teacher Resource Book 12, ISBN: 0-7726-5049-7

Mathematics 3109C Study Guide

Resources

http://mathforum.org
http://edHelper.com
http://www.purplemath.com/index.htm
http://www.educationindex.com/math/
http://www.learner.org/exhibits/dailymath/resources.html
http://www.statcan.ca
http://cba.ca
http://www.termcanada.com
http://www.canadamortgage.com
www.calculatorz.com/united/amortschedule.cgi
www.rbc.com
www.cibc.com
www.scotiabank.com
www.kanetix.com

VIII.   <u>Recommended Evaluation</u>

Written Notes	10%
Assignments	10%
Test(s)	30%
Final Exam *(entire course)*	<u>50%</u>
	100%

Personal Finance Statistics

Unit 1 - Personal Finance

Outcomes

1.1   Solve problems involving different types of life insurance including term and whole-life insurance.

1.1.1   Appropriately use terms associated with life insurance including: premium, beneficiary, policy, and insurer.

1.1.2   Use tables to calculate the costs of term and whole-life insurance that are dependent on the characteristics of the insured person.

Notes for Teaching and Learning

In this **Exploration**, students will become familiar with term and whole-life insurance. Students should become aware that they need to plan for their future and life insurance is a part of personal finance.

Students may need some guidance in reading the tables on pages 14 - 17 of the textbook. After completing this section, students should know that smokers pay higher premiums than non-smokers and men pay higher premiums than women.

Encourage students to access life expectancy tables, available online at www.statcan.ca. Look under Health, Vital Statistics. CANSIM table 102-0125

Students are not required to complete the **Chapter Project** or **Project Activity**. The instructor, however, could use the **Chapter Project** for Personal Finance as an assessment in the place of a chapter test. Some modifications may be required and it may be best suited for students working in pairs.

Students should be familiar with the vocabulary in **New Terms**, but they are not required to memorize the definitions.

Suggestions for Assessment

Study Guide questions 1.1 to 1.3 will meet the objectives of
Outcome 1.1.

Problems similar to the following can be used for assessment.

A male smoker aged 25 buys a $50,000 whole-life policy.

a) What will he pay per year?
b) What will he pay per month?
c) What is the total difference in premium over a year when
    you pay per month?
d) Why do people like to pay on a monthly basis?
e) How much less would this person pay per year if he did
    not smoke?

Resources

Essentials of Mathematics 12,
Life Insurance, pages 11,
13 - 23

Teacher Resource Book 12,
pages 17 - 24

www.termcanada.com

www.statcan.ca

Outcomes

1.2   Solve problems involving the following different types of mortgages: closed, open, fixed rate and variable rate.

    1.2.1   Appropriately use the following terms associated with mortgages: amortization period, equity, interest, principal and unpaid balance.

    1.2.2   Define the following terms: closed mortgage, fix-rate mortgage, open mortgage and variable rate mortgage.

    1.2.3   Determine the payments to be made on a given type of mortgage when different repayment schedules are used.

Notes for Teaching and Learning

Although amortization schedules are completed in **Example 1**, on page 29 of the text, the ability to complete these schedules is <u>not</u> required for this course. It is important, however, for students to be able to **read** amortization tables and see how, initially, most of the mortgage payment goes towards interest.

The website <u>www.canadamortgage.com</u> allows students to look at amortization schedules as well as see the effects that different amortization periods have on monthly payments.

The instructor should talk to students to determine if they understand the impact on principal and interest payments when the term of the mortgage is changed or when the interest rate is increased or decreased.

Depending on the mortgage calculator used, answers to **Notebook Assignment** may vary slightly.

Unit 1 - Personal Finance

Suggestions for Assessment

Study Guide questions 1.4 and 1.5 will meet the objectives of Outcome 1.2.

Problems similar to the following can be used for assessment.

1. What is the difference between an open and closed mortgage?

2. If you obtain a mortgage of $80,000 for 20 years at a rate of 6.75%, find the monthly payment. What would the monthly payment be if you borrowed this amount for 15 years?

3. How much interest would be paid for the first and second months in (2) above?

Resources

Essentials of Mathematics 12, Calculating Mortgage Payments, pages 24 - 33

Teacher Resource Book 12, pages 25 - 29

www.canadamortgage.com

www.calculatorz.com/united/amortschedule.cgi

www.rbc.com
www.cibc.com
www.scotiabank.com

Outcomes

1.3   Use an online mortgage calculator to determine the payment changes that would occur by changing amortization periods and payment options of a given mortgage.

   1.3.1   Explain different payment options that are available for mortgages and the advantages and disadvantages of each.

Notes for Teaching and Learning

Since this exploration considers the different payment options available, the instructor should encourage students to access the site www.canadamortage.com. This site readily demonstrates how the different payment options can dramatically change the amount of interest paid on a mortgage.

Students should use an on-line mortgage calculator and not a spreadsheet program to complete the questions in this chapter.

Suggestions for Assessment

Study Guide question 1.6 will meet the objectives of
Outcome 1.3.

