Adult Basic Education
Mathematics

Mathematics 1104C

Systems of Equations and Inequalities, and Matrices

Study Guide

Prerequisites: Mathematics 1104A, Mathematics 1104B

Credit Value: 1

Text: *Mathematics 11.* Alexander and Kelly; Addison-Wesley, 1998.

Required Mathematics Courses
[Degree and Technical Profile/Business-Related College Profile]

Mathematics 1104A
Mathematics 1104B
Mathematics 1104C
Mathematics 2104A
Mathematics 2104B
Mathematics 2104C
Mathematics 3104A
Mathematics 3104B
Mathematics 3104C

Table of Contents

To the Student

I. <u>Introduction to Mathematics 1104C</u>

The two topics in this course are systems of equations and inequalities, and matrices. Systems of equations in two variables are solved by two methods: graphing and substitution. Although you can complete some of the exercises by using a graphing calculator, it is recommended that you graph several systems by hand to ensure that the process is understood. You must have the ability to translate word problems into equations. If you have difficulty in this area, it would be helpful to discuss the word problems and compare equations with other students. You should be organized in your approach and show each step that you follow to solve a word problem.

You will first graph linear inequalities in two variables and then progress to a <u>system</u> of linear inequalities. Again, you will solve word problems using systems of linear inequalities.

The second topic in this courses gives an introduction to matrices. Matrices are frequently used to represent real world data. You will be introduced to the terminology and then matrix calculations: addition, subtraction and multiplication. **Appendix A** in this Study Guide provides the notes and problems for matrices. If necessary, see your instructor for further explanation or more practice problems.

II. <u>Resources</u>

You will require the following:

- *Addison Wesley Mathematics 11*, Western Canadian edition Textbook
- Scientific calculator
- graph paper
- Access to a TI-83 Plus graphing calculator (see your instructor)
  and/or *Graphmatica* or *Winplot* graphing software

<u>Notes concerning the textbook</u>:

Glossary: Knowledge of mathematical terms is essential to understand concepts and correctly interpret questions. Written explanations will be part of the work you submit for evaluation, and appropriate use of vocabulary will be required.

Your text for this course includes a Glossary where definitions for mathematical terms are found. Be sure you understand such definitions and can explain them in your own words. Where appropriate, you should include examples or sketches to support your definitions.

To the Student

Examples: You are instructed to study carefully the **Examples** in each section and see your instructor if you have any questions. These **Examples** provide full solutions to problems that can be of great use when answering assigned **Exercises**.

Notes concerning technology:

It is important that you have a **scientific** calculator for your individual use. Ensure that the calculator used has the word “scientific” on it as there are calculators designed for calculation in other areas such as business or statistics which would not have the functions needed for study in this area. Scientific calculators are sold everywhere and are fairly inexpensive. You should have access to the manual for any calculator that you use. It is a tool that can greatly assist the study of mathematics but, as with any tool, the more efficient its use, the better the progress.

You will require access to some sort of technology in order to meet some of the outcomes in this course. Since technology has become a significant tool in the study of Mathematics, your textbook encourages you to become proficient in its use by providing you with step-by-step exercises that will teach you about the useful functions of the TI-83 Plus Graphing calculator. **See your instructor concerning this**. Please note that a graphing calculator is not essential for success in this course but it is useful.

While graphing calculators and graphing software (*Graphmatica* or *Winplot*) are useful tools, they cannot provide the same understanding that comes from working paper and pencil exercises.

III. Study Guide

This Study Guide is required at all times. It will guide you through the course and you should take care to complete each unit of study in the order given in this Guide. Often, at the beginning of each unit, you will be instructed to see your instructor for **Prerequisite** exercises. Please do not skip this step! It should only take a few minutes for you and your instructor to discover what, if any, prerequisite skills need review.

To be successful, you should read the **References and Notes** first and then, when indicated by the ►► symbols, complete the **Work to Submit** problems. Many times you will be directed to see your instructor, and this is vital, especially in a Mathematics course. If you only have a hazy idea about what you just completed, nothing will be gained by continuing on to the next set of problems.

To the Student

Reading for this Unit: In this box, you will find the name of the text, and the chapters, sections and pages used to cover the material for this unit. As a preliminary step, skim the referenced section, looking at the name of the section, and noting each category. Once you have completed this overview, you are ready to begin.

References and Notes

This left hand column guides you through the material to read from the text.

It will also refer to specific **Examples** found in each section. You are directed to study these **Examples** carefully and see your instructor if you have any questions. The **Examples** are important in that they not only explain and demonstrate a concept, but also provide techniques or strategies that can be used in the assigned questions.

The symbols ▶▶ direct you to the column on the right which contains the work to complete and submit to your instructor. You will be evaluated on this material.

Since the answers to **Discussing the Ideas** and **Communicating the Ideas** are not found in the back of the student text, you **must** have these sections corrected by your instructor **before** going on to the next question.

This column will also contain general **Notes** which are intended to give extra information and are not usually specific to any one question.

Work to Submit

There are four basic categories included in this column that correspond to the same categories in the sections of the text. They are **Investigate, Discussing the Ideas**, **Exercises**, and **Communicating the Ideas**.

Investigate: This section looks at the thinking behind new concepts. The answers to its questions are found in the back of the text.

Discussing the Ideas: This section requires you to write a response which clarifies and demonstrates your understanding of the concepts introduced. The answers to these questions are not in the student text and will be provided when you see your instructor.

