Adult Basic Education

Mathematics

Mathematics 2104A

Solving Linear Systems
Statistics

Study Guide

Prerequisites: Mathematics 1104A, 1104B, 1104C

Credit Value: 1

Text: *Mathematics 10*. Alexander and Kelly; Addison-Wesley, 1998.
Mathematics 11. Alexander and Kelly; Addison-Wesley, 1998.

Required Mathematics Courses
[Degree and Technical Profile/Business-Related College Profile]

Mathematics 1104A
Mathematics 1104B
Mathematics 1104C
Mathematics 2104A
Mathematics 2104B
Mathematics 2104C
Mathematics 3104A
Mathematics 3104B
Mathematics 3104C

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To the Student

I. Introduction to Mathematics 2104A

In this course, you will solve linear systems of equations using addition and subtraction. Using this technique to solve a system of equations with three or more unknowns may be very cumbersome. Instead, you will learn to use matrices for these complex situations. You will write systems of equations in matrix form and, using technology, find the inverse matrix and then solve the system. The material for this topic is found in *Mathematics 11* and Appendix A, at the end of the Study Guide. You will look at linear systems and classify them as consistent (one solution or many solutions) or inconsistent (no solution).

You will also be introduced to inferential statistics and examine various sampling methods and potential sources of bias associated with these methods. You will make inferences about samples based on population data and make inferences about populations based on data from samples.

II. Resources

You will require the following:

- *Addison Wesley Mathematics 10*, Western Canadian edition Textbook
- *Addison Wesley Mathematics 11*, Western Canadian edition Textbook
- Scientific calculator
- graph paper
- Access to a TI-83 Plus graphing calculator (see your instructor) and/or *Graphmatica* or *Winplot* graphing software

Notes concerning the textbook:

Glossary: Knowledge of mathematical terms is essential to understand concepts and correctly interpret questions. Written explanations will be part of the work you submit for evaluation, and appropriate use of vocabulary will be required.

Your text for this course includes a Glossary where definitions for mathematical terms are found. Be sure you understand such definitions and can explain them in your own words. Where appropriate, you should include examples or sketches to support your definitions.

Examples: You are instructed to study carefully the **Examples** in each section and see your instructor if you have any questions. These **Examples** provide full solutions to problems that can be of great use when answering assigned **Exercises**.

To the Student

<u>Notes concerning technology</u>:

It is important that you have a **scientific** calculator for your individual use. Ensure that the calculator used has the word “scientific” on it as there are calculators designed for calculation in other areas such as business or statistics which would not have the functions needed for study in this area. Scientific calculators are sold everywhere and are fairly inexpensive. You should have access to the manual for any calculator that you use. It is a tool that can greatly assist the study of mathematics but, as with any tool, the more efficient its use, the better the progress.

You will require access to some sort of technology in order to meet some of the outcomes in this course. Since technology has become a significant tool in the study of Mathematics, your textbook encourages you to become proficient in its use by providing you with step-by-step exercises that will teach you about the useful functions of the TI-83 Plus Graphing calculator. **See your instructor concerning this**. Please note that a graphing calculator is not essential for success in this course but it is useful.

While graphing calculators and graphing software (*Graphmatica* or *Winplot*) are useful tools, they cannot provide the same understanding that comes from working paper and pencil exercises.

III. <u>Study Guide</u>

This Study Guide is required at all times. It will guide you through the course and you should take care to complete each unit of study in the order given in this Guide. Often, at the beginning of each unit, you will be instructed to see your instructor for **Prerequisites** exercises. Please do not skip this step! It should take only a few minutes for you and your instructor to discover what, if any, prerequisite skills need review.

To be successful, you should read the **References and Notes** first and then, when indicated by the ⧉⧉ symbols, complete the **Work to Submit** problems. Many times you will be directed to see your instructor, and this is vital, especially in a Mathematics course. If you have only a hazy idea about what you just completed, nothing will be gained by continuing on to the next set of problems.

