

Adult Basic Education  
**Mathematics**

---

# **Mathematics 1104B**

## **Real Numbers, Right Triangle Trigonometry, and Functions**

### **Curriculum Guide**

**Prerequisite:** Mathematics 1104A

**Credit Value:** 1

**Required Mathematics Courses**

**[Degree and Technical Profile/Business-Related College Profile]**

Mathematics 1104A

**Mathematics 1104B**

Mathematics 1104C

Mathematics 2104A

Mathematics 2104B

Mathematics 2104C

Mathematics 3104A

Mathematics 3104B

Mathematics 3104C



## Table of Contents

To the Instructor .....	v
Introduction to Mathematics 1104B .....	v
Prerequisites .....	v
Textbook .....	v
Technology .....	vi
Curriculum Guides .....	vii
Study Guides .....	viii
Resources .....	viii
Recommended Evaluation .....	ix
Unit 1 - Real Numbers .....	Page 2
Unit 2 - Right Triangle Trigonometry .....	Page 10
Unit 3 - Functions .....	Page 12



## To the Instructor

### I. Introduction to Mathematics 1104B

This course has three topics. The first unit, *Real Numbers*, uses the Pythagorean Theorem to apply work with radicals and introduce irrational numbers. Using diagrams, the relationships among natural numbers, whole numbers, integers, rational numbers and irrational numbers are presented. While solving problems involving triangles whose side lengths are irrational numbers, special triangles are introduced. These triangles will be important when studying trigonometry. Lastly, radicals are added, subtracted and expressed in simplest form.

*Right Triangle Trigonometry*, the second unit, develops the definitions and applications of tangent, sine and cosine. Students may need some introductory work on this topic since the textbook assumes that this is a review of previously learned material.

The third unit, *Functions*, expresses functions in different forms (words, a table of values, set of ordered pairs, a graph or an equation). Information about a function is found by analyzing its graph. The graphs and equations for linear functions, in particular, are studied in detail.

### II. Prerequisites

Students taking this course should have prerequisite skills to use the Pythagorean Theorem to determine the side lengths of right triangles. Students must be able to simplify radicals and recognize equivalent radicals. They should also be familiar with similar triangles and be able to sketch a triangle when given some of its measures. Students should be able to identify and describe situations that involve dependency, be familiar with the meaning of functions, and be able to identify slopes and intercepts from the equations of a straight lines.

### III. Textbook

Most of the concepts are introduced, developed and explained in the **Examples**. The instructor must insist that students carefully study and understand each **Example** before moving on to the **Exercises**. In the Study Guide, students are directed to see the instructor if there are any difficulties.

## To the Instructor

There are four basic categories included in each section of the textbook which require the student to complete questions:

1. Investigate
2. Discussing the Ideas
3. Exercises
4. Communicating the Ideas

**Investigate:** This section looks at the thinking behind new concepts. The answers to its questions are found in the back of the text.

**Discussing the Ideas:** This section requires the student to write a response which clarifies and demonstrates understanding of the concepts introduced. The answers to these questions are not in the student text but are in the *Teacher's Resource Book*. Therefore, in the Study Guide, the student is directed to see the instructor for correction. This will offer the instructor some perspective on the extent of the student's understanding. If necessary, reinforcement or remedial work can be introduced. Students should not be given the answer key for this section as the opportunity to assess the student's understanding is then lost.

**Exercises:** This section helps the student reinforce understanding of the concepts introduced. There are three levels of **Exercises**:

- A:** direct application of concepts introduced;
- B:** multi-step problem solving and some real-life situations;
- C:** problems of a more challenging nature.

The answers to the **Exercises** questions are found in the back of the text.

**Communicating the Ideas:** This section helps confirm the student's understanding of a particular lesson by requiring a clearly written explanation. The answers to **Communicating the Ideas** are not in the student text, but are in the *Teacher's Resource Book*. In the Study Guide students are asked to see the instructor for correction.

## IV. Technology

It is important that students have a **scientific** calculator and its manual for their individual use. Ensure that the calculator used has the word "scientific" on it as there are calculators designed for calculation in other areas such as business or statistics which would not have the functions needed for study in this area.