Resources

Essentials of Mathematics 12,
Exploring Mortgage
Payments, pages 34 - 41

Teacher Resource Book 12,
pages 30 - 33

www.canadamortgage.com

Outcomes	Notes for Teaching and Learning
1.4 Calculate the Gross Debt Service Ratio (GDSR) using given data.	Students should be made aware of the factors that banks consider in determining whether or not they will lend the money for a mortgage.
1.4.1 Determine eligibility for mortgage loans using GDSR.	The instructor should provide students with copies of the form **Affordability Chart** (on page 489 of *Essentials of Mathematics 12*) to help students organize their information as they work through this **Exploration**.
1.4.2 List the factors that affect the calculation of GDSR.	Students should be encouraged to check online or with their municipality to find the property tax rate.

Suggestions for Assessment

Study Guide question 1.7 will meet the objectives of Outcome 1.4.

In the Study Guide, students have been assigned Practice Exercise 1, *Calculating Gross Debt Service Ratio*.

Problems similar to the following can be used for assessment.

Karen wants to buy a house worth $90,500.00. She makes a down payment of $4000.00. Monthly property taxes are $165.00 and heating costs are $95.00 per month. Calculate the maximum affordable price, the monthly mortgage payment and the gross debt service ratio if the bank will finance the house at 6.5% for 25 years. Karen's gross monthly income is $2800.00.

Resources

Essentials of Mathematics 12, Gross Debt Ratio, pages 44 - 53

Affordability Chart, page 489

Teacher Resource Book 12 pages 37 - 40

Appendix, Practice Exercise 1, *Calculating Gross Debt Service Ratio*

Unit 1 - Personal Finance

Outcomes

1.5   Solve problems involving different types of property insurance.

1.5.1   Appropriately use the following terms associated with property insurance: market value, replacement value, metro, protected, semi-protected and unprotected.

1.5.2   Identify the restrictions and areas of coverage provided by different types of property insurance and calculate the costs.

Notes for Teaching and Learning

Students may need some guidance when reading the rate tables for insurance policies. (Tables 5 and 6, *Essentials of Mathematics 12*, pages 55 and 57)

The instructor should provide brochures from various insurance agencies.

The insurance website, www.kanetix.com provides insurance rates for different levels of coverage.

Suggestions for Assessment

Study Guide questions 1.8 and 1.9 will meet the objectives of Outcome 1.5.

Problems similar to the following can be used for assessment.

1. What is the cost for homeowner's insurance for a house that has a replacement cost of $120,000?

2. What are some of the factors in determining the cost of homeowner's or tenant's insurance?

3. A couple has a house they would like to insure for $250,000.00. They have not had any claims in the past and would like a comprehensive package. If they get a $500.00 deductible, what will they pay for the insurance?

4. Have students find out what it would cost them to have insurance for an apartment in which they might live. Discuss what the contents would be worth, then have them calculate the cost of the insurance.

Resources

Essentials of Mathematics 12, Property Insurance, pages 54 - 60

Teacher Resource Book 12, pages 41 - 43

www.kanetix.com

Outcomes	Notes for Teaching and Learning
1.6 Determine the additional costs and fees associated with purchasing a home. 　1.6.1 Prepare a budget which itemizes the additional costs and fees involved in buying a home.	The instructor should provide copies of the form the **Additional Costs in Purchasing a Home** (*Essentials of Mathematics 12*, page 490).

Suggestions for Assessment

Study Guide question 1.10 will meet the objectives of Outcome 1.6.

On pages 66 - 68, **Chapter Review**, questions 1 - 7, could be used for a test.

Mortgage and **Property Insurance**, in **Case Study** on page 70, could be assigned for an assessment.

In the Study Guide, students have been assigned Practice Exercise 2, *Additional Costs in Purchasing a Home.* The instructor could use this worksheet for a homework assignment.

The **Chapter Project**, with modifications, would be suitable as a chapter evaluation.

Problems similar to the following can be used for assessment.

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1. Describe six additional costs in purchasing a home, besides the cost of the home itself.

2. If property taxes have been paid for the year ($3085.00) and the purchasers of a home take possession of the property in August, what portion of the property taxes will be paid by the purchaser?

3. If a buyer increases the homeowner's insurance premium to $450 from $350 per year and pays for seven of the 12 months in the year, what is the additional cost for the buyer?

Resources

Essentials of Mathematics 12, Additional Costs in Purchasing a Home, pages 61 - 65

Chapter Review, pages 66 -68

Case Study, page 70

Appendix, Practice Exercise 2, *Additional Costs in Purchasing a Home*

Teacher Resource Book 12, pages 44 - 46

Unit 2 - Statistics

Outcomes

2.1   Determine the percentile rank of an item in a set of sample data.

2.2   Distinguish between *percentile rank* and *percent.*

Notes for Teaching and Learning

Students are expected to draw and validate inferences throughout this unit.

Omit the **Chapter Project** and **Project Activity**.

This unit lends itself to a project where students are given a set of data that they use to find a percentile rank, standard deviation, correlation coefficient, etc.

P_{23}, P_{50} and P_{75} are special percentile ranks that represent the lower quartile, the mean and the upper quartile, respectively.

The instructor should ensure that students can correctly use the formula for percentile rank.

Suggestions for Assessment

Study Guide questions 2.1 to 2.3 will meet the objectives of Outcomes 2.1 and 2.2.

In the Study Guide, students have been assigned Practice Exercise 3, *Percentile Rank*, which provides extra practice on this topic.

