Adult Basic Education

Mathematics

Mathematics 2104A

Solving Linear Systems
Statistics

Curriculum Guide

Prerequisites: Mathematics 1104A, 1104B, 1104C

Credit Value: 1

Required Mathematics Courses
[Degree and Technical Profile/ Business-Related College Profile]

Mathematics 1104A
Mathematics 1104B
Mathematics 1104C
Mathematics 2104A
Mathematics 2104B
Mathematics 2104C
Mathematics 3104A
Mathematics 3104B
Mathematics 3104C

Table of Contents

To the Instructor

I. Introduction to Mathematics 2104A

In this course, students will solve linear systems of equations using addition and subtraction. Using this technique to solve a system of equations with three or more unknowns is very cumbersome. Students will learn to use matrices for these complex situations. Students will write systems of equations in matrix form and, using technology, find the inverse matrix and then solve the system.

Students will look at linear systems and classify them as consistent (one solution or many solutions) or inconsistent (no solution). The material for this topic is found in *Mathematics 11* and Appendix A, at the end of the Study Guide.

Students will also be introduced to inferential statistics and examine various sampling methods and potential sources of bias associated with these methods. Students will make inferences about samples based on population data and make inferences about populations based on data from samples.

II. Prerequisites

Students should be able to add and subtract equations and multiply equations by a constant in order to obtain equal or opposite coefficients. Students should be able to graph systems of linear equations and identify parallel or coincident lines from their graph or equations.

Students should be familiar with matrices and matrix addition and multiplication. Students should also understand the terms inverse, multiplicative inverse and identity.

III. Textbook

Most of the concepts are introduced, developed and explained in the **Examples**. The instructor must insist that students carefully study and understand each **Example** before moving on to the **Exercises**. In the Study Guide, students are directed to see the instructor if there are any difficulties.

To the Instructor

There are four basic categories included in each section of the textbook which require the student to complete questions:

1. Investigate
2. Discussing the Ideas
3. Exercises
4. Communicating the Ideas

Investigate: This section looks at the thinking behind new concepts. The answers to its questions are found in the back of the text.

Discussing the Ideas: This section requires the student to write a response which clarifies and demonstrates understanding of the concepts introduced. The answers to these questions are not in the student text but are in the *Teacher's Resource Book*. Therefore, in the Study Guide, the student is directed to see the instructor for correction. This will offer the instructor some perspective on the extent of the student's understanding. If necessary, reinforcement or remedial work can be introduced. Students should not be given the answer key for this section as the opportunity to assess the student's understanding is then lost.

Exercises: This section helps the student reinforce understanding of the concepts introduced. There are three levels of **Exercises**:

- **A:** direct application of concepts introduced;
- **B:** multi-step problem solving and some real-life situations;
- **C:** problems of a more challenging nature.

The answers to the **Exercises** questions are found in the back of the text.

Communicating the Ideas: This section helps confirm the student's understanding of a particular lesson by requiring a clearly written explanation. The answers to **Communicating the Ideas** are not in the student text, but are in the *Teacher's Resource Book*. In the Study Guide students are asked to see the instructor for correction.

IV. Technology

It is important that students have a **scientific** calculator and its manual for their individual use. Ensure that the calculator used has the word "scientific" on it as there are calculators designed for calculation in other areas such as business or statistics which would not have the functions needed for study in this area.

To the Instructor

A graphing calculator should be **available** to the students since the text provides many opportunities for its use. The *Teacher's Resource Book* suggests many occasions to utilize a graphing calculator. These suggestions are outlined where there is the heading *Integrating Technology*. In the Study Guide, students are directed to see the instructor when a graphing calculator is required. The *Teacher's Resource Book* contains a module called **Graphing Calculator Handbook** which will help the instructor and student get acquainted with some of the main features of the TI-83 Plus graphing calculator.