To the Student

Reading for this Unit: In this box, you will find the name of the text, and the chapters, sections and pages used to cover the material for this unit. As a preliminary step, skim the referenced section, looking at the name of the section, and noting each category. Once you have completed this overview, you are ready to begin.

References and Notes	Work to Submit
This left hand column guides you through the material to read from the text. It will also refer to specific **Examples** found in each section. You are directed to study these **Examples** carefully and see your instructor if you have any questions. The **Examples** are important in that they not only explain and demonstrate a concept, but also provide techniques or strategies that can be used in the assigned questions. The symbols ▶▶ direct you to the column on the right which contains the work to complete and submit to your instructor. You will be evaluated on this material. Since the answers to **Discussing the Ideas** and **Communicating the Ideas** are not found in the back of the student text, you **must** have these sections corrected by your instructor **before** going on to the next question. This column will also contain general **Notes** which are intended to give extra information and are not usually specific to any one question.	There are four basic categories included in this column that correspond to the same categories in the sections of the text. They are **Investigate, Discussing the Ideas**, **Exercises**, and **Communicating the Ideas**. **Investigate:** This section looks at the thinking behind new concepts. The answers to its questions are found in the back of the text. **Discussing the Ideas**: This section requires you to write a response which clarifies and demonstrates your understanding of the concepts introduced. The answers to these questions are not in the student text and will be provided when you see your instructor. **Exercises**: This section helps to reinforce your understanding of the concepts introduced. There are three levels of **Exercises**: **A:** direct application of concepts introduced **B:** multi-step problem solving and some real-life situations **C:** problems of a more challenging nature The answers to the **Exercises** questions are found in the back of the text. **Communicating the Ideas:** This section helps confirm your understanding of the lesson of the section. If you can write a response, and explain it clearly to someone else, this means that you have understood the topic. The answers to these questions are not in the student text and will be provided when you see your instructor This column will also contain **Notes** which give information about specific questions.

IV. Recommended Evaluation

Written Notes	10%
Assignments	10%
Test(s)	30%
Final Exam *(entire course)*	50%
	100%

The overall pass mark for the course is 50%.

Unit 1 - Solving Linear Systems

To meet the objectives of this unit, students should complete the following:

Reading for this unit: *Mathematics 11*

Chapter 5: Section 5.2: pages 310 - 318
Section 5.3: pages 319 - 322
Section 5.6: pages 337 - 343

Mathematics 2104A Study Guide: Appendix: pages 17 - 26

References and Notes	Work to Submit
Read **Section 5.2**.	
Answer the following questions.	1.1 Investigate, page 310 Answer questions 1 - 6.
	1.2 **Discussing the Ideas**, page 315 Answer questions 1 - 4.
	1.3 **Exercises**, pages 315 - 318 Answer questions 1 - 3. (*See notes below on these questions*.) Answer questions 5 - 8. (*See note below on questions 7 and 8*.) Answer questions 12 and 19. (*See note below on question 12*.)
	Question 1: You don't have to solve each system to see if $(-1, 1)$ is a solution.
	Questions 2 and **3**: Don't forget to multiply <u>both</u> sides of the equation by a constant.
	Questions 7 and **8**: These exercises are very similar to **Example 4**.
	Question 12: Expand the equations in g) and h) and rearrange the terms in all equations so that it will be easier to add or subtract vertically.

Unit 1 - Solving Linear Systems

References and Notes	Work to Submit
Read **Section 5.3**. Study and work through **Examples 1**, **2** and **3**. Answer the following questions. ▶▶	1.4 **Discussing the Ideas**, page 321 Answer questions 1 - 5. 1.5 **Exercises**, pages 321 and 322 Answer questions 1 - 3. (*See note below on question 1*.) Answer questions 4 and 5. (*See note below on these questions*.) Answer questions 6 and 7. **Question 1**: There is more than one way to answer this question. You could express each equation in the form $y = mx + b$ and look at the m and b values. **Example 3** gives another technique that you could follow. **Questions 4 and 5**: Make sure that you understand the meaning of *consistent* and *inconsistent* before you complete these exercises.
Read **Section 5.6**. **Visualizing**, page 337, gives an explanation on what a linear equation in three variables represents.	1.6 See your instructor for **Prerequisites** exercises.