Resources

Essentials of Mathematics 12, Percentiles, pages 381 - 392

Teacher Resource Book 12, pages 245 - 251

Appendix, Practice Exercise 3, *Percentile Rank*

Outcomes

2.3   Calculate the standard deviation of a set of data.

2.4   Use standard deviation to describe the variability within a set of data.

Notes for Teaching and Learning

The instructor should discuss the meaning of standard deviation with the students. Students should be made aware that standard deviation is a measure of the variability of data; it measures the extent to which the data are spread from the mean. The larger the standard deviation, the more widely dispersed the data is around the mean. The smaller the standard deviation, the closer that data is to the mean.

Students may need some explanation of the formula for standard deviation, especially the $\sum$ symbol.

The formula for standard deviation is

$$s = \sqrt{\frac{\sum\left(x - \bar{x}\right)^2}{n - 1}}$$

The instructor should point out to students that even though the variance is called the **mean** of the squares, the sum is divided by $(n-1)$, <u>not</u> n as one might suppose. (See **Example 2**, **Step 6** on page 395.)
The theoretical justification for this is beyond the scope of this course.

Students should be encouraged to use a table similar to the table on page 395 when calculating standard deviation. After students can efficiently use the formula, the instructor could advise them how to use a graphing calculator to check the standard deviation they have calculated.

The instructor should show students the steps:
2nd [LIST] MATH 7: stdDev ({42, 53, 59, 66}] ENTER

Suggestions for Assessment

Study Guide questions 2.4 and 2.5 will meet the objectives of Outcomes 2.3 and 2.4.

Resources

Essentials of Mathematics 12, Standard Deviation, pages 393 - 400

*Teacher Resource Book 12*, pages 252 - 257

Outcomes

2.5   Explain the variability of data using standard deviation and the normal curve.

   2.5.1   Draw and label normal curves.

Notes for Teaching and Learning

Students should be able to draw and label a normal curve with the correct percentages and standard deviations.

The graph in the textbook has percentages that are rounded in order to simplify calculations. The instructor should show students a normal curve similar to the one given below. In this curve, it is shown that 0.15% of all data fall above 3 standard deviations and 0.15% fall below 3 standard deviations.
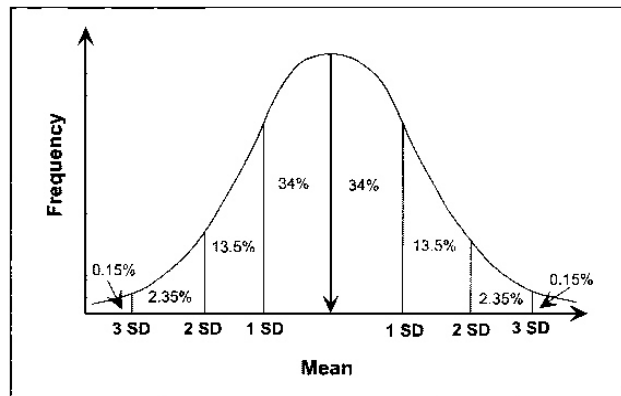
The instructor should ensure that students are aware that the answers to 7c), d) and e) on page 413 of the textbook are incorrect. The correct answers are:
c)   68%, 34,000
d)   1 bar
e)   1 bar

Suggestions for Assessment

Study Guide questions 2.6 to 2.9 will meet the objectives of Outcome 2.5.

Practice Exercise 4, *Standard Deviation and the Normal Curve* could be used for homework. Students have been assigned this worksheet in the Study Guide.

Resources

Essentials of Mathematics 12, The Distribution of Data, pages 401 - 413

Teacher Resource Book 12, pages 258 - 262

Appendix, Practice Exercise 4, *Standard Deviation and the Normal Curve*

Outcomes

2.6   Describe the relationship between the correlation coefficient and two data sets.

2.6.1   Create scatterplots when given two sets of data.

Notes for Teaching and Learning

The instructor should point out that not all correlation is linear, but for the purpose of this course, only linear correlation will be considered. The data used to calculate the correlation coefficient can be expressed in two ways; either in table format or two-dimension scatterplot.

Examples and questions should focus on the meaning rather than the computation of correlation coefficient.

In the Appendix there are **Review Notes** which provide a summary on correlation.

Unit 2 - Statistics

Suggestions for Assessment

Study Guide questions 2.10 to 2.16 will meet the objectives of Outcome 2.6.

Practice Exercise 5, *Correlation*, could be used as an assessment. This worksheet has been assigned in the Study Guide.

Students have been assigned questions from **Chapter Review** and **Case Study**. The instructor may choose to use these sections as a chapter assessment.