Graphing software such as *Graphmatica* or *Winplot* can also be used if the students don't have access to a graphing calculator but do have access to a computer. The textbook doesn't offer the same guidance for graphing with these tools as it does for a graphing calculator, but each software program does have a HELP feature to answer questions.

V. <u>Curriculum Guides</u>

Each new ABE Mathematics course has a Curriculum Guide for the instructor and a Study Guide for the student. The Curriculum Guide includes the specific curriculum outcomes for the course. Suggestions for teaching, learning, and assessment are provided to support student achievement of the outcomes. Each course is divided into units. Each unit comprises a **two-page layout of four columns** as illustrated in the figure below. In some cases the four-column spread continues to the next two-page layout.

Curriculum Guide Organization:
The Two-Page, Four-Column Spread

Unit Number - Unit Title	
Outcomes Specific curriculum outcomes for the unit.	**Notes for Teaching and Learning** Suggested activities, elaboration of outcomes, and background information.

Unit Number - Unit Title	
Suggestions for Assessment Suggestions for assessing students' achievement of outcomes.	**Resources** Authorized and recommended resources that address outcomes.

To the Instructor

VI. Study Guides

The Study Guide provides the student with the name of the text(s) required for the course and specifies the sections and pages that the student will need to refer to in order to complete the required work for the course. It guides the student through the course by assigning relevant reading and providing questions and/or assigning questions from the text or some other resource. Sometimes it also provides important points for students to note. (See the *To the Student* section of the Study Guide for a more detailed explanation of the use of the Study Guides.) The Study Guides are designed to give students some degree of independence in their work. Instructors should note, however, that there is much material in the Curriculum Guides in the *Notes for Teaching and Learning* and *Suggestions for Assessment* columns that is not included in the Study Guide and instructors will need to review this information and decide how to include it.

VII. Resources

Essential Resources

Addison Wesley Mathematics 11 (Western Canadian edition)
ISBN:0-201-34624-9

Addison Wesley Mathematics 10 (Western Canadian edition)
ISBN:0-201-34619-2

Mathematics 11 Teacher's Resource Book (Western Canadian edition)
ISBN: 0-201-34626-5

Mathematics 10 Teacher's Resource Book (Western Canadian edition)
ISBN: 0-201-34621-4

Math 2104A Study Guide

Recommended Resources

*Mathematics 11 Independent Study Guide (*Western Canadian edition)
ISBN: 0-201-34625-7

*Mathematics 10 Independent Study Guide (*Western Canadian edition)
ISBN: 0-201-34620-6

Center for Distance Learning and Innovation: http://www.cdli.ca

Winplot: http://math.exeter.edu/rparris/winplot.html
(Free graphing software)

Graphmatica (Evaluation software available on CD-ROM contained in *Teacher's Resource Book*)

CD Rom accompanying *Teacher's Resource Book*
This CD contains selected solutions from the text and self test solutions from the *Independent Study Guide*.

Other Resources

Math Links: http://mathforum.org

http://www.purplemath.com

http://www.sosmath.com/index.html

http://www.math.com/

http://spot.pcc.edu/~ssimonds/winplot
(Free videos concerning Winplot)

http://www.pearsoned.ca/school/math/math/

VIII. Recommended Evaluation

Written Notes	10%
Assignments	10%
Test(s)	30%
Final Exam *(entire course)*	50%
	100%

The overall pass mark for the course is 50%.

Solving Linear Systems
Statistics

Outcomes

1.1 Solve linear systems of equations by using addition and subtraction.

Notes for Teaching and Learning

Two basic properties of linear systems are introduced in this section. These properties provide a method of efficiently solving equations algebraically.

Note: Assign review questions from **Prerequisites**, *Teacher's Resource Book*, Chapter 5, page 8.

When answering question 1, page 315, students should realize that they don't have to solve each system to see if $(-1, 1)$ is a solution.

Students should realize that the method of addition and subtraction works for linear systems only.

Students should be reminded that when they are solving linear systems, both sides of an equation must be multiplied by the constant.