Unit 1 - Solving Linear Systems

References and Notes	Work to Submit
Answer the following question: ▶▶	1.7 **Investigate**, page 337 Answer questions 1, 2 and 3.
Study **Examples 1, 2** and **3.** Work though all of the given steps. The initial equation in **Example 3** is a quadratic. However, you will note that when values are substituted for *n*, you will have a linear equation with three unknowns.	
Answer the following questions. ▶▶	1.8 **Discussing the Ideas**, page 341 Answer questions 1 - 5. 1.9 **Exercises**, page 341 - 343 Answer questions 1 - 8. (*See note below on question 8*.) Answer questions 9 -11. (*See note below on these questions*.) Answer question 12. (*See note below on question 12*.) **Question 8:** Try this: Write an *expression* $ax + by + cz$ with <u>any</u> non-zero values of a, b, and c that you choose. Substitute the given values of x, y and z in the equation and *simplify*. Write an equation with the *expression* on the left side and the *simplification* on the right side.

References and Notes	Work to Submit
	Questions 9 -11: See your instructor if you have difficulty putting these word problems into equations. **Question 12:** You can use a graphing calculator to graph the function that you find. Use the CALCULATE menu to confirm your answer in part b. Use the window settings $^{-}2 \le X \le 10$ and $^{-}1000 \le Y \le 40\ 000$.
You will use matrices to solve systems of linear equations. You will use the **Appendix** and other resources provided by your instructor to complete this section. You may need to review the work that you did on matrices in *Mathematics 1104C* before you start. You must have access to a TI-83 for <u>all</u> of this topic. Read **Appendix**, page 17. Answer the following questions.	1.10 **Exercises**, page 17 Answer questions 1 - 3. (*See note below on question 3*.) **Question 3**: You should find that AI = IA. However, recall that, in general, multiplication of matrices is <u>not</u> commutative. In other words, given two matrices A and B, AB $\neq$ BA.

Unit 1 - Solving Linear Systems

References and Notes

Read **Appendix**, page 18.

You will practise finding the inverse of square matrices. This is a critical step when using matrices to solve systems of equations.

Answer the following questions.
▶▶

Work to Submit

1.11 **Exercises**, page 19
Answer questions 4 and 5.
(*See note below on question 5*.)

Question 5: In order to determine whether or not $B = A^{-1}$, find AB or BA. If the result is the identity matrix, then you know that they are inverses of each other.
(Or, $B = A^{-1}$ and $A = B^{-1}$)

Read **Appendix**, pages 20 and 21. Carefully study **Examples 2** and **3**. Make sure that you understand how to write a system of equations in matrix form.

On the bottom of pages 20 and 21, you are asked to find (A)(X). When you do this multiplication, the result should be the left hand side of the original system of equations.

Note that you must have the same number of equations as there are variables before you can solve a system of equations.

References and Notes	Work to Submit
Read **Appendix**, pages 22 to 25.	
Carefully study **Examples 4** and **5.**	
Ask your instructor for more explanation if you do not understand the importance of the inverse matrix and its purpose when solving a system of equations.	
Also, see your instructor if you are having difficulty using the TI-83 to work through **Examples 4** and **5.**	
Answer the following questions. ▶▶	1.12 **Exercises**, page 25 Answer questions 6 and 7.
Study **Example 6**, page 26.	
Answer the following questions. ▶▶	1.13 **Exercises**, page 26 Answer questions 8, 9 and 10.