## To the Instructor

A graphing calculator should be **available** to the students since the text provides many opportunities for its use. The *Teacher's Resource Book* suggests many occasions to utilize a graphing calculator. These suggestions are outlined where there is the heading *Integrating Technology*. In the Study Guide, students are directed to see the instructor when a graphing calculator is required. The *Teacher's Resource Book* contains a module called **Graphing Calculator Handbook** which will help the instructor and student get acquainted with some of the main features of the TI-83 Plus graphing calculator.

Graphing software such as *Graphmatica* or *Winplot* can also be used if students don't have access to a graphing calculator but do have access to a computer. The textbook doesn't offer the same guidance for graphing with these tools as it does for a graphing calculator but each software program does have a HELP feature to answer questions.

## V. Curriculum Guides

Each new ABE Mathematics course has a Curriculum Guide for the instructor and a Study Guide for the student. The Curriculum Guide includes the specific curriculum outcomes for the course. Suggestions for teaching, learning, and assessment are provided to support student achievement of the outcomes. Each course is divided into units. Each unit comprises a **two-page layout of four columns** as illustrated in the figure below. In some cases the four-column spread continues to the next two-page layout.

### Curriculum Guide Organization: The Two-Page, Four-Column Spread

Unit Number - Unit Title		Unit Number - Unit Title	
<b>Outcomes</b>  Specific curriculum outcomes for the unit.	<b>Notes for Teaching and Learning</b>  Suggested activities, elaboration of outcomes, and background information.	<b>Suggestions for Assessment</b>  Suggestions for assessing students' achievement of outcomes.	<b>Resources</b>  Authorized and recommended resources that address outcomes.

## To the Instructor

### VI. Study Guides

The Study Guide provides the student with the name of the text(s) required for the course and specifies the sections and pages that the student will need to refer to in order to complete the required work for the course. It guides the student through the course by assigning relevant reading and providing questions and/or assigning questions from the text or some other resource. Sometimes it also provides important points for students to note. (See the *To the Student* section of the Study Guide for a more detailed explanation of the use of the Study Guides.) The Study Guides are designed to give students some degree of independence in their work. Instructors should note, however, that there is much material in the Curriculum Guides in the *Notes for Teaching and Learning* and *Suggestions for Assessment* columns that is not included in the Study Guide and instructors will need to review this information and decide how to include it.

### VII. Resources

#### *Essential Resources*

*Addison Wesley Mathematics 10* (Western Canadian edition)  
ISBN:0-201-34619-2

*Mathematics 10 Teacher's Resource Book* (Western Canadian edition)  
ISBN: 0-201-34621-4

Math 1104B Study Guide

#### *Recommended Resources*

*Mathematics 10 Independent Study Guide* (Western Canadian edition)  
ISBN: 0-201-34620-6

*Center for Distance Learning and Innovation*: <http://www.cdli.ca>

*Winplot*: <http://math.exeter.edu/rparris/winplot.html>  
(Free graphing software)

*Graphmatica* (Evaluation software available on CD-ROM contained in  
*Teacher's Resource Book*)



## To the Instructor

### *Other Resources*

Math Links: <http://mathforum.org>

<http://www.purplemath.com>

<http://www.sosmath.com/index.html>

<http://www.math.com/>

<http://spot.pcc.edu/~ssimonds/winplot>  
(Free videos concerning Winplot)

<http://www.pearsoned.ca/school/math/math/>

### **VIII. Recommended Evaluation**

Written Notes	10%
Assignments	10%
Test(s)	30%
Final Exam ( <i>entire course</i> )	<u>50%</u>
	100%

**The overall pass mark for the course is 50%.**



# **Real Numbers, Right Triangle Trigonometry, and Functions**

## Unit 1 - Real Numbers

### Outcomes

1.1 Use the Pythagorean Theorem to solve problems involving right triangles.

### Notes for Teaching and Learning

This unit reviews the Pythagorean Theorem, works with radicals and introduces irrational numbers.

Note: Omit the proof ( $\sqrt{2}$  is not a rational number.) on pages 97 and 98.