The instructor should ensure that students rearrange terms to add or subtract vertically, and, if necessary, expand and collect like terms. (See question 12 on page 317.)
The instructor should ask students to complete question 19, page 318, by using a graphing calculator.

Suggestions for Assessment

Study Guide questions 1.1 to 1.3 will meet the objectives of Outcome 1.1.

Resources

Mathematics 11,
Section 5.2,
Solving Linear Systems by Addition or Subtraction, pages 310 - 318

Mathematics 11,
Teacher's Resource Book,
Chapter 5, pages 8 - 10

Mathematics 11,
Independent Study Guide,
pages 66 and 67

Outcomes

1.2 Classify linear systems as consistent or inconsistent.

1.2.1 Define the terms *inconsistent system of equations* and *consistent system of equations*.

1.2.2 Without graphing, identify equations that are consistent or inconsistent .

Notes for Teaching and Learning

Note: Assign review questions from **Prerequisites**, *Teacher's Resource Book*, Chapter 5, page 11.

Students should know the meaning of the terms *consistent* and *inconsistent* before completing the **Exercises**.

By the time students finish this section, they should be able to look at a system of equations (and rearrange, if necessary) and state which possibility is true: one solution, no solutions or many solutions.

If students have graphing calculators, they can use them to graph each equation after it is written in the form $y = mx + b$.

Suggestions for Assessment

Study Guide questions 1.4 and 1.5 will meet the objectives of Outcome 1.2.

Resources

Mathematics 11,
Section 5.3,
Number of Solutions of a Linear System,
pages 319 - 322

Mathematics 11,
Teacher's Resource Book,
Chapter 5, pages 11 - 13

Mathematics 11,
Independent Study Guide,
page 67

Outcomes

1.3 Solve linear systems of equations in three variables.

Notes for Teaching and Learning

Assign **Prerequisites**, page 21, *Teacher's Resource Book*, Chapter 5.

Although students have mastered techniques of solving systems of two equations and two unknowns, they may be intimidated by systems of equations with <u>three</u> variables.

The instructor should point out that each system with three equations and three unknowns is reduced to an equivalent system with two equations and two unknowns by using the familiar addition and subtraction technique.

The key is to consider the equations two at a time and select the variable that can be most easily eliminated from two pairs of equations.

Students should be encouraged to write neatly so that addition and subtraction can be performed easily.

If students are having difficulty with question 8 in **Exercises**, the instructor should suggest that they write an expression $ax + by + cz$ with <u>any</u> value for a, b, and c. Substitute the given value of x, y, and z into the expression and simplify. Write an expression on the left side and the simplification on the right.

Students may need some guidance with the word problems in questions 9 and 10.

Suggestions for Assessment

Study Guide questions 1.6 to 1.9 will meet the objectives of Outcome 1.3.

Resources

Mathematics 11, Section 5.6, Solving Linear Systems in Three Variables, pages 337 - 343.

Mathematics 11, *Teacher's Resource Book*, Chapter 5, pages 21 - 24

Mathematics 11, *Independent Study Guide*, page 68

Outcomes

1.4 Solve systems of equations using matrices.

1.4.1 Demonstrate an understanding of the conditionals under which matrices have identities and inverses.

1.4.2 Demonstrate an understanding of the relationship between algebraic and matrix equations.

1.4.3 Use the graphing calculator correctly and efficiently.

1.4.4 Write systems of equations in matrix form.

1.4.5 Solve systems of equations using inverse matrices and a graphing calculator.

Notes on Teaching and Learning

If necessary, students should review the Matrices section that they studied in *Mathematics 1104C*.

The Appendix in the Study Guide provides the notes and assigned questions for this topic.

The instructor should consult other textbooks for extra problems.

Students should be carefully guided through this topic. The instructor must ensure that students are able to use the TI-83 correctly and efficiently. Students should understand the significance of the inverse matrix and how it is used to produce the identity matrix. For this course, students will use a TI-83 to find the inverse of all square matrices.