Unit 2 - Statistics

To meet the objectives of this unit, students should complete the following:

Reading for this unit:	*Mathematics 10*		
	Chapter 9:	Section 9.1:	pages 530 - 535
		Mathematics File: pages 536 and 537	
		Section 9.2:	pages 538 - 542
		Mathematics File: page 543	
		Linking Ideas: page 544	
		Section 9.3:	pages 545 - 554
		Section 9.4:	pages 555 - 559

References and Notes	**Work to Submit**
Read **Section 9.1**.	
Answer the following questions. ▶▶	2.1 **Investigate**, page 530 Answer questions 1 - 5.
Read page 531, study **Example** on page 532 and answer the following questions. ▶▶	2.2 Define the following terms: i) *population* ii) *sample* iii) *bias* 2.3 **Discussing the Ideas,** page 533 Answer questions 1 - 4.
See your instructor to have **Discussing the Ideas** corrected before moving on to the **Exercises.**	

References and Notes	Work to Submit
	2.4 **Exercises**, pages 533 and 534 Answer questions 1 - 11. (*See note below on question 6*.) **Question 6:** Include segments of the population that would bias a survey in <u>any</u> direction. 2.5 **Communicating the Ideas**, page 535 2.6 Define the term *random numbers*.
Read **Mathematics File**, 'Using a Graphing Calculator' only. Your instructor will have the program called RANDNUM. You can enter this program on your calculator. Alternatively, you can use the unit-to-unit link cable which comes with the TI-83 calculator to copy this program from one calculator to another. **Mathematics File**, pages 536 and 537, lists four different ways to generate random numbers. You will be using the graphing calculator <u>only</u>, although you should <u>read</u> the other 3 methods.	

References and Notes	Work to Submit
The TI-83 simulates the results of experiments with objects such as spinners, dice or coins. Press MATH, PRB, randInt, to get: randInt (lowest number, highest number, number of trials). This command will generate a list of random integers that are greater than or equal to the lower boundary and less than or equal to the upper boundary. The length of the list corresponds to the number of trials requested. For example, randInt (1, 10, 5) will create a list of 5 random digits ≥ 1, but ≤ 10. To simulate the rolling of a die, use randInt with a lower boundary of 1 and an upper boundary of 6. To simulate the flipping of a coin, set the lower boundary to 0 and the upper to 1, with 0 representing heads and 1 representing tails. Answer the following questions. ▶▶	2.7 **Mathematics Files**, pages 536 and 537 Answer questions 1, 2, 3, 4 and 5.

References and Notes	Work to Submit
Read **Section 9.2**.	2.8 See your instructor for **Prerequisites** problems before completing this section.
Study **Example** on page 539.	
Answer the following questions.	2.9 Describe the following sampling methods: i)) Cluster sample ii) Convenience sample iii) Self-selected sample iv) Simple random sample v) Stratified random sample vi) Systematic sample
	2.10 **Discussing the Ideas**, page 539 Answer questions 1, 2, 3 and 4.
See your instructor about **Discussing the Ideas** before completing the **Exercises.**	
	2.11 **Exercises**, pages 540 - 542 Answer questions 1- 6, 9, 10, 12, 13 and 14. (*See note below on questions 13 and 14*.)
	Questions 13 and 14: Choose one issue from either of those questions and design a survey.

References and Notes	Work to Submit
Before completing **Section 9.3**, read **Mathematics File** on page 543. Do **not** make the sampling box, but read it so that you will have an understanding of its use.	
Read **Linking Ideas**, question 1 on page 544. Make sure that you have the SMPLSIM3 program working on your calculator before you start **Section 9.3**.	
Answer the following question. ▶▶	2.12 **Linking Ideas**, page 544 Answer question 1.
Read **Section 9.3**.	
In this section you will know information about a given population. You will simulate the sampling of this population and put the results in boxplots.	
Answer the following questions. ▶▶	2.13 Define the following terms: i) *binomial outcome* ii) *binomial population* 2.14 **Investigate**, page 545 Answer questions 1, 3, 5 and 6.

References and Notes	Work to Submit
Do not confuse the percents used in the sentence following Step 4 on page 547. The 90% refers to the probability of something happening (in this case, there being between 12 and 17 marked items); while the 70% refers to the percent of marked items in the population.	
Answer the following questions. ▶▶	2.15 **Investigate**, page 547 Answer questions 1, 2, 3 and 4. (*See note below on question 1*.) **Question 1**: Generate 100 samples with 10% of beads marked and 100 samples with 30% of beads marked.
Study **Examples 1** and **2**. Work through the given solutions. If you place a ruler vertically on the boxplots it will be easier to read the values.	
Answer the following questions. ▶▶	2.16 **Discussing the Ideas**, page 549 Answer questions 1, 2 and 3.