When completing the **Exercises**, students should be encouraged to express lengths as square roots, rather than decimals, and draw a diagram when one is not provided.

The instructor could introduce Pythagorean Triples in this unit. A Pythagorean Triple is a set of three numbers,  $a$ ,  $b$  and  $c$  where  $a^2 + b^2 = c^2$ .

If students have access to the internet, [www.cdli.ca](http://www.cdli.ca) has several interactive lessons which may prove to be helpful.

## Unit 1 - Real Numbers

### Suggestions for Assessment

Study Guide question 1.1 will meet the objectives of Outcome 1.1.

The CDLI website has an interactive demonstration on the Pythagorean Theorem. There is also a lesson on Pythagorean Triples. This topic is not addressed in *Mathematics 10*. It is, however, a straightforward extension from Pythagorean Theorem.

### Resources

*Mathematics 10*,  
Section 2.4, The  
Pythagorean Theorem,  
pages 96, 99 and 100

*Mathematics 10*,  
*Teacher's Resource Book*,  
Chapter 2, pages 14 and  
15

*Mathematics 10*,  
*Independent Study Guide*,  
page 34

[www.cdli.ca](http://www.cdli.ca) , Math 1204,  
Unit 05, Section 02

## Unit 1 - Real Numbers

### Outcomes

1.2 Recognize numbers as being natural, whole, integral, rational or irrational, and show that they are real numbers.

### Notes for Teaching and Learning

This section illustrates that the set of real numbers is made up of two discrete sets; rational numbers and irrational numbers.

Students should be assigned **Prerequisites**, *Teacher's Resource Book*, Chapter 2, page 16.

The instructor may need to explain the different classifications of numbers as illustrated in the box, **The Real Numbers**, on page 103 of *Mathematics 10*.

The *Teacher's Resource Book* has a section, called *Integrating Technology*, which provides many hints on using a computer, graphing calculator and scientific calculator. Page 17 of the *Teacher's Resource Book* explains how the student could use a calculator to find the answer to question 2 in **Exercises**, page 104 in *Mathematics 10*.

## Unit 1 - Real Numbers

### Suggestions for Assessment

Study Guide questions 1.2 and 1.3 will meet the objectives of Outcome 1.2.

### Resources

*Mathematics 10*,  
Section 2.5, Irrational  
Numbers, pages 101 - 105

*Mathematics 10*,  
*Teacher's Resource Book*  
Chapter 2, pages 16 and  
17

*Mathematics 10*,  
*Independent Study Guide*  
pages 34 and 35

## Unit 1 - Real Numbers

### Outcomes

1.3 Solve problems involving triangles whose side lengths are irrational numbers.

### Notes for Teaching and Learning

In this section, students will be looking at an application of radicals; finding the side lengths of triangles when the lengths are irrational numbers.

This topic is a brief introduction to the *special* 30-60-90 and 45-45-90 triangles which students will study in more detail in Mathematics 2104C.

**Example 2** on page 109 is omitted for this course.

The instructor should choose problems that find lengths of sides, but **omit** problems which find the **area** of a triangle.



## Unit 1 - Real Numbers

### Suggestions for Assessment

Study Guide questions 1.4 and 1.5 will meet the objectives of Outcome 1.3.

### Resources

*Mathematics 10*,  
Section 2.6, Relating the  
Sides of Special  
Triangles, pages 108 -  
113

*Mathematics 10*,  
*Teacher's Resource Book*,  
Chapter 2, pages 18 - 20

*Mathematics 10*,  
*Independent Study Guide*,  
Chapter 2, page 35

## Unit 1 - Real Numbers

### Outcomes

1.4 Add and subtract radicals.

1.4.1 Simplify radicals.

1.4.2 Define the term *like radicals*.

### Notes for Teaching and Learning

**Note: Sections 2.7 and 2.8, *Mathematics 10*, are omitted for this course.**

Since students will be adding and subtracting radicals in this section, **Prerequisites** exercises, page 28, *Teacher's Resource Book* should be assigned.

For **Investigate** on page 129, question 4, the instructor should ensure that students use the diagrams and the Pythagorean Theorem to determine the required lengths.