The instructor should explain to students that the inverse matrix can be found by using pencil and paper; but, except for a 2 x 2 matrix, the method is very time consuming.

Suggestions for Assessment

Study Guide questions 1.10 to 1.13 will meet the objectives of Outcome 1.4.

Resources

Appendix, pages 21 - 30

Outcomes

2.1 Make decisions about the validity of survey results by considering sampling techniques and possible biases.

2.1.1 Define the following terms: *population*, *sample* and *bias*.

Notes for Teaching and Learning

If there is more than one student studying this unit, it may be more profitable if they work as a group for many of the exercises and discussions.

The **Prerequisites** exercises and **Investigate** questions are designed to be completed in small groups.

The instructor should review how to express a number as a percent.

Ensure that students know the difference between a *population* and a *sample*.

When completing question 6 in **Exercises**, page 534, students should be encouraged to think of ways that a survey could be biased in *any* direction.

Suggestions for Assessment

Study Guide questions 2.1 to 2.5 will meet the objectives of Outcome 2.1.

Resources

Mathematics 10, Section 9.1, The Nature of Surveys, pages 530 - 535

Mathematics 10, *Teacher's Resource Book*, Chapter 9, pages 4 - 5

Mathematics 10, *Independent Study Guide*, page 101

Outcomes

2.2 Apply sampling methods that will result in an unbiased sample from a given population.

2.2.1 Use a graphing calculator to generate random numbers.

2.2.2 Describe the following six sampling methods:
I) simple random sample
ii) systematic sample
iii) convenience sample
iv) stratified random sample
v) cluster sample
vi) self-selected sample

Notes for Teaching and Learning

To generate random numbers, students will use only one of the methods described in **Mathematics File**.

The RANDNUM program for the TI-83 graphing calculator is on Master 9.7. This program is also on the CD-ROM that is with the *Teacher's Resource Book.* Connecting a TI-GRAPH LINK cable between a computer and TI-83 would be the most efficient method of entering the program into the graphing calculators. Also, a unit-to-unit link cable, which is included with all new TI-83 calculators, allows communication with another TI-83.

Instructors should explain to students that the RANDNUM program on the graphing calculator (as well as the other methods mentioned in **Mathematics File**) simulates the results of experiments with spinners, dice, coins or cards.

The TI-83 generates random numbers using the following command: randInt (lowest number, highest number, number of trials).

See *Integrating Technology* on page 5 of the *Teacher's Resource Book*, Chapter 9, for detailed instructions.

Before completing **Section 9.2**, the instructor should assign **Prerequisites** exercises for review. Ensure that students are familiar with ways in which a sample could be biased.

Suggestions for Assessment

Study Guide questions 2.6 to 2.11 will meet the objectives of Outcome 2.2.

Resources

Mathematics 10, Section 9.2, Methods of Sampling, pages 536 - 542

Mathematics 10, Teacher's Resource Book, Chapter 9, pages 5 - 8

Mathematics 10, Independent Study Guide, page 101

Outcomes

2.3 Make inferences about samples based on data from populations.

2.3.1 Use a graphing calculator to simulate experiments with binomial outcomes.

2.3.2 Use boxplots to represent data and to estimate probabilities about random samples.

Notes for Teaching and Learning

Students should read **Mathematics File** on page 543 even though they will not be constructing the sampling box described there. However, if students understand how and why the sampling box works, they should have a better appreciation of what the program SMPLSIM3 generates.

The SMPLSIMI program for the TI-83 graphing calculator is on Master 9.7, as well as on the CD-ROM that comes with the *Teacher's Resource Book.*

Students can also use the randBin command on the TI-83 calculator to simulate an experiment involving a binomial population. The *Teacher's Resource Book*, Chapter 9, page 9, provides more details on using this command in the section *Integrating Technology*.

Section 9.3 uses boxplots to make estimates about *samples* when information about a *population* is known. Students may need some guidance in reading a boxplot and in creating a boxplot. Students should be encouraged to place a ruler vertically on the boxplots to help them read the values.