References and Notes	Work to Submit
	2.17 **Exercises**, page 549 - 551 Answer questions 1 - 7. (*See note below on these questions*.) Answer questions 9, 10, 11, 14 and 15. **Note**: You must use the appropriate boxplots from pages 552 - 554 to complete these exercises. You should calculate the percent of the sample to check whether your estimate is reasonable.
Read **Section 9.4**. In this section the procedure of **Section 9.3** is reversed. You are given the number of marked items in a sample, the sample size and then you will interpret boxplots to make an inference about the population. On page 556, study **Example** and work through the given solution. You will notice that the interval for the estimated percent in the population narrows as the sample size increases.	

References and Notes	Work to Submit
You can check your inference in **Example a)** by calculating $\frac{12}{40}$ as a percent, 30%. Why is this percent close to the median (middle number) of the percents estimated for the boxplots?	
Answer the following questions. ▶▶	2.18 **Exercises**, page 557 Answer questions 1 - 9 and 12.

Appendix A

Matrices

Identity Matrix

The *identity element* for real number multiplication is 1. In matrix theory there is an ***Identity Matrix*** which, when multiplied by any other matrix, leaves that matrix unchanged.

Exercises:

1. Using Guess and Check, attempt to generate the 2 × 2 Identity Matrix, I, that when multiplied by $\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ leaves this matrix unchanged.

2. Use the TI-83 to determine the 3 × 3 Identity Matrix.

3. Create any 2 × 2 matrix and name it **A**. Explore the solutions to **AI** and **IA**. Write a few sentences explaining the answers you obtained.

You should notice that the identity matrix <u>**must**</u> be square.

If A = $\begin{bmatrix} 4 & 3 & 1 \\ 0 & 1 & 2 \\ 6 & -9 & 1 \end{bmatrix}$, the identity matrix, I, is $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

The identity matrix is a square matrix with 1's on the main diagonal. (top left to bottom right) and all other elements are 0.

<u>Inverse Matrix</u>

Any real number (except 0) has a multiplicative inverse. A number multiplied by its inverse will yield 1, the multiplicative <u>identity</u>. For example, the multiplicative inverse of $\frac{3}{7}$ is $\frac{7}{3}$. We know that $\frac{3}{7} \times \frac{7}{3} = 1$. Similarly, most square matrices have inverses. The inverse of matrix A is denoted by A^{-1}. A matrix multiplied by its inverse will yield the identity matrix.

$$AA^{-1} = I \text{ and } A^{-1}A = I$$

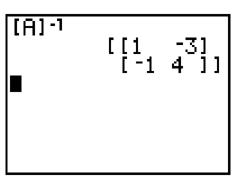
The inverse of a matrix can be calculated by hand and, except for a 2 × 2 matrix, is very time consuming. For this course, we will find the inverse of matrices by using a TI-83 graphing calculator.

<u>Use a TI-83 to get the inverse of 2 × 2 and 3 × 3 or any square matrix</u>.

Example 1:

$A = \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$. When you enter A in your TI-83, you should see the following display:

To find A^{-1}, press: [2nd], [MATRIX], [ENTER],[x^{-1}], [ENTER].
You should see the following display:



Multiply AA^{-1} or $A^{-1}A$. You will get the identity matrix.

Exercises:

4. Use a TI-83 to get the inverse of each of the following. In each case, show that $AA^{-1} = I$ and $A^{-1}A = I$.

a) $A = \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$ b) $A = \begin{bmatrix} 1 & 2 \\ 3 & 7 \end{bmatrix}$

c) $A = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix}$ d) $A = \begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

e) $A = \begin{bmatrix} 1 & 3 & -5 \\ 2 & 0 & 1 \\ 1 & -2 & 4 \end{bmatrix}$ f) $A = \begin{bmatrix} -1 & 2 & 0 \\ 4 & -3 & 1 \\ 2 & 1 & 3 \end{bmatrix}$