Exercises: This section helps to reinforce your understanding of the concepts introduced. There are three levels of **Exercises**:
A: direct application of concepts introduced
B: multi-step problem solving and some real-life situations
C: problems of a more challenging nature
The answers to the **Exercises** questions are found in the back of the text.

Communicating the Ideas: This section helps confirm your understanding of the lesson of the section. If you can write a response, and explain it clearly to someone else, this means that you have understood the topic. The answers to these questions are not in the student text and will be provided when you see your instructor

This column will also contain **Notes** which give information about specific questions.

IV. Recommended Evaluation

Written Notes	10%
Assignments	10%
Test(s)	30%
Final Exam *(entire course)*	50%
	100%

The overall pass mark for the course is 50%.

Unit 1 - Systems of Equations and Inequalities

To fulfill the objectives of this unit, students should complete the following:

Reading for this unit:	*Mathematics 11*		
	Chapter 5:	Section 5.1:	pages 300 - 309
		Section 5.4:	pages 323 - 325
		Section 5.5:	pages 333 - 335
		Section 5.7:	pages 348 - 351
		Section 5.8:	pages 352 - 359

References and Notes	Work to Submit
Quad paper or graph paper is required for this unit.	
As you progress through this unit, see your instructor for guidance on what problems are best solved with a graphing calculator.	
Carefully read **Section 5.1**. Study the example on pages 302 and 303, and **Examples 1** and **2**.	
Answer the following questions. ▶▶	1.1 Define the term *linear system of equations*.
	1.2 **Discussing the Ideas**, page 305 Answer questions 1, 2 and 4.

Unit 1 - Systems of Equations and Inequalities

References and Notes	Work to Submit
	1.3 **Exercises**, pages 305 and 306 Answer questions 1 - 6. (*See note below on question 6*.) **Question 6**: In this exercise you have to work backwards. The equations of <u>any</u> two lines that pass through the given point will be a system which works. As you may realize, there is an infinite number of systems of equations that will answer the question
Read pages 308 and 309 and, using a TI-83 graphing calculator, follow the steps. Answer the following questions.	1.4 **Exploring with a Graphing Calculator**, pages 308 and 309 Answer questions 1, 2, 3 and 4. (*See notes below on questions 2 and 4*.) **Question 2**: You may have to adjust your viewing window. Your starting window could be ZStandard (to obtain this window, press: ZOOM, 6). **Question 4**: This exercise will help you see both the power and limits of technology. A graph can be misleading, therefore it may be necessary to zoom in on an apparent point of intersection to see if it really exists.

Unit 1 - Systems of Equations and Inequalities

References and Notes	Work to Submit
Read **Section 5.4**. Carefully study **Examples 1** and **2**. Answer the following questions. ▶▶	1.5 **Discussing the Ideas**, page 325 Complete a) and b). 1.6 **Exercises**, pages 325 Answer questions 1 - 6.
Read **Section 5.5**. **Examples 1** and **2** are solved using the addition-subtraction method. Since you will not study this method until *Math 2104A*, you should rework **Examples 1** and **2** using the substitution method. Answer the following questions. ▶▶	1.7 **Exercises**, pages 334 and 335 Answer questions 1 - 3, 5 - 8 and 10. **Note**: These problems can be solved using one or two variables. It may be easier to use two variables and use the substitution method which you used when you solved **Examples 1** and **2**.
Read **Section 5.7**. Carefully read the description of the graphs of $y > x$ and $y < x$ on page 348.	

References and Notes	**Work to Submit**
Study **Examples 1** and **2**.	
Answer the following questions. ▶▶	1.8 **Discussing the Ideas**, page 350 Answer questions 1, 2 and 3.
	1.9 **Exercises**, pages 350 and 351 Answer questions 1 - 3. (*See note below on question 3*.) Answer questions 4 and 6. (*See note below on question 4*.)
	Question 3: Your instructor can give you **Master 5.1**, from the *Teacher's Resource Book*, which has instructions for using the TI-83 to graph an inequality. You could try to reproduce the screens shown in question 3.
	Question 4: To graph the corresponding equation, you could use any method. However, if you choose to rewrite the inequality so that it is in the form $y > mx + b$ or $y < mx + b$, you should recall that when dividing or multiplying by a negative number, you must change the direction of the inequality sign.
Read **Section 5.8**. **Section 5.8** deals with graphing systems of linear inequalities. (A system is two or more inequalities.) The solution of these systems is the area where the half-planes overlap.	

Unit 1 - Systems of Equations and Inequalities

References and Notes	Work to Submit
Study **Examples 1** and **2**. Answer the following questions. ▶▶	1.10 **Discussing the Ideas**, page 354 Answer questions 1 and 2. 1.11 **Exercises**, pages 354 - 356 Answer question 1. (*See note below on question 1*.) Answer questions 2, 3 and 4. (*See note below on questions 3 and 4*.) Answer questions 6, 8 and 9. (*See note below on question 9*.) **Question 1**: Choose a point in the shaded area. Substitute its coordinates into the equation which defines a boundary. Check to see whether the left side of the equation is less than or greater than the right side. Note that more than one inequality is required to describe each region. **Questions 3 and 4:** Graph each region defined, but you do not have to determine the area. **Question 9:** This problem is similar to **Example 2** on page 353. If you put motorcycles on the *x*-axis and bicycles on the *y*-axis, the vertices of the shaded region should be (0, 30), (10, 30), (20, 20) and (20, 0).