The instructor may need to explain that the diagram in **Example 1** on page 548 is really composed of several boxplots. There is a single boxplot on page 546.

Suggestions for Assessment

Study Guide questions 2.12 to 2.17 will meet the objectives of Outcome 2.3.

Resources

Mathematics 10, Section 9.3, From Population to Sample, pages 543 - 554

Mathematics 10, Teacher's Resource Book, Chapter 9, pages 9 - 12

Mathematics 10, Independent Study Guide, pages 101 and 102

Outcomes

2.4 Make inferences about populations based on data from samples.

2.4.1 Interpret boxplots to make an inference about the population.

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Notes for Teaching and Learning

In Section 9.4 the procedure of the previous section is reversed. Here, the number of marked items in a sample and the sample size are given. Boxplots are interpreted to make inferences about the population.

The instructor should check the students' understanding of the poll reported on page 555 by having them describe the results in their own words. There are several questions in the *Teacher's Resource Book*, Chapter 9, page 13, which will also help students verbalize their understanding of the poll results.

The instructor should advise the students to check their inferences in **Example a)** on page 556 by calculating $\frac{12}{40}$ as a percent, 30%. Students should note that this is close to the median (middle number) of the percents estimated for the boxplots.

Suggestions for Assessment

Study Guide question 2.18 will meet the objectives of Outcome 2.4.

In the *Teacher's Resource Book*, each chapter section has questions under **Supplementary Examples** and **Assessing the Outcome**.

Questions 1 to 6 on the Self-Test in the *Independent Study Guide* could be used as homework, assignment or test questions.

Carefully chosen questions from Written Test and Multiple Choice Test, Masters 9.3 - 9.6, could be used for assessment.

Resources

Mathematics 10, Section 9.4, From Sample to Population, pages 555 - 559

Mathematics 10, Teacher's Resource Book, Chapter 9, pages 13 - 15 Masters 9.3 - 9.6

Mathematics 10, Independent Study Guide, pages 102 and 103

Appendix

Matrices

Identity Matrix

The *identity element* for real number multiplication is 1. In matrix theory there is an ***Identity Matrix*** which, when multiplied by any other matrix, leaves that matrix unchanged.

Exercises:

1. Using Guess and Check, attempt to generate the 2 × 2 Identity Matrix, I, that when multiplied by $\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ leaves this matrix unchanged.

2. Use the TI-83 to determine the 3 × 3 Identity Matrix.

3. Create any 2 × 2 matrix and name it **A**. Explore the solutions to **AI** and **IA**. Write a few sentences explaining the answers you obtained.

You should notice that the identity matrix **<u>must</u>** be square.

If A = $\begin{bmatrix} 4 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & -9 & 1 \end{bmatrix}$, the identity matrix, I, is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

The identity matrix is a square matrix with 1's on the main diagonal. (top left to bottom right) and all other elements are 0.

Inverse Matrix

Any real number (except 0) has a multiplicative inverse. A number multiplied by its inverse will yield 1, the multiplicative identity. For example, the multiplicative inverse of $\frac{3}{7}$ is $\frac{7}{3}$. We know that $\frac{3}{7} \times \frac{7}{3} = 1$. Similarly, most square matrices have inverses. The inverse of matrix A is denoted by A^{-1}. A matrix multiplied by its inverse will yield the identity matrix.

$$AA^{-1} = I \text{ and } A^{-1}A = I$$

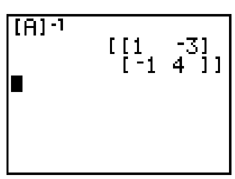
The inverse of a matrix can be calculated by hand and, except for a 2 × 2 matrix, is very time consuming. For this course, we will find the inverse of matrices by using a TI-83 graphing calculator.

Use a TI-83 to get the inverse of 2 × 2 and 3 × 3 or any square matrix.