5. In exercises a) - d), determine whether or not $B = A^{-1}$.

a) $A = \begin{bmatrix} 5 & -2 \\ -2 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 \\ 2 & 5 \end{bmatrix}$

b) $A = \begin{bmatrix} 3 & -4 \\ 5 & -7 \end{bmatrix}$ $B = \begin{bmatrix} 7 & -4 \\ 5 & -2 \end{bmatrix}$

c) $A = \begin{bmatrix} 1 & -2 & 3 \\ 2 & -5 & 7 \\ -1 & 3 & -5 \end{bmatrix}$ $B = \begin{bmatrix} 4 & -1 & 1 \\ 3 & -2 & -1 \\ 1 & -1 & -1 \end{bmatrix}$

d) $A = \begin{bmatrix} 1 & -1 & 3 \\ 3 & -4 & 8 \\ -2 & 3 & -4 \end{bmatrix}$ $B = \begin{bmatrix} 8 & -5 & -4 \\ 4 & -2 & -1 \\ -1 & 1 & 1 \end{bmatrix}$

Using matrices to solve a system of equations

Before we can use matrices to solve a system of equations, we must be able to write the system in matrix form. To do this, we write one matrix containing the coefficients of the variables, one matrix containing the variables, and one matrix containing the constant terms.

Example 2:

Given the following system of equations:

$3r + s + t = 1.1$
$-2r + 2s - 3t = 3.4$
$r + 5s + 2t = 5.3$

The coefficient matrix, A, is $\begin{bmatrix} 3 & 1 & 1 \\ -2 & 2 & -3 \\ 1 & 5 & 2 \end{bmatrix}$.

The variable matrix, X, is $\begin{bmatrix} r \\ s \\ t \end{bmatrix}$

The values matrix, B, is $\begin{bmatrix} 1.1 \\ 3.4 \\ 5.3 \end{bmatrix}$

The matrix form could be written $(A)\ (X) = B$, which gives us ;

$$\begin{bmatrix} 3 & 1 & 1 \\ -2 & 2 & -3 \\ 1 & 5 & 2 \end{bmatrix} \begin{bmatrix} r \\ s \\ t \end{bmatrix} = \begin{bmatrix} 1.1 \\ 3.4 \\ 5.3 \end{bmatrix}$$

Multiply: (A)(X). What is the result?

To write the matrix form of a system of equations, there must be the same number of rows as there are variables. If any variables or equations are missing, they must be filled in with zero coefficients.

Example 3:

The system,
$$\begin{aligned} 2x + 3y &= 5 \\ -2y + z &= -1 \\ x + z &= 2 \end{aligned}$$

when written with all variables included in every equation becomes:

$$2x + 3y + 0z = 5$$
$$0x - 2y + z = -1$$
$$x + 0y + z = 2$$

For this example, A = $\begin{bmatrix} 2 & 3 & 0 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$, X = $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and B = $\begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix}$

The matrix form could be written (A) (X) = B, which is:

$$\begin{bmatrix} 2 & 3 & 0 \\ 0 & -2 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ -1 \\ 2 \end{bmatrix}$$

Again, what is the result when you multiply (A)(X)?

Look at the matrix form $AX = B$.

If you multiply both sides by the inverse, A^{-1}, you will get

$(A^{-1})\,(A)(X) = (A^{-1})(B)$

$(\,I\,)\,(X) = (A^{-1})(B)$

$X = (A^{-1})(B)$

This will give a one column matrix, X, whose elements are the solution to the system.

Solving a Matrix Equation

Before we begin solving matrix equations, think back to solving regular equations such as:

$$\frac{3}{4}x = 6$$

In that case, we multiplied both sides of the equation by the **inverse** of the coefficient which gave:

$$\begin{aligned} \frac{4}{3}\left(\frac{3}{4}x\right) &= \frac{4}{3}(6) \\ 1x &= 8 \\ x &= 8 \end{aligned}$$

Notice that we wanted to get the coefficient of x to be 1, the **identity** for multiplication.