## Unit 1 - Real Numbers

### Suggestions for Assessment

Study Guide questions 1.6 to 1.10 will meet the objectives of Outcome 1.4.

Each section in the *Teacher's Resource Book* has extra questions in **Supplementary Examples** and **Assessing the Outcome**.

Selected questions can be chosen from the Written Test and Multiple Choice Test on Masters 2.5 - 2.8.

The CDLI site has several useful interactive lessons called Square Roots and their Properties.

### Resources

*Mathematics 10*,  
Section 2.9, Adding and  
Subtracting Radicals,  
pages 129 - 134

*Mathematics 10*,  
*Teacher's Resource Book*,  
Chapter 2,  
pages 28 and 29,  
Masters 2.5 - 2.8

*Mathematics 10*,  
*Independent Study Guide*,  
Chapter 2, page 36

[www.cdli.ca](http://www.cdli.ca) , Math 1204,  
Unit 05, Section 03

## Unit 2 - Right Triangle Trigonometry

### Outcomes

2.1 Use the tangent, sine and cosine ratios to calculate the lengths of sides and the measures of angles in right triangles.

2.1.1 Use a calculator to find trigonometric values when angles are known.

2.1.2 Use a calculator to find angles when trigonometric values are known.

2.1.3 Use trigonometry to solve problems including angle of elevation and angle of depression problems.

2.1.4 Use trigonometric ratios to solve right triangles.

### Notes for Teaching and Learning

This section develops the definitions and applications of the three primary trigonometric ratios: sine, cosine and tangent.

The textbook assumes that students have already been taught this material in Grade 9 and therefore the topic is developed here as a review. Since students have **not** previously learned this topic, the instructor should spend some extra time explaining the meaning of these trigonometric ratios.

The instructor should use other mathematics textbooks and check the CDLI site ([www.cdli.ca](http://www.cdli.ca)) for some problems and lessons which may prove useful to students. Since trigonometry is such an important and large topic in Academic Math, students need to start with a solid understanding of the basic trigonometric ratios.

The instructor should ensure that students know that  $\tan^{-1} x$  is the inverse of the tangent function and  $(\tan x)^{-1}$  is the reciprocal.

Students need to be reminded to have their scientific or graphing calculator in degree mode.

Assign exercises from **Prerequisites**, *Teacher's Resource Book*, Chapter 8, pages 4, 7 and 10.

## Unit 2 - Right Triangle Trigonometry

### Suggestions for Assessment

Study Guide questions 2.1 to 2.9 will meet the objectives of Outcome 2.1.

Each section in the *Teacher's Resource Book* has extra questions in **Supplementary Examples** and **Assessing the Outcome**.

Questions from the Multiple Choice Test and Written Test in Masters 8.3 - 8.6 should be carefully chosen for an assessment of this unit.

### Resources

*Mathematics 10*,  
Section 8.1, The Tangent  
Ratio, pages 452 - 459

Section 8.2, The Sine and  
Cosine Ratios, page 460 -  
467.

Section 8.3, Solving Right  
Triangles, pages 470 -  
477

*Mathematics 10*,  
*Teacher's Resource Book*,  
Chapter 8, pages 4 -12  
Masters 8.3 - 8.6

*Mathematics 10*,  
*Independent Study Guide*,  
Chapter 8, pages 91 - 93

[www.cdli.ca](http://www.cdli.ca) , Math 1204,  
Unit 5, Section 04,  
Lesson 02

Unit 5, Section 04,  
Lessons 01 and 02

Unit 5, Section 7,  
Lesson 01 (Review)

## Unit 3 - Functions

### Outcomes

- 3.1 Express a function
- i) as a table of values
  - ii) as an equation
  - iii) as a set of ordered pairs
  - iv) as a graph
  - v) in words.
- 3.1.1 Define the term *function*.

### Notes for Teaching and Learning

**Note:** As a review question, the instructor should ask the student to describe a situation where one quantity depends on another.

This chapter introduces the topic of functions. Since students will meet functions frequently in all of their mathematics courses, it is vital that they get a good understanding of this topic now.

The instructor should ask students to describe a function in their own words. Students should understand the meaning of *valid* in the context of functions.