Resources

Essentials of Mathematics 12, Correlation Coefficient, pages 416 - 426

Chapter Review, pages 271 - 273

Case Study, pages 275 and 276

Teacher Resource Book 12, pages 266 - 270

Appendix, **Review Notes**

Appendix, Practice Exercise 5, *Correlation*

Appendix

Practice Exercise 1: Calculating Gross Debt Service Ratio

A person decided to buy a house worth $85,000.00. The down payment will be $6000.00, the monthly property taxes are $120.00, and the heating costs are $110.00 per month. Calculate the maximum affordable price, the monthly mortgage payment, and the gross debt ratio, if the bank will finance the house at 7.5% for 25 years. The gross monthly income is $2500.00

The formula for gross debt service ratio and the Interest Rate Factor Table are on page 45 of *Essentials of Mathematics 12*. Make a copy of the Affordability Chart on page 489 of the same textbook

Answer Key Practice Exercise 1:   Calculating Gross Debt Service Ratio

The formula	Your calculations
Gross monthly household income	$ 2,500.00
Multiply by 32%	× 0.32
Total affordable household expenses	= $ 800.00
Subtract	
Monthly property taxes	– 120.00
Monthly heating costs	– 110.00
One half of condo/strata fees (if applicable)	
Monthly mortgage payment your household can afford:	= $ 570.00
To calculate total mortgage amount, divide by estimated interest rate factor which corresponds to your interest rate (see table below).	0.00732
Maximum amount of mortgage you can afford:	= $77,868.85
Add your cash down payment	+ 6,000.00
Your maximum affordable price	$83,868.85
Actual mortgage payment (= 0.000732 × [$85,000.00 – $6,000.00])	$578.28
Gross debt service ratio	32.3%

Interest Rate Factor Table					
Rate	**Factor**	**Rate**	**Factor**	**Rate**	**Factor**
6.0%	0.00640	8.0%	0.00763	10.0%	0.00894
6.5%	0.00670	8.5%	0.00795	10.5%	0.00928
7.0%	0.00700	9.0%	0.00828	11.0%	0.00963
7.5%	0.00732	9.5%	0.00861	11.5%	0.00997

This table is based on 25-year amortization. The figures used in this table do not necessarily reflect current market rates.

Practice Exercise 2: Additional Costs in Purchasing a Home

Complete the chart *Additional Costs in Purchasing a Home* for each of the following problems.

1.   The Jones family lives in Portage la Prairie and is relocating to Winnipeg. They purchase a house for $140,000.00 and hire a mover to move their belongings. The mover charges $1700.00. A lawyer is hired to look after legalities for a fee of $900.00. An appraisal is done of the property for a cost of $425.00. The possession date is July 9, with the first payment due on July 15. The interest adjustment is $105.00. Property taxes are $2150.00 for which the Jones family has agreed to pay for the months of July - December.

Before moving, the Jones family wants to redo the yard for $2500.00, and replace the fridge and stove for $630.00 and $825.00, respectively. They have agreed to split the cost of the appliances with the sellers. They want to replace the drapes and vertical blinds for $1200.00 and paint the bedrooms at a cost of $680.00. The cost to hook up the phone is $25.00 and to activate the natural gas is $55.00. Their annual premium for insurance increases to $540.00 from $325.00 per year and they pay the additional for the remaining six months.

2.   The Smiths live in Winnipeg and Mr. Smith has accepted a job in Brandon.

• They purchased a house in Brandon for $150,000 and hired a mover to move their belongings. The moving charges are $1800.00.

• Their lawyer fees are $1000.00.

• An appraisal was done on their property for a fee of $140.00. A survey of the property was done for a fee of $375.00. They called in a house inspector to assess the condition of the new house. The inspector charged $400.00 and recommended that the siding be replaced.

• The Smiths' possession date is August 5th. The interest adjustment is $457.00. Annual property taxes are $2850.00 and the Smiths agree to pay for the five months from August to December.

• Before moving in, the Smiths want to build a fence and re-side the house at a cost of $5000.00, replace the carpet in the living room at a cost of $4200.00, and paint the master bedroom and the kitchen at a cost of $650.00.

• The stove has to be replaced at a cost of $850.00.

• They increase their annual premium to $590.00 from $425.00 per year and pay the additional amount for the remaining five months of the policy year.

• The cost to hook up the phone is $65.00 and the cost to activate the natural gas is $45.00.

Answer Key for Practice Exercise 2: Additional Costs in Purchasing a Home

1.

Appraisal Fees	$	425.00
Inspection Costs	$	
Property Survey	$	
Insurance - High Ratio Mortgage	$	107.50
Home Insurance	$	105.00
Interest Adjustment	$	1,075.00
Prepaid Property Taxes and Utilities	$	900.00
Legal Fees and Disbursements	$	
Sales Tax	$	
Moving Expenses	$	1,700.00
Services Charges	$	80.00
Immediate Repairs	$	2,500.00
Appliances	$	727.50
Decorating Costs	$	1,880.00
Total Additional Costs	$	9,500.00

Home Insurance: ($540.00 − $325.00) (6/12) = $107.50
Prepaid Taxes: ($2150.00) (6/12) = $1075.00
Appliances: ($630.00 + $825.00) (6/12) = $727.50

Practice Exercise 3: Percentile Rank

1. 500 job applicants wrote an exam. Maria scored 65%. 380 applicants scored lower than she did, and 23 other applicants also scored 65%. What is Maria's percentile rank?

The following notation is used to represent percentile rank: P_{79} is equivalent to the 79th percentile.

2. What percent of students scored less than P_{28}?

3. What percent of students scored more than P_{28}?

4. A total of 3286 students wrote a university entrance examination. Student X and 432 other students had a score of 891 out of 1200. There were 2279 students who scored less than 891.

In order to gain entrance, a given student needed a percentile ranking of 70 or better.

a) What was the percentile ranking of Student X?

b) Did Student X score high enough to gain entrance into the university?