Example 1:

$A = \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$. When you enter A in your TI-83, you should see the following display:

To find A^{-1}, press: [2nd], [MATRIX], [ENTER],[x^{-1}], [ENTER].
You should see the following display:



Multiply AA^{-1} or $A^{-1}A$. You will get the identity matrix.

Exercises:

4. Use a TI-83 to get the inverse of each of the following. In each case, show that $AA^{-1} = I$ and $A^{-1}A = I$.

a) $A = \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

c) $A = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$ d) $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

e) $A = \begin{bmatrix} 1 & 3 & -5 \\ 2 & 0 & 1 \\ 1 & -2 & 4 \end{bmatrix}$ f) $A = \begin{bmatrix} -1 & 2 & 0 \\ 4 & -3 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

5. In exercises a) - d), determine whether or not $B = A^{-1}$.

a) $A = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$

b) $A = \begin{bmatrix} 3 & -4 \\ 5 & -7 \end{bmatrix}$ $B = \begin{bmatrix} 7 & -4 \\ 5 & -2 \end{bmatrix}$

c) $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -5 & 7 \\ -1 & 3 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -1 & 1 \\ 3 & -2 & -1 \\ 1 & -1 & -1 \end{bmatrix}$

d) $A = \begin{bmatrix} 1 & -1 & 3 \\ 3 & -4 & 8 \\ -2 & 3 & -4 \end{bmatrix}$ $B = \begin{bmatrix} 8 & -5 & -4 \\ 4 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

Using matrices to solve a system of equations

Before we can use matrices to solve a system of equations, we must be able to write the system in matrix form. To do this, we write one matrix containing the coefficients of the variables, one matrix containing the variables, and one matrix containing the constant terms.

Example 2:

Given the following system of equations:

$3r + s + t = 1.1$
$-2r + 2s - 3t = 3.4$
$r + 5s + 2t = 5.3$

The coefficient matrix, A, is $\begin{bmatrix} 3 & 1 & 1 \\ -2 & 2 & -3 \\ 1 & 5 & 2 \end{bmatrix}$.

The variable matrix, X, is $\begin{bmatrix} r \\ s \\ t \end{bmatrix}$

The values matrix, B, is $\begin{bmatrix} 1.1 \\ 3.4 \\ 5.3 \end{bmatrix}$

The matrix form could be written $(A)\ (X) = B$, which gives us ;

$$\begin{bmatrix} 3 & 1 & 1 \\ -2 & 2 & -3 \\ 1 & 5 & 2 \end{bmatrix} \begin{bmatrix} r \\ s \\ t \end{bmatrix} = \begin{bmatrix} 1.1 \\ 3.4 \\ 5.3 \end{bmatrix}$$

Multiply: (A)(X). What is the result?

To write the matrix form of a system of equations, there must be the same number of rows as there are variables. If any variables or equations are missing, they must be filled in with zero coefficients.

Example 3:

The system,
$$2x + 3y = 5$$
$$-2y + z = -1$$
$$x + z = 2$$

when written with all variables included in every equation becomes:

$$2x + 3y + 0z = 5$$
$$0x - 2y + z = -1$$
$$x + 0y + z = 2$$

For this example, A = $\begin{bmatrix} 2 & 3 & 0 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix}$

The matrix form could be written (A) (X) = B, which is:

$$\begin{bmatrix} 2 & 3 & 0 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix}$$

Again, what is the result when you multiply (A)(X)?

Look at the matrix form $AX = B$.

If you multiply both sides by the inverse, A^{-1}, you will get

$(A^{-1})(A)(X) = (A^{-1})(B)$

$(I)(X) = (A^{-1})(B)$

$X = (A^{-1})(B)$

This will give a one column matrix, X, whose elements are the solution to the system.

Solving a Matrix Equation

Before we begin solving matrix equations, think back to solving regular equations such as:

$$\frac{3}{4}x = 6$$

In that case, we multiplied both sides of the equation by the **inverse** of the coefficient which gave:

$$\frac{4}{3}\left(\frac{3}{4}x\right) = \frac{4}{3}(6)$$
$$1x = 8$$
$$x = 8$$

Notice that we wanted to get the coefficient of x to be 1, the **identity** for multiplication.