References and Notes	Work to Submit
Read **Maximum - Minimum Problems** on pages 358 and 359.	
Answer the following questions. ▶▶	1.12 **Maximum-Minimum Problems**, pages 358 and 359 Answer questions 1 - 3. (*See note below on questions 1 and 2*.)
	Questions 1 and 2: In order to determine how many of each kind of a ball should be made to yield the maximum profit, you only need to find the profit represented by the coordinates of <u>each</u> vertex of the region. In the graph on page 358, there are 3 vertices (0, 15), (30, 0) and (20, 10). Don't forget that footballs are *x* values and soccer balls are *y* values. If there is a profit of $10.00 per football and $15.00 per soccer ball, the total profit for <u>each</u> of the vertices would be: (0) ($10) + (15) ($15) = $225.00 (30) ($10) + (0) ($15) = $300.00 (20) ($10) + (10) $15) = $350.00 Therefore, 20 footballs and 10 soccer balls will yield maximum profit. Use the coordinates of the same three vertices to find the answers to question 2**.**

Unit 2 - Matrices

To fulfill the objectives of this unit, students should complete the following:

Reading for this unit: Appendix A
Matrices: pages 11 - 26

References and Notes	Work to Submit
Your instructor may have other resources which may be helpful with this topic.	
Read *Appendix A*, **Matrices**, pages 11 - 26, in this Study Guide.	
Study pages 11 and 12.	
Answer the following questions. ▶▶	2.1 **Exercises**: Give an example of each of the following matrices: i) square ii) row iii) column iv) zero v) identity
Practise entering matrices into the TI-83 calculator. See your instructor if you need help with this.	
Answer the following questions. ▶▶	2.2 **Exercises:** Answer questions 1, 2, 3 and 4 on page 13.
See your instructor to have these questions corrected.	

Unit 2 - Matrices

References and Notes	Work to Submit
Read **Matrix Addition**, page 14. Again, use a graphing calculator to try and discover the rules for matrix addition and subtraction. See your instructor for additional resources if necessary. Enter the matrices on page 14, Question 5a) into your TI-83 calculator. Using these matrices, you should find a pattern and be able to state a rule for matrix addition and subtraction. Answer the following questions.	2.3 **Exercises**: Answer questions 5 and 6 on pages 14 - 16. Note: Use paper and pencil and the rules you developed in question 5a).
Read **Scalar multiplication,** page 17. Answer the following questions.	2.4 **Exercises**: Answer questions 7 - 12 on pages 17 and 18.
Read **Matrix multiplication** on pages 19 - 21. Work through each of the given steps. Answer the following questions.	2.5 **Exercises**: Answer questions 13 - 18 on pages 22 - 26.

Appendix A

Matrices

Matrices

A <u>matrix</u> is a rectangular array of numbers within brackets. The array is used to represent real world data and solve real world problems. Simply put, it is a way to arrange data in a table form.

Any table that has rows and columns is a matrix. Databases are examples of matrices used to organize information in matrix form. It is difficult to read through a newspaper and not see examples of matrices. Below is an example.

MJAHL est. 1967 MARITIME JUNIOR A HOCKEY LEAGUE

MARITIME JUNIOR A HOCKEY LEAGUE
1999-2000
FINAL STANDINGS

MAURICE BENT DIVISION

Team	GP	W	L	T	OL	F	A	P
Halifax	52	38	10	4	2	296	178	82
Antigonish	52	34	13	5	1	273	181	74
Truro	52	32	17	3	0	231	177	67
East Hants	52	8	35	9	1	134	140	26
Amherst	52	8	41	3	1	159	279	19

ROGER MEEK DIVISION

Team	GP	W	L	T	OL	F	A	P
Campbellton	52	37	14	1	1	285	192	76
Summerside	52	24	25	3	1	198	229	52
Charlottetown*	52	19	31	2	0	198	247	40
Moncton	52	18	32	2	1	183	231	40

* - awarded third place on more wins

You should become familiar with the following terms:

- *element* - individual number in a matrix
- *row* - horizontal group of numbers in a matrix
- *column* - vertical group of numbers in a matrix
- *dimension* - number of rows and columns in a matrix. If a matrix has m rows and n columns, it is a ($m \times n$) matrix (pronounce m by n matrix).
- *naming a matrix* - usually given a capital letter, e.g. A, B, X, I

There are different types of matrices.

- Square matrix: has the same number of rows as columns

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 12 & 6 & 4 \\ 5 & 9 & -4 \end{bmatrix}$$ A has dimensions (3 × 3).

- Row matrix: has only one row

$$B = \begin{bmatrix} 5 & 0 & -3 & 8 \end{bmatrix}$$ B is a (1 × 4) matrix. B has 1 row and 4 columns.

- Column matrix: has only one column

$$D = \begin{bmatrix} 2 \\ 6 \\ 0 \\ -4 \end{bmatrix}$$ D is a (4 × 1) matrix. D has 4 rows and 1 column.

- Zero matrix: all elements are zeros

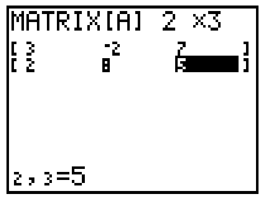
$$L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$ L is a (2 × 2) matrix.

- Identity matrix: a square matrix with 1's on the main diagonal (top left to bottom right) and all other elements are zeros.