5. Two students from different schools were comparing their recent test results. These results are summarized in the following table.

Student	# of scores below student score	# of scores equal to student score	# of students in each class
A	18	1	24
B	16	5	26

a) Determine which student did better by calculating the percentile rank for each.

b) Explain the meaning of each percentile rank.

Answer Key for Practice Exercise 3: Percentile Rank

1. $B = 380$
        $E = (23 + 1) = 24$
        $n = 500$

Percentile rank $= \left(\dfrac{380 + 0.5\,(24)}{500} \right) \times 100 \;=\; 78.4$

Round up all decimals to the next whole number, so percentile rank = 79.

Therefore, 79% of all applicants scored lower than Maria, or Maria scored better than 79% of all applicants.

4. a)   Percentile Rank $= \left(\dfrac{B + 0.5E}{n} \right) \times 100$

$B = 2279$
$E = 433\,(432 + 1)$
$n = 3286$

Percentile Rank $= \left(\dfrac{2279 + 0.5(433)}{3286} \right) \times 100$
                   $= 75.9$ or
                   $= $ 76th percentile

Therefore, this student scored better than 76% of all other students.

b) Yes, this student scored high enough to gain entrance into the university.

5. a)   Student A $= \dfrac{18 + 0.5\,(2)}{24} \times 100$
                      $= 79.16$ or 80th percentile

         Student B $= \dfrac{16 + 0.5(6)}{26} \times 100$
                      $= 73.07$ or 74th percentile

Therefore, Student A did better.

b) Student A scored better than 80% of all other students in the class.
    Student B scored better than 74% of all other students in the class.

Practice Exercise 4: Standard Deviation and Normal Curve

1. Five hockey fans were surveyed to determine the number of hockey games they attended throughout the season. The results were 6, 7, 8, 8, 10.

a) Calculate the mean.

b) Complete the table.

No. of Games	Difference from Mean	Square of Difference
(x)	$(x - \bar{x})$	$(x - \bar{x})^2$
6		
7		
8		
8		
10		

c) Find the sum of the squares.

d) Divide the sum of squares by $n - 1$.

e) Find Sx, the square root of this number.

2. A volleyball coach measured the heights (cm) of the players on the team and found them to be:

172   176   176   178   180   181   181   182   184   184   187

a) Determine the range.

b) Determine the mean.

c) Complete the table and determine the standard deviation for this set of data.

Heights in cm x	Difference from mean $x - \bar{x}$	Square of the difference $(x - \bar{x})^2$
Sum of the squares $\sum(x - \bar{x})^2$	

3. The normal curve:
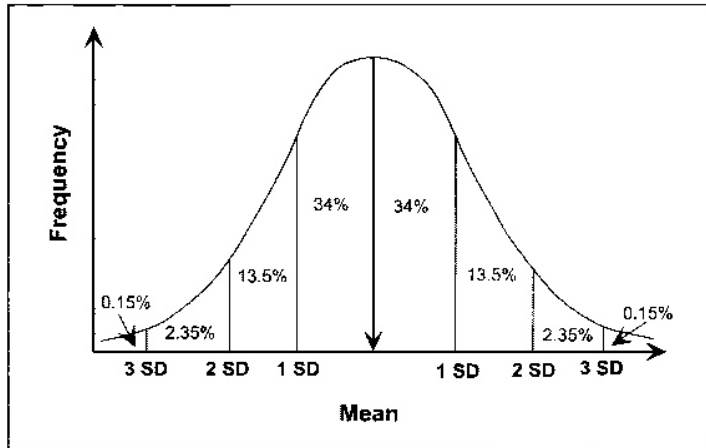
The graph shows that 68% (34% + 34%) of all the data will fall within one standard deviation of the mean. 95% of all the data will fall within two standard deviations of the mean and 99.7% of all data are within 3 standard deviations of the mean. Approximately 5% of all data will fall more than two standard deviations from the mean.

A hospital finds that the average stay is five days, with a standard deviation of one day. Assume a normal distribution.

a) What percent of patients stay at the hospital between four and five days?

b) If 200 patients are admitted today, how many will stay more than six days?

4. A football coach graphed the weight of each player on his team and found that the data were distributed normally. The mean was 100 kg and the standard deviation was 10 kg.

a) What percent of the players weigh between 90 kg and 110 kg?

b) What percent of the players weigh more than 120 kg or less than 80 kg?

c) If there are 45 players on the team, how many weigh more than 110 kg?

5. The personnel manager of a business employing 2000 people found the average term of employment was 22 years with a standard deviation of 7 years. Assuming that the data are distributed normally:

a) What percent of the employees will work more than 15 years?

b) How many employees will work between 15 and 29 years?

c) How many employees will work longer than 29 years?

Answer Key for Practice Exercise 4: Standard Deviation and Normal Curve

1. Five hockey fans were surveyed to determine the number of hockey games they attended throughout the season. The results were 6, 7, 8, 8, 10.

a) Calculate the mean: $\bar{x} = \dfrac{39}{5} = 7.8$

b) Complete the table:

No. of Games	Difference from Mean	Square of Difference
(x)	$(x - \bar{x})$	$(x - \bar{x})^2$
6	⁻1.8	3.24
7	⁻0.8	0.64
8	0.2	0.04
8	0.2	0.04
10	2.2	4.84

c) Find the sum of the squares:

$3.24 + 0.64 + 0.04 + 0.04 + 4.84 = 8.8$

d) Divide the sum of squares by $n - 1$: $\dfrac{8.8}{4} = 2.2$

e) Find the square root of this number: $S_x = \sqrt{2.2} = 1.48$

2.
a) The range is 15 cm.
b) The mean is 180.1 cm.