Now let us apply this idea to matrix equations. Even though the matrix form of a system is introduced to help solve more complex systems, we will start by applying the technique to a simple system in two variables. The technique would be the same regardless of the number of variables.

Example 4:
Let us consider the following system:

$$-2x = y = -1$$
$$4x + y = -5$$

Rewrite the system as: $-2x - y = -1$
$4x + y = -5$

Write the system in matrix form:

$$\begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$$

We now multiply both sides by the **inverse of the coefficient matrix**, just as we multiplied the coefficient and inverse of the coefficient in the equation above.

The inverse of a matrix is written with a superscript of -1, thus the inverse of our coefficient matrix is written as:

$\begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}^{-1}$, and the equation can be written as:

$$\begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -5 \end{bmatrix}$$

Since the two matrices on the left are inverses of each other, their product will give the **identity matrix** and our equation will look like this:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -5 \end{bmatrix}$$

To solve the equation, you need to determine the following inverse: $\begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}^{-1}$

Use your TI-83 calculator, which can calculate the inverse of a matrix.

The inverse matrix will be: $\begin{bmatrix} .5 & .5 \\ -2 & -1 \end{bmatrix}$

You now have the inverse matrix that you desired. Write:

$$\begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} .5 & .5 \\ -2 & -1 \end{bmatrix}$$

Solving the original equation gives:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} .5 & .5 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} (.5 \times -1) + (.5 \times -5) \\ (-2 \times -1) + (-1 \times -5) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$$

So the solution to the original system is $x = {}^{-}3$ and y = 7. You should check this by substituting into the original system.

Example 5:

Given: $x - 2y = 7$
$3x + 4y = 1$,

solve for x and y. Write the system in matrix form first.

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}, \qquad B = \begin{bmatrix} 7 \\ 1 \end{bmatrix}, \qquad X = \begin{bmatrix} x \\ y \end{bmatrix}$$

Write: (A)(X) = B

X = (A^{-1})(B)

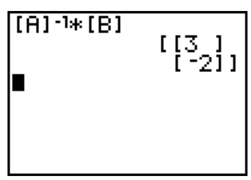
Enter Matrix A.

Enter Matrix B.

Press [2nd] [QUIT] and then the following:

Press [Matrix A], [ENTER],[x^{-1}] [×] [MATRIX] [▾][2:B],[ENTER]

You should see the following window:



The solution is (3, -2).

Exercises:

Solve the following systems using matrices:

6. $3x + 2y = 2$
$4x + 5y = 12$

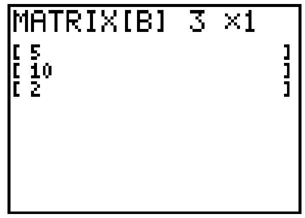
7. $3x + 2y = 19$
$5x - 7y = 5$

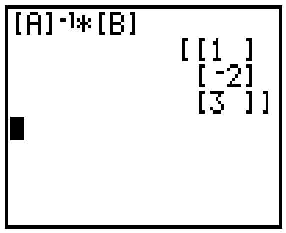
Matrices become more practical as the number of variables increases.

Example 6:

To solve the following system, you will get the windows as shown:

$4x + y + z = 5$
$2x - y + 2z = 10$
$x - 2y - z = 2$





Work through the necessary steps on your TI-83 to solve this system.

Exercises:

Solve the following systems using matrices:

8. $x - 3y - 2z = 9$
   $3x + 2y + 6z = 20$
   $4x - y + 3z = 25$

9. $x + y + z = {}^{-}2$
   $2x - y - z = {}^{-}1$
   $3x + 2y + 4z = {}^{-}15$

10. $3x + 2y - 5z = -9$
    $x - 3y + 4z = 23$
    $2x + y - 3z = -4$