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ I is a (3 × 3) matrix.

You should explore the **matrix** feature on the TI-83 graphing calculator.

Example: If you want to enter the matrix A = $\begin{bmatrix} 3 & -2 & 7 \\ 2 & 8 & 5 \end{bmatrix}$ into the TI-83, use the following steps: **matrix** ▸ **edit** enter the dimensions **2** × **3** then enter the elements in matrix **A.**



1. Given matrix A = $\begin{bmatrix} -1 & 5 & 6 & 8 \\ 3 & 4 & 6 & -2 \\ 12 & 1 & 3 & -1 \end{bmatrix}$

   a) State the dimensions of matrix A.
   b) What is the element in row 2, column 3?
   c) What is the element in row 3, column 4?

2. A store sells two types of sneakers, cross-trainers and court sneakers. In June, the store sold 50 cross-trainers and 30 court sneakers, while in July they sold 80 cross-trainers and 90 court sneakers. Represent this information in a rectangular array (or matrix form). ***Hint***: Let the Rows represent the type of sneaker and let the Columns represent the months (type × month).

3. A music store compared the sales of Rap music CD’s to Classical music over 3 months. In November, the store sold 70 Rap CD’s and 100 Classical CD’s. In December, the sold 120 Rap CD’s and 90 Classical CD’s. Finally in January there were 80 Rap CD’s and 60 Classical CD’s sold. Represent this information in matrix form.

4. A store sells three sneaker brands; Nike, Reebok and Adidas. In May, there were 60 Nike, 30 Reebok and 40 Adidas pairs sold. In June, there were 70 Nike, 80 Reebok and 25 Adidas pairs sold. Represent this information in matrix form (brand × month).

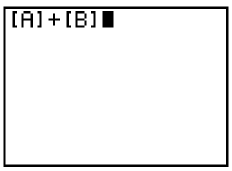
Matrix Addition

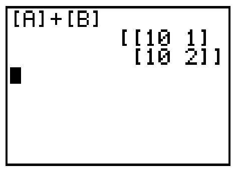
Use a TI-83 to try and discover the rules as to when matrices can be added or subtracted. See your instructor if a graphing calculator is not on hand. Use the problems below to discover the rules.

You should be able to deduce that only elements in matching positions in each matrix can be added. Therefore the matrices <u>*must*</u> *have the same dimensions for them to be added or subtracted.*

Enter the matrices A and B, from question 5a) below, into the TI-83 as shown earlier. Once the matrices have been entered, press **2nd quit** to return to the home screen. To add the matrices:

Press **matrix 1**: **A** press **enter + matrix** ▾ down to **2:B** press **enter**





Note: the same procedure can be followed if matrices are to be multiplied.

5. Use the following problems to complete the table on the next page, if possible. Once the table is completed, look for a pattern and state a rule for matrix addition or subtraction.

a) Find A + B $A = \begin{bmatrix} 6 & -2 \\ 5 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 3 \\ 5 & -2 \end{bmatrix}$

b) Find A + B $A = \begin{bmatrix} 2 & 3 \\ 5 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -6 & 5 \\ -2 & 3 & -4 \end{bmatrix}$

c) Find A − B $A = \begin{bmatrix} 4 & 7 \\ 2 & -1 \\ 0 & 5 \end{bmatrix}$ $B = \begin{bmatrix} -3 & 1 \\ -8 & 4 \\ -5 & 1 \end{bmatrix}$

d) Find A + B $A = \begin{bmatrix} 2 & 0 & -1 \\ 3 & -4 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 0 \\ 5 & 2 \end{bmatrix}$

e) Find A + B $A = \begin{bmatrix} 2 & 1 \\ 5 & 2 \end{bmatrix}$ $B = \begin{bmatrix} 2 & 4 \end{bmatrix}$

f) Find A − B $A = \begin{bmatrix} 4 & 6 & -8 \\ -2 & 5 & 0 \end{bmatrix}$ $B = \begin{bmatrix} 18 & 3 & 12 \\ 1 & 0 & 5 \end{bmatrix}$

g) Complete the table:

Dimensions of matrix A	Dimensions of matrix B	Dimensions of answer
a)		
b)		
c)		
d)		
e)		
f)		

6. Simplify: (Use paper and pencil and the rule you developed in Question 5. DO NOT use a graphing calculator.)

a) $\begin{bmatrix} 4 & 1 \\ -2 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ 2 & 3 \end{bmatrix}$

b) $\begin{bmatrix} -2 & -3 \\ -2 & 10 \end{bmatrix} + \begin{bmatrix} -2 & 3 \\ 2 & -8 \end{bmatrix}$

c) $\begin{bmatrix} 3 & 61 \\ 23 & 14 \end{bmatrix} + \begin{bmatrix} 0 & -2 \\ 6 & 4 \end{bmatrix}$

d) $\begin{bmatrix} 12 & 18 \\ 21 & -14 \end{bmatrix} + \begin{bmatrix} 4 & 6 \\ -3 & -6 \end{bmatrix}$

e) $\begin{bmatrix} 5 & 1 \\ -4 & 7 \end{bmatrix} + \begin{bmatrix} 8 & 2 \\ 4 & -5 \end{bmatrix}$

f) $\begin{bmatrix} 11 & 1 \\ 16 & -4 \end{bmatrix} + \begin{bmatrix} 3 & -1 \\ 4 & -6 \end{bmatrix}$