Heights in cm x	Difference from mean $x - \bar{x}$	Square of the difference $(x - \bar{x})^2$
172	−8.1	65.61
176	−4.1	16.81
176	−4.1	16.81
178	−2.1	4.41
180	−0.1	0.01
181	0.9	0.81
181	0.9	0.81
182	1.9	3.61
184	3.9	15.21
184	3.9	15.21
187	6.9	47.61
Sum of the squares $\sum (x - \bar{x})^2$		186.91

$$S_x = \sqrt{\frac{186.91}{10}} = 4.32 \text{ cm}$$

3.
a) 34%

b) 16% of 200 = 32
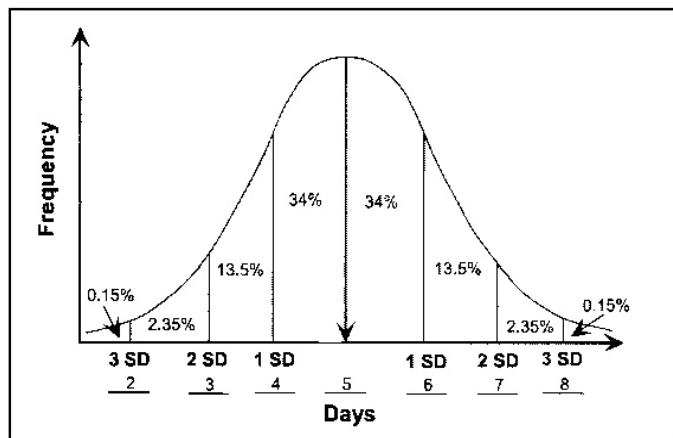
4.
a) 68% (34% + 34%)

b) approximately 5% (2.5% + 2.5%)

c) 45 × 16 % = 7.2 or approximately 7 players
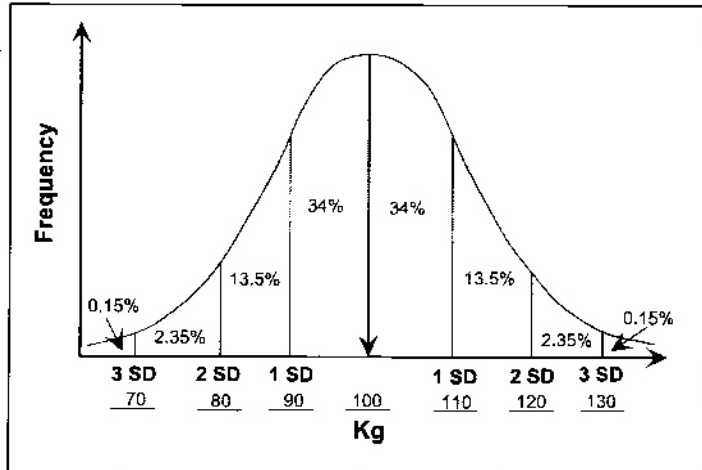
5.
a) 84% (34% + 34% + 13.5 + 2.35% + 0.15%)

b) 2000 × 68 % = 1360 employees

c) 2000 × 16% = 320 employees
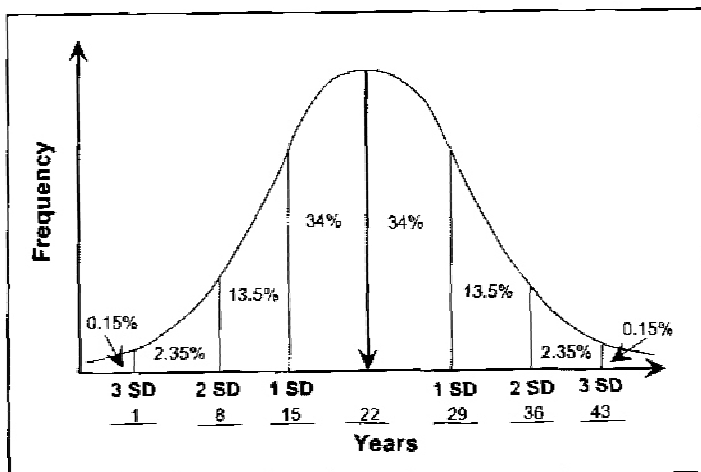
Practice Exercise 5: Correlation

1. Predict whether each of the following will exhibit positive, negative or zero correlation. In addition, write a statement that describes this relationship. The first one has been done for you.

a) population of a country versus number of houses
   Answer: A strong positive correlation. As the population of a country increases, the number of houses increases.