**Section 5.1** simply shows the different ways that a function can be expressed.

Students must be encouraged to use grid paper and ruler and to label both axes as well as the graph.

Students should understand why the function in **Example 1**, page 248, is graphed with points (not joined) and the function in **Example 2**, page 249, is graphed with a line (points are joined).

Using **Examples 1** and **2** ( and other examples if necessary) the instructor should discuss *dependent* and *independent* variables and which axis is used for each variable.

The instructor should discuss *step function* with students. (See question 4, page 250.) Question 7, page 251 could be graphed as a *step function*. If students choose to do this, they should not be expected to write an equation.

## Unit 3 - Functions

### Suggestions for Assessment

Study Guide questions 3.1 and 3.3 will meet the objectives of Outcome 3.1.

### Resources

*Mathematics 10*, Section 5.1, What is a Function?  
pages 246 - 253

*Mathematics 10*,  
*Teacher's Resource Book*,  
Chapter 5, pages 4 - 7

*Mathematics 10*,  
*Independent Study Guide*,  
page 62

## Unit 3 - Functions

### Outcomes

3.2 Identify the domain, range, slope and intercepts from the graph of a function which represents real situations.

3.2.1 Define *domain* of a function.

3.2.2 Define *range* of a function.

### Notes for Teaching and Learning

Note: Assign review questions from **Prerequisites**, *Teacher's Resource Book*, page 7.

In this section, students will find the domain and range by looking at the graph of a function. This is easier than starting with the equation of the function.

Since students may have difficulty with these concepts, the instructor should have extra examples available for discussion before students start **Exercises** on page 258. Again, the terms *valid* and *restricted domain* may need more explanation and examples.

Students may need some guidance when working through **Example 2** on pages 257 and 258.

If graphing calculators are available, students could use one to draw the graphs in **Linking Ideas**, page 265, and find the intercepts by using the TRACE feature. Again, the instructor may need to work through this page with students. A graphing calculator is not necessary to answer the questions.



## Unit 3 - Functions

### Suggestions for Assessment

Study Guide questions 3.4 to 3.7 will meet the objectives of Outcome 3.2.

### Resources

*Mathematics 10*, Section 5.2, Interpreting Graphs of Functions, pages 255 - 265

*Mathematics 10*,  
*Teacher's Resource Book*,  
Chapter 5, pages 7 - 12

*Mathematics 10*,  
*Independent Study Guide*,  
pages 62 and 63

## Unit 3 - Functions

### Outcomes

3.3 Given graphs and tables of values, identify the rules which describe them and then express the rules as equations.

### Notes for Teaching and Learning

**Note:** Assign review questions from **Prerequisites**, *Teacher's Resource Book*, page 12.

The textbook assumes that students have done arithmetic sequences and uses this approach to find the equation which describes the data. Since these students have not covered arithmetic sequences, they can find the equations using other strategies.

There are **Supplementary Examples** in Chapter 5, *Teacher's Resource Book*, page 13, which the instructor should provide for extra practice.

## Unit 3 - Functions

### Suggestions for Assessment

Study Guide questions 3.8 and 3.9 will meet the objectives of Outcome 3.3.

### Resources

*Mathematics 10*,  
Section 5.3 , Finding  
Relationships in Data,  
pages 266 - 272

*Mathematics 10*,  
*Teacher's Resource Book*,  
Chapter 5, pages 12 - 16

*Mathematics 10*,  
*Independent Study Guide*,  
page 64

## Unit 3 - Functions

### Outcomes

3.4 Given a linear function, find its graph and equation.

3.4.1 Given the graph of a linear function, explain what each of *slope*, *intercept*, *domain* and *range* represents.

### Notes for Teaching and Learning

**Note:** Assign review questions from **Prerequisites**, *Teacher's Resource Book*, page 16.

In this section, students will study the graph and equation of linear functions.

The instructor should introduce the terms, *discrete* and *line of best fit*.

The instructor should ensure sure that students understand the set notation used in **Example 2**, part c, page 276.

## Unit 3 - Functions

### Suggestions for Assessment

Study Guide questions 3.10 to 3.13 will meet the objectives of Outcome 3.4.