g) $\begin{bmatrix} 1 & 2 \\ -3 & 5 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 0 & 5 \\ 2 & 4 \\ 3 & -2 \end{bmatrix}$

h) $\begin{bmatrix} 0 & -2 & 4 \\ 3 & -2 & -6 \end{bmatrix} + \begin{bmatrix} 1 & -5 & 0 \\ 3 & -6 & -2 \end{bmatrix}$

i) $\begin{bmatrix} 1 & 5 & 3 \\ 2 & 7 & -1 \end{bmatrix} + \begin{bmatrix} 5 & 7 & 1 \\ 8 & -6 & 11 \end{bmatrix}$

j) $\begin{bmatrix} 7 & 5 & 2 \\ 9 & 1 & 0 \\ 3 & 6 & 4 \end{bmatrix} + \begin{bmatrix} 5 & 2 & 8 \\ 1 & 0 & 6 \\ 9 & 11 & -2 \end{bmatrix}$

Multiplication

Scalar Multiplication

7. If A = $\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix}$, find A + A.

$$A + A = \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix} =$$

Now, find 2A.

$$2A = 2\begin{bmatrix} 4 & 3 \\ -1 & 0 \end{bmatrix} =$$

What do you notice?

8. With the matrix below, evaluate 3A.

$$A = \begin{bmatrix} 3 & -2 \\ 5 & 1 \end{bmatrix}$$

9. A store sells two types of sneakers, cross-trainers and court sneakers. In June, the store sold 50 cross-trainers and 30 court sneakers, while in July they sold 80 cross-trainers and 90 court sneakers. This information is represented in matrix form below. If sales were twice the original projections, represent this solution in matrix form.

$$A = \begin{array}{r} \\ \text{Trainers} \\ \text{Court} \end{array} \begin{array}{c} \begin{array}{cc} \text{June} & \text{July} \end{array} \\ \begin{bmatrix} 50 & 80 \\ 30 & 90 \end{bmatrix} \end{array}$$

10. Does scalar multiplication change the dimensions of a matrix? Give examples in your answer.

11. If $J = \begin{bmatrix} 5 & 3 & 1 \\ 1 & 2 & 0 \end{bmatrix}$, $F = \begin{bmatrix} 1 & -4 & 3 \\ 6 & 1 & 5 \end{bmatrix}$, and $M = \begin{bmatrix} 4 & 5 & -7 \\ 2 & 3 & 8 \end{bmatrix}$,

evaluate the following:

a) J − M b) 2J + F c) 3F + 2M d) J − F + 2M

Zero Matrix

For the addition of matrices, one special matrix, the zero matrix, plays a role similar to the number zero.

For example, $\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}$

and for scalar multiplication, $x \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

12. If $K = \begin{bmatrix} 3 & 4 \\ 2 & 1 \\ 0 & 9 \end{bmatrix}$ and $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$, evaluate the following:

a) K + A b) 3A

Matrix Multiplication

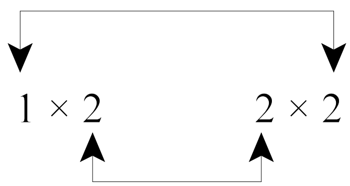
Example 1: Read and complete the accompanying exercises. Formulate a rule for matrix multiplication.

Multiply A × B.

A = $\begin{bmatrix} 1 & 5 \end{bmatrix}$ B = $\begin{bmatrix} -4 & 2 \\ 5 & 1 \end{bmatrix}$

First, we must see if these matrices can be multiplied. Write the dimensions of each.

<u>Matrix A</u>	<u>Matrix B</u>
row × column	row × column



Look at the given dimensions (1 × 2) and (2 × 2).
If the two <u>inside</u> numbers are the <u>same</u> then the matrices <u>can be multiplied</u>. The *result* will be a matrix with dimensions determined by the *outer* numbers. For the above example, the inner numbers are both 2 and thus multiplication can be done. The outer numbers are 1 and 2 and thus dimensions of the solution matrix is (1 × 2) (1 row and 2 columns).

$\begin{bmatrix} \_ & \_ \end{bmatrix}$

To fill in these blanks, name their positions. The first blank is in the **1st row**, **1st column** position. To get the element that goes in this blank multiply the elements in the **1st row** of matrix A by the elements in the **1st column** of matrix B.
ie: **(1 × −4) + (5 × 5) = −4 + 25 = 21**.   The first blank is the element **21.**

The second blank has the position, **1st row 2nd column**. The element that goes here comes from multiplying the elements in the **1st row** of matrix A and the **2nd column** of matrix B.
ie**:**   **(1 × 2) + (5 × 1) = 7**. The second blank is the element **7**.

A × B = $\begin{bmatrix} 21 & 7 \end{bmatrix}$

Example 2:

If P = $\begin{bmatrix} 4 & 1 \\ 0 & -2 \end{bmatrix}$ and Q = $\begin{bmatrix} 0 & -4 & 3 \\ 1 & 5 & 2 \end{bmatrix}$ find P × Q.

First, check the dimensions of P and Q and decide whether it is possible to multiply them, and if so, what the dimensions of the product matrix will be.

<u>Matrix P</u> row × column

<u>Matrix Q</u> row × column

2 × 2   2 × 3

The two *inside* numbers are the same, therefore the matrices <u>can</u> be multiplied. The two *outside* numbers are 2 and 3, therefore the dimensions of the product matrix is (2 × 3).