b) number of students in a classroom versus the day of the week

c) person's age versus person's shoe size

d) selling price of lumber versus the amount of lumber purchased

e) number of kilometres on a vehicle's odometer versus age of the vehicle

f) number of hours studied for a test versus score on that test

g) number of red M&Ms in 10 bags versus the number of pages in 10 books

h) number of staples in a stapler versus the room number the stapler is in
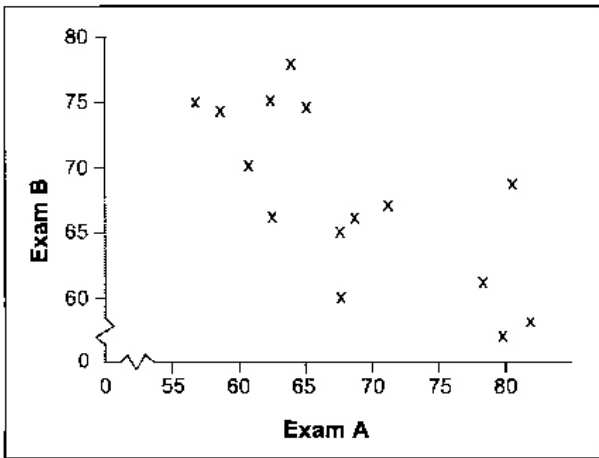

2. For each of the following graphs, state the type of correlation that exists (strong positive, weak positive, etc.), describe the relationship between the two variables, and then choose the number that best describes this correlation.

a) a male's arm span (cm) compared to his height (cm)
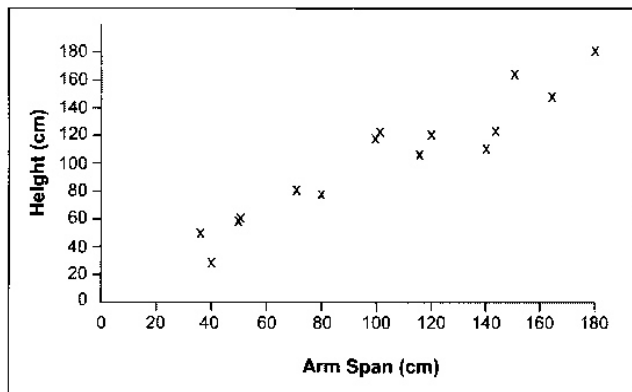
r-value choices
0.97
0.12
−0.12
−0.97

b) scores on Exam A and the scores on Exam B

r-value choices
0.75
0.05
−0.05
−0.75

3. In the following problem, complete each of the following:

a) create a scatterplot with appropriate x- and y-scales labeled
b) from your scatterplot, determine the type of linear correlation (+, −, or 0)
c) in a statement, describe the relationship between the two variables

The hours each student studied for an exam versus the student's scores:

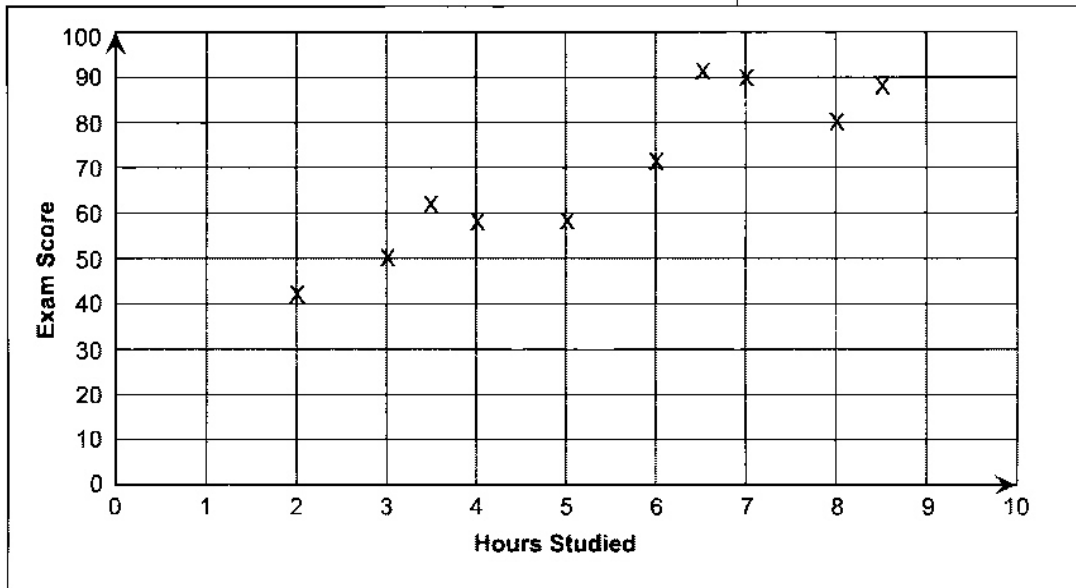| Student	Hours Studied	Exam Mark
1	8.5	88
2	2	44
3	3.5	63
4	4	58
5	7	89
6	6.5	92
7	3	50
8	8	81
9	5	57
10	6	72

Answer Key for Practice Exercise 5: Correlation

1.
a) Positive correlation: As the population of a country increases, the number of residential housing units increases.

b) Zero correlation: There is no relationship between the number of students in a classroom and the day of the week.

c) Positive correlation: There is a slight positive correlation between a person's age and shoe size.

d) Negative correlation: As the selling price of lumber increases, the volume of lumber purchased decreases.

e) Positive correlation: As the number of kilometres on a vehicle's odometer increases, the age of the vehicle increases.

f) Positive correlation: As the number of hours studied for a test increases, the test score increases.

g) Zero correlation: There is no relationship between the number of red M&Ms in 10 bags and the number of pages in 10 books.

h) Zero correlation: There is no relationship between the number of staples in a stapler and the number of the room the stapler is in.