Now let us apply this idea to matrix equations. Even though the matrix form of a system is introduced to help solve more complex systems, we will start by applying the technique to a simple system in two variables. The technique would be the same regardless of the number of variables.

Example 4:
Let us consider the following system:

$-2x = y - 1$
$4x + y = -5$

Rewrite the system as: $-2x - y = -1$
$4x + y = -5$

Write the system in matrix form:

$$\begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -5 \end{bmatrix}$$

We now multiply both sides by the **inverse of the coefficient matrix**, just as we multiplied the coefficient and inverse of the coefficient in the equation above.

The inverse of a matrix is written with a superscript of -1, thus the inverse of our coefficient matrix is written as:

$$\begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}^{-1},$$

and the equation can be written as:

$$\begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}^{-1}\begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}^{-1}\begin{bmatrix} -1 \\ -5 \end{bmatrix}$$

Since the two matrices on the left are inverses of each other, their product will give the **identity matrix** and our equation will look like this:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}^{-1}\begin{bmatrix} -1 \\ -5 \end{bmatrix}$$

To solve the equation, you need to determine the following inverse: $\begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}^{-1}$

Use your TI-83 calculator, which can calculate the inverse of a matrix.

The inverse matrix will be: $\begin{bmatrix} .5 & .5 \\ -2 & -1 \end{bmatrix}$

You now have the inverse matrix that you desired. Write:

$$\begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} .5 & .5 \\ -2 & -1 \end{bmatrix}$$

Solving the original equation gives:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ 4 & 1 \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} .5 & .5 \\ -2 & -1 \end{bmatrix} \begin{bmatrix} -1 \\ -5 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} (.5 \times -1) + (.5 \times -5) \\ (-2 \times -1) + (-1 \times -5) \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$$

So the solution to the original system is $x = -3$ and y = 7. You should check this by substituting into the original system.

Example 5:

Given: $x - 2y = 7$
$3x + 4y = 1$,

solve for x and y. Write the system in matrix form first.

$$A = \begin{bmatrix} 1 & -2 \\ 3 & 4 \end{bmatrix}, \qquad B = \begin{bmatrix} 7 \\ 1 \end{bmatrix}, \qquad X = \begin{bmatrix} x \\ y \end{bmatrix}$$

Write: (A)(X) = B

X = (A^{-1})(B)

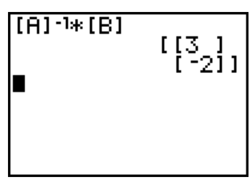
Enter Matrix A.

Enter Matrix B.

Press [2nd] [QUIT] and then the following:

Press [Matrix A], [ENTER],[x^{-1}] [×] [MATRIX] [▾][2:B],[ENTER]

You should see the following window:



The solution is (3, −2).

Exercises:

Solve the following systems using matrices:

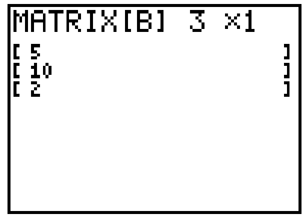
6.	$3x + 2y = 2$ $4x + 5y = 12$	7.	$3x + 2y = 19$ $5x - 7y = 5$

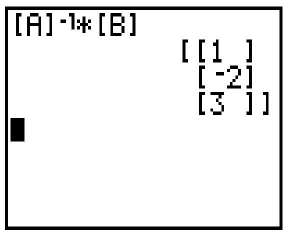
Matrices become more practical as the number of variables increases.

Example 6:

To solve the following system, you will get the windows as shown:

$4x + y + z = 5$
$2x - y + 2z = 10$
$x - 2y - z = 2$





Work through the necessary steps on your TI-83 to solve this system.

Exercises:

Solve the following systems using matrices:

8. $x - 3y - 2z = 9$
   $3x + 2y + 6z = 20$
   $4x - y + 3z = 25$

9. $x + y + z = -2$
   $2x - y - z = -1$
   $3x + 2y + 4z = -15$

10. $3x + 2y - 5z = -9$
    $x - 3y + 4z = 23$
    $2x + y - 3z = -4$