P × Q = $\begin{bmatrix} \_ & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}$

The first blank is in the **1st row**, **1st column** position. Therefore, you must multiply the elements in the **1st row** of matrix P by the elements in the **1st column** of matrix Q.

ie: (4 × 0) + (1 × 1) = 1. The first blank is the element 1.

P × Q = $\begin{bmatrix} \underline{1} & \_ & \_ \\ \_ & \_ & \_ \end{bmatrix}$

The element which goes in the **1st row** and the **2nd column** position is found by multiplying the elements in the **1st row** of matrix P and the **2nd column** of matrix Q.

ie: (4 × −4) × (1 × 5) = −16 + 5 = −11

P × Q = $\begin{bmatrix} 1 & -11 & \_ \\ \_ & \_ & \_ \end{bmatrix}$

Similarly: the element in the **1st row**, **3rd column** is found by multiplying the elements in **row 1** of matrix P by **column 3** of matrix Q.
ie: (4 × 3) + (1 × 2) = 12 + 2 = 14

$$P \times Q = \begin{bmatrix} 1 & -11 & 14 \\ \_ & \_ & \_ \end{bmatrix}$$

The element in the **2nd row**, **1st column** is found by multiplying the elements in the **2nd row** of matrix P, and the **1st** column of matrix Q.

ie: (0 × 0) + (−2 × 1) = −2

$$P \times Q = \begin{bmatrix} 1 & -11 & 14 \\ -2 & \_ & \_ \end{bmatrix}$$

Using the pattern established above, complete the other two blanks in the table.

Example 3: If $L = \begin{bmatrix} 1 & 4 & 0 \\ 9 & -3 & -1 \end{bmatrix}$ and $M = \begin{bmatrix} 4 & 1 & 9 \\ 0 & 9 & 4 \end{bmatrix}$

Find L × M.

First, check dimensions of L and M and decide whether it is possible to multiply them.

Matrix L	Matrix M
row × column	row × column
2 × 3	2 × 3

The two *inside* numbers are *different*: 2 and 3, therefore the matrices cannot be multiplied.
End of problem!

13. Use the following problems to complete the accompanying table. When possible, find the solution for A × B. Use paper and pencil. You may use your graphing calculator to check your answers.

a) $A = \begin{bmatrix} 2 & -1 \\ -4 & 3 \end{bmatrix}$ $B = \begin{bmatrix} -2 & 0 \\ 7 & -3 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 5 \end{bmatrix}$ $B = \begin{bmatrix} -4 & 2 \\ 5 & 1 \end{bmatrix}$

c) $A = \begin{bmatrix} 4 & -6 \\ 3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 3 & -4 \end{bmatrix}$ d) $A = \begin{bmatrix} 3 & 1 \\ 4 & 7 \end{bmatrix}$ $B = \begin{bmatrix} 4 & 9 \\ 6 & -1 \\ 0 & 0 \end{bmatrix}$

e) $A = \begin{bmatrix} 0 & 3 \\ 5 & -1 \end{bmatrix}$ $B = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$ f) $A = \begin{bmatrix} 8 \\ -4 \end{bmatrix}$ $B = \begin{bmatrix} -1 & 4 \\ 3 & -7 \end{bmatrix}$

g) $A = \begin{bmatrix} 6 & 0 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 1 & -2 \\ -3 & 6 & -1 \\ 3 & 2 & 7 \end{bmatrix}$

Dimensions of A	Dimensions of B	Dimensions of A × B
a)		
b)		
c)		
d)		
e)		
f)		
g)		

14. In the above series of problems, find the solution matrix for **B × A**. Are the answers the same as those for **A × B**? What does this tell you about matrix multiplication?

Example

A store sells three sneaker brands: Nike, Reebok and Adidas. In May there were 60 Nike, 30 Reebok and 40 Adidas pairs sold. In June there were 70 Nike, 80 Reebok and 25 Adidas pairs sold. Represent this information in matrix form (brand × month). The price of the sneakers is Nike $90, Reebok $70, and Adidas $85. Write this in matrix form (price × brand). Finally multiply these matrices to determine the revenue generated each month.

<u>Solution</u>:

$$\begin{matrix} & N \quad R \quad A \\ & \begin{bmatrix} 90 & 70 & 85 \end{bmatrix} \end{matrix} \begin{matrix} \text{May} \quad \text{June} \\ \begin{bmatrix} 60 & 70 \\ 30 & 80 \\ 40 & 25 \end{bmatrix} \end{matrix}$$

Price × Brand **Brand × Month**

$$= \begin{bmatrix} 10{,}900 & 14{,}025 \end{bmatrix}$$

This matrix tells us that $10,900 was generated by all brands in May and $14,025 in June.

15. Two outlets of an electronics store sell 3 comparable items. Use matrix multiplication to show the total revenue that these items could generate in each store when they are sold at the regular price and at the sale price.

Number of items in each store

	TV's	Stereos	Cameras
Carbonear	85	100	60
Pasadena	70	120	90

Prices of items

	Regular	Sale Price
TV's	$450	$300
Stereos	$320	$250
Cameras	$280	$170

16. a) Write the following information from the CFL in matrix form and label it matrix A.