2.a) A very strong positive correlation. There is a very strong positive correlation between a male's arm span and his height. This suggests that the longer a male's arm span, the taller he is; or the taller he is, the longer his arm span. $R = +0.97$

b) A relatively strong negative correlation. There is a relatively strong negative correlation between the scores on Exam A and Exam B. This suggests that the better a student scored on Exam A, the poorer the student scored on Exam B; or the poorer a student scored on Exam A, the better the student scored on Exam B. $r = -0.75$

3.

a)

b)   A strong positive correlation.

c)   There is a strong positive correlation between the number of hours studied and the exam score for each student. This suggests that the longer a person studied, the better he or she did on the exam.

Review Notes: Positive, Negative and Zero Correlations

Positive Correlation: The population of various cities (on the *x*-axis) is plotted versus the number of fast-food restaurants in the city (on the *y*-axis). Generally, the more people there are living in a particular city, the more fast-food restaurants there are.

As can be seen, most of the points tend to fall along a straight line that rises from left to right.

This represents a fairly strong positive correlation. In other words, we can conclude that the larger the population, the greater the number of fast-food restaurants.
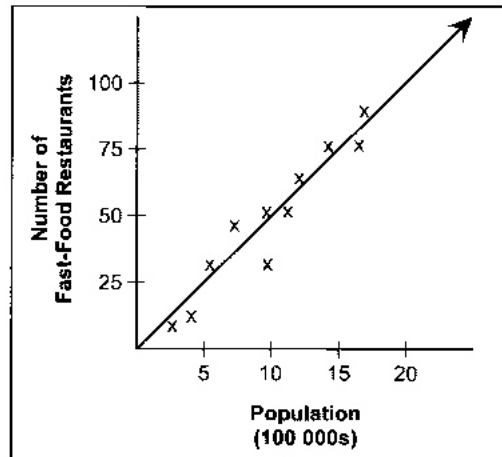
Negative Correlation: means that as one variable *increases*, another variable *decreases*, or that as one variable *decreases*, another variable *increases*. The *r*-value used to describe a negative correlation falls between the values of 0 and −1. Stronger negative correlation is indicated by an *r*-value closer to −1. Weaker negative correlation is indicated by an *r*-value closer to 0. This can be illustrated by the following example.

A golfer's average scores are plotted versus the money earned in tournaments. Generally, the lower the golf score, the lower the earnings.

As can be seen, most of the points tend to fall along a straight line that falls from left to right.

It represents a fairly strong negative correlation. In other words, we can conclude that the lower the average score, the greater the earnings.
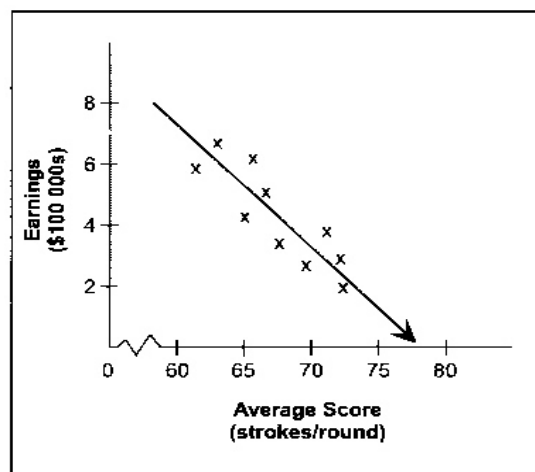
Zero Correlation: A person's street address (house number) is plotted verus her mass in kg.   As can be seen, there is no obvious positive or negative correlation between house number and mass. When calculated, the value of r will lie between -1 and $+1$ (if not, a calculation error has occurred).
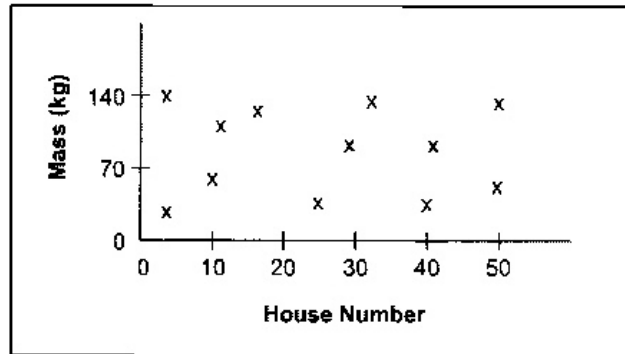
Statistics Canada / **Statistique Canada**

Life expectancy at birth, by sex, by province

	Males	Females
Canada	Years	Years
1920 - 22	59	61
1930 - 32	60	62
1940 - 42	63	66
1950 - 52	66	71
1960 - 62	68	74
1970 - 72	69	76
1980 - 82	72	79
1990 - 92	75	81
1990 - 92		
Newfoundland and Labrador	74	80
Prince Edward Island	73	81
Nova Scotia	74	80
New Brunswick	74	81
Quebec	74	81
Ontario	75	81
Manitoba	75	81
Saskatchewan	75	82
Alberta	75	81
British Columbia	75	81

Source: Statistics Canada
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