### Resources

*Mathematics 10*,  
Section 5.4, Linear  
Functions: Part I, pages  
274 - 284

*Mathematics 10*,  
*Teacher's Resource Book*,  
Chapter 5, pages 16 - 20

*Mathematics 10*,  
*Independent Study Guide*,  
pages 64 and 65

## Unit 3 - Functions

### Outcomes

3.5 Determine direct variation and constant of proportionality in functions.

3.5.1 Define *constant of proportionality*.

### Notes for Teaching and Learning

**Note:** Assign review questions from **Prerequisites**, *Teacher's Resource Book*, page 23.

In this section, students are introduced to the term *constant of proportionality* for linear functions that are direct variations.

The instructor should ensure that students understand that a) not all linear functions show direct variation and b) the graph of every direct variation passes through (0,0).

## Unit 3 - Functions

### Suggestions for Assessment

Study Guide questions 3.14 to 3.17 will meet the objectives of Outcome 3.5.

### Resources

*Mathematics 10*,  
Section 5.5, Linear  
Functions: Part II, pages  
290 - 298

*Mathematics 10*,  
*Teacher's Resource Book*,  
Chapter 5, pages 23 - 26

*Mathematics 10*,  
*Independent Study Guide*,  
pages 65 and 66

## Unit 3 - Functions

### Outcomes

3.6 Use function notation to evaluate, write and graph functions.

3.7 Simplify expressions, solve equations and graph functions which involve absolute value.

3.7.1 Define the term *absolute value*.

### Notes for Teaching and Learning

**Notes:** Assign review questions from **Prerequisites**, *Teacher's Resource Book*, page 28.

This section introduces function notation and its advantages when evaluating a function for different values of  $x$ .

Students often have difficulty when substituting expressions for the variable in the function. **Example 4**, page 306, is this type of problem. If necessary, the instructor could assign **Supplementary Example**, *Teacher's Resource Book*, page 28 for extra practice before students move on to the **Exercises**.



## Unit 3 - Functions

### Suggestions for Assessment

Study Guide questions 3.18 to 3.21 will meet the objectives of Outcomes 3.6 and 3.7.

### Resources

*Mathematics 10*,  
Section 5.6, Function  
Notation, pages 304 - 311

*Mathematics 10*,  
*Teacher's Resource Book*,  
Chapter 5, pages 28 - 30

*Mathematics 10*,  
*Independent Study Guide*,  
page 66

## Unit 3 - Functions

### Outcomes

- 3.8 Express a relation
- i) as a table of values
  - ii) as an equation
  - iii) as a set of ordered pairs
  - iv) as a graph
  - v) in words.
- 3.8.1 Define the term *relation*.
- 3.8.2 Identify relations that are functions.
- 3.8.3 State the Vertical Line Test.

### Notes for Teaching and Learning

**Note:** Assign review questions from **Prerequisites**, *Teacher's Resource Book*, page 31.

This section introduces relations. The instructor should ensure that students understand that functions are a subset of relations. This could be illustrated with a diagram. Students could be helped in their understanding of relations if the instructor drew mappings of different types of relations. (See page 31 in *Teacher's Resource Book*.)

Students have been assigned **Exercise 9**, page 318 in *Mathematics 10*. This question could be answered with the help of a graphing calculator. The instructor should explain to students how to use a graphing calculator to graph relations that are not functions.

## Unit 3 - Functions

### Suggestions for Assessment

Study Guide questions 3.22 to 3.25 will meet the objective of Outcome 3.8.

Each section in the *Teacher's Resource Book* has extra questions in **Supplementary Examples** and **Assessing the Outcome**.

Questions from Written Test and Multiple Choice Test in Masters 5.2 - 5.6 could be used for an assessment of Unit 3.

### Resources

*Mathematics 10*,  
Section 5.7, Relations,  
pages 312 - 318

*Mathematics 10*,  
*Teacher's Resource Book*,  
Chapter 5, pages 31 - 33  
Masters 5.2 - 5.6

*Mathematics 10*,  
*Independent Study Guide*  
page 66