	W	L	T
Toronto	6	1	1
Montreal	5	2	1
B.C.	3	3	2
Calgary	2	6	0

Matrix B represents the points awarded for a win, a loss and a tie.

$$\textbf{B} = \begin{matrix} & \textbf{Points} \\ \textbf{W} & \\ \textbf{L} & \\ \textbf{T} & \end{matrix}\!\!\begin{matrix} \\ \begin{bmatrix} \textbf{2} \\ \textbf{0} \\ \textbf{1} \end{bmatrix} \end{matrix}$$

b) What are the dimensions of matrix A and matrix B?

c) Calculate **A × B**.

d) What do the elements in the product matrix represent?

17. Your teacher keeps a record of your marks in matrix form with rows representing students and columns representing the test results in %. Class tests/assignments are worth 60% of the term mark while the final exam is worth the remaining 40%. There are five class tests worth 60% ÷ 5 = 12% (.12) each. The final exam is worth 40% (.40).

a) Enter the information below into matrix **A** in the TI-83.

b) Create matrix **B** (6 × 1) representing the values of the tests.
Enter this into matrix **B** in the TI-83.

c) Calculate **A** × **B**.

d) What do the elements in the product matrix represent?

	#1	#2	#3	#4	#5	Final
Anderson, N.	75	59	88	79	91	85
Balcom, P.	53	49	62	59	70	60
Davis, T.	63	82	84	76	89	92
Hunt, S.	92	94	90	89	95	96
Noonan, L.	83	76	87	83	55	62

18. Multiply. Use paper and pencil first. Check your answers on your TI-83.

a) $\begin{bmatrix} 3 & 4 \\ -1 & 1 \end{bmatrix} \times \begin{bmatrix} 3 & 2 \\ -1 & 4 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 3 \\ -1 & 5 \end{bmatrix} \times \begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$

c) $\begin{bmatrix} 2 & -4 \\ 1 & 9 \end{bmatrix} \times \begin{bmatrix} 3 & -2 \\ -1 & 4 \end{bmatrix}$

d) $\begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \times \begin{bmatrix} 4 & 6 \\ -3 & -6 \end{bmatrix}$

e) $\begin{bmatrix} 3 & -2 \\ 5 & -4 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ 2 & 4 \end{bmatrix}$

f) $\begin{bmatrix} 1 & 2 \\ -3 & 5 \\ 4 & 6 \end{bmatrix} \times \begin{bmatrix} 0 & 5 \\ 2 & 4 \\ 3 & -2 \end{bmatrix}$

g) $\begin{bmatrix} 0 & -2 & 4 \\ 3 & -2 & -6 \end{bmatrix} \times \begin{bmatrix} 1 & -5 & 0 \\ 3 & -6 & -2 \end{bmatrix}$

h) $\begin{bmatrix} 1 & 2 \\ -3 & 5 \end{bmatrix} \times \begin{bmatrix} 2 & 3 & -4 \\ 1 & 2 & 1 \end{bmatrix}$

i) $\begin{bmatrix} 2 & 5 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ 4 & 2 \end{bmatrix}$

j) $\begin{bmatrix} 7 & 5 & 2 \\ 9 & 1 & 0 \\ 3 & 6 & 4 \end{bmatrix} \times \begin{bmatrix} 5 & 2 & 8 \\ 1 & 0 & 6 \\ 9 & 11 & -2 \end{bmatrix}$

Answers for Selected Problems

1. a) 3×4
   b) 6
   c) -1

2.

$$A = \begin{array}{r} \\ \text{Trainers} \\ \text{Court} \end{array} \begin{array}{c} \begin{array}{cc} \text{June} & \text{July} \end{array} \\ \begin{bmatrix} 50 & 80 \\ 30 & 90 \end{bmatrix} \end{array}$$

3

$$B = \begin{array}{r} \\ \text{Rap} \\ \text{Classical} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Nov} & \text{Dec} & \text{Jan} \end{array} \\ \begin{bmatrix} 70 & 120 & 80 \\ 100 & 90 & 60 \end{bmatrix} \end{array}$$

4.

$$A = \begin{array}{r} \\ \text{Nike} \\ \text{Reebok} \\ \text{Adidas} \end{array} \begin{array}{c} \begin{array}{cc} \text{May} & \text{June} \end{array} \\ \begin{bmatrix} 60 & 70 \\ 30 & 80 \\ 40 & 25 \end{bmatrix} \end{array}$$

5. a) $\begin{bmatrix} 10 & 1 \\ 10 & 2 \end{bmatrix}$

   b) Cannot be added because dimensions are different.

c) $\begin{bmatrix} 7 & 6 \\ 10 & -5 \\ 5 & 4 \end{bmatrix}$

d) Cannot be added.

e) Cannot be added.

f) $\begin{bmatrix} -14 & 3 & -20 \\ -3 & 5 & -5 \end{bmatrix}$

6.

a) $\begin{bmatrix} 4 & 4 \\ 0 & 3 \end{bmatrix}$

b) $\begin{bmatrix} -4 & 0 \\ 0 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 3 & 59 \\ 29 & 18 \end{bmatrix}$

d) $\begin{bmatrix} 16 & 24 \\ 18 & -20 \end{bmatrix}$

e) $\begin{bmatrix} 13 & 3 \\ 0 & 2 \end{bmatrix}$

f) $\begin{bmatrix} 14 & 0 \\ 20 & -10 \end{bmatrix}$

g) $\begin{bmatrix} 1 & 7 \\ -1 & 9 \\ 7 & 4 \end{bmatrix}$

h) $\begin{bmatrix} 1 & -7 & 4 \\ 6 & -8 & -8 \end{bmatrix}$

i) $\begin{bmatrix} 6 & 12 & 4 \\ 10 & 1 & 10 \end{bmatrix}$

j) $\begin{bmatrix} 12 & 7 & 10 \\ 10 & 1 & 6 \\ 12 & 17 & 2 \end{bmatrix}$

7. $A + A = \begin{bmatrix} 8 & 6 \\ -2 & 0 \end{bmatrix}$ $2A = \begin{bmatrix} 8 & 6 \\ -2 & 0 \end{bmatrix}$

8. $3A = \begin{bmatrix} 9 & -6 \\ 15 & 3 \end{bmatrix}$

9. $2A = 2 \times \begin{array}{c} \\ \text{Trainers} \\ \text{Court} \end{array} \begin{array}{c} \begin{matrix} \text{June} & \text{July} \end{matrix} \\ \begin{bmatrix} 50 & 80 \\ 30 & 90 \end{bmatrix} \end{array} = \begin{array}{c} \begin{matrix} \text{June} & \text{July} \end{matrix} \\ \begin{bmatrix} 100 & 160 \\ 60 & 180 \end{bmatrix} \end{array}$

11. a) $\begin{bmatrix} 1 & -2 & 8 \\ -1 & -1 & -8 \end{bmatrix}$ b) $\begin{bmatrix} 11 & 2 & 5 \\ 8 & 5 & 5 \end{bmatrix}$

c) $\begin{bmatrix} 11 & -2 & -5 \\ 22 & 9 & 31 \end{bmatrix}$ d) $\begin{bmatrix} 12 & 17 & -16 \\ -1 & 7 & 11 \end{bmatrix}$

12. a) $\begin{bmatrix} 3 & 4 \\ 2 & 1 \\ 0 & 9 \end{bmatrix}$ b) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

13. a) $\begin{bmatrix} -11 & 3 \\ 29 & -9 \end{bmatrix}$ b) $\begin{bmatrix} 21 & 7 \end{bmatrix}$ c) Cannot be multiplied d) Cannot be multiplied

e) $\begin{bmatrix} 9 \\ -8 \end{bmatrix}$ f) Cannot be multiplied g) $\begin{bmatrix} 24 & 2 & -26 \end{bmatrix}$

15.

$$N = \begin{array}{c} \\ \text{Carbonear} \\ \text{Pasadena} \end{array} \begin{array}{c} \begin{array}{ccc} \text{TV's} & \text{Stereos} & \text{Cameras} \end{array} \\ \begin{bmatrix} 85 & 100 & 60 \\ 70 & 120 & 90 \end{bmatrix} \end{array}$$

$$P = \begin{array}{c} \\ \text{TV's} \\ \text{Stereos} \\ \text{Cameras} \end{array} \begin{array}{c} \begin{array}{cc} \text{Regular} & \text{Sales} \end{array} \\ \begin{bmatrix} \$450 & \$300 \\ \$320 & \$250 \\ \$280 & \$170 \end{bmatrix} \end{array}$$

$$N \times P = \begin{bmatrix} \$87{,}050 & \$60{,}700 \\ \$95{,}100 & \$66{,}300 \end{bmatrix}$$

16.

c)

$$A = \begin{array}{c} \\ \textbf{Toronto} \\ \textbf{Montreal} \\ \textbf{B. C.} \\ \textbf{Calgary} \end{array} \begin{array}{c} \begin{array}{ccc} \textbf{W} & \textbf{L} & \textbf{T} \end{array} \\ \begin{bmatrix} 6 & 1 & 1 \\ 5 & 2 & 1 \\ 3 & 3 & 2 \\ 2 & 6 & 0 \end{bmatrix} \end{array}$$

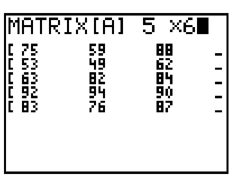
$$\textbf{Points}$$

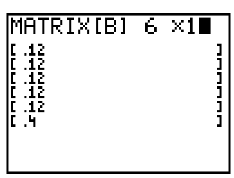
$$B = \begin{array}{c} \textbf{W} \\ \textbf{L} \\ \textbf{T} \end{array} \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$$

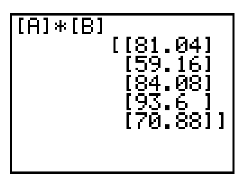
$$A \times B = \begin{bmatrix} 13 \\ 11 \\ 8 \\ 4 \end{bmatrix}$$

d) Total points for each team.

17.c)







d) Mark for each student.

18. a) $\begin{bmatrix} 5 & 22 \\ -4 & 2 \end{bmatrix}$ b) $\begin{bmatrix} 9 & 11 \\ 7 & 21 \end{bmatrix}$ c) $\begin{bmatrix} 10 & -20 \\ -6 & 34 \end{bmatrix}$ d) $\begin{bmatrix} 5 & 6 \\ 0 & -6 \end{bmatrix}$

e) $\begin{bmatrix} -1 & -17 \\ -3 & -31 \end{bmatrix}$ f) no solution g) no solution h) $\begin{bmatrix} 4 & 7 & -2 \\ -1 & 1 & 17 \end{bmatrix}$

i) $\begin{bmatrix} 22 & 4 \end{bmatrix}$ j) $\begin{bmatrix} 58 & 36 & 82 \\ 46 & 18 & 78 \\ 57 & 50 & 52 \end{bmatrix}$