Adult Basic Education

Mathematics

Mathematics 2104B

Rational Expressions, Trigonometric Applications, and Functions

Curriculum Guide

Prerequisites: Mathematics 1104A, 1104B, 1104C
Mathematics 2104A

Credit Value: 1

Required Mathematics Courses
[Degree and Technical Profile/ Business-Related College Profile]

Mathematics 1104A
Mathematics 1104B
Mathematics 1104C
Mathematics 2104A
Mathematics 2104B
Mathematics 2104C
Mathematics 3104A
Mathematics 3104B
Mathematics 3104C

Table of Contents

To the Instructor

I. Introduction to Mathematics 2104B

In the first unit, *Rational Expressions*, students will evaluate and simplify rational expressions, and identify restrictions on values for variables. Students will multiply, divide, add and subtract rational expressions which have monomial, binomial or trinomial denominators.

In the second unit, the definitions for the sine and cosine ratios are developed in terms of the coordinates (x, y). This definition is more general than the right-triangle definition developed in *Mathematics 1104B* (Sections 8.1 - 8.3, *Mathematics 10*). Students will apply the Sine Law and Cosine Law to solve problems involving unknown sides and angles of triangles.

The final unit covers the concepts of transformations of functions which are revisited throughout this course and all remaining courses in Academic Math. Students will study vertical and horizontal translations and stretches of graphs of functions. Graphs which are reflected in the x-axis and y-axis are introduced. Finally, students will examine what happens when the various transformations are combined.

II. Prerequisites

To be successful in this course, students should be able to find values of a variable that make an expression equal to zero; multiply, divide, add and subtract rational numbers; and find least common multiples.

Students must be able to determine the sine and cosine of acute angles and solve simple trigonometric ratios.

Finally, students should be able to recognize and graph many different type of functions. They must be able to substitute variables into functions, solve for y in a given function, factor polynomials and find the zeros of a function written in factored form.

III. Textbook

Most of the concepts are introduced, developed and explained in the **Examples**. The instructor must insist that students carefully study and understand each **Example** before moving on to the **Exercises**. In the Study Guide, students are directed to see the instructor if there are any difficulties.

To the Instructor

There are four basic categories included in each section of the textbook which require the student to complete questions:

1. Investigate
2. Discussing the Ideas
3. Exercises
4. Communicating the Ideas

Investigate: This section looks at the thinking behind new concepts. The answers to its questions are found in the back of the text.

Discussing the Ideas: This section requires the student to write a response which clarifies and demonstrates understanding of the concepts introduced. The answers to these questions are not in the student text but are in the *Teacher's Resource Book*. Therefore, in the Study Guide, the student is directed to see the instructor for correction. This will offer the instructor some perspective on the extent of the student's understanding. If necessary, reinforcement or remedial work can be introduced. Students should not be given the answer key for this section as the opportunity to assess the student's understanding is then lost.

Exercises: This section helps the student reinforce understanding of the concepts introduced. There are three levels of **Exercises**:

- **A:** direct application of concepts introduced;
- **B:** multi-step problem solving and some real-life situations;
- **C:** problems of a more challenging nature.

The answers to the **Exercises** questions are found in the back of the text.

Communicating the Ideas: This section helps confirm the student's understanding of a particular lesson by requiring a clearly written explanation. The answers to **Communicating the Ideas** are not in the student text, but are in the *Teacher's Resource Book*. In the Study Guide students are asked to see the instructor for correction.

IV. Technology

It is important that students have a **scientific** calculator and its manual for their individual use. Ensure that the calculator used has the word "scientific" on it as there are calculators designed for calculation in other areas such as business or statistics which would not have the functions needed for study in this area.

To the Instructor

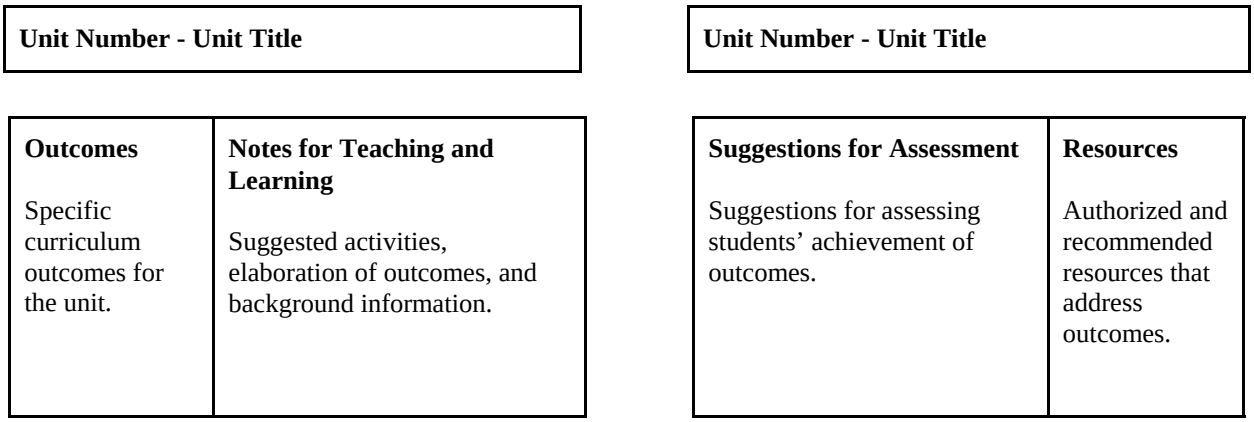
A graphing calculator should be **available** to the students since the text provides many opportunities for its use. The *Teacher's Resource Book* suggests many occasions to utilize a graphing calculator. These suggestions are outlined where there is the heading *Integrating Technology*. In the Study Guide, students are directed to see the instructor when a graphing calculator is required. The *Teacher's Resource Book* contains a module called **Graphing Calculator Handbook** which will help the instructor and student get acquainted with some of the main features of the TI-83 Plus graphing calculator.

Graphing software such as *Graphmatica* or *Winplot* can also be used if the students don't have access to a graphing calculator but do have access to a computer. The textbook doesn't offer the same guidance for graphing with these tools as it does for a graphing calculator, but each software program does have a HELP feature to answer questions.

V. Curriculum Guides

Each new ABE Mathematics course has a Curriculum Guide for the instructor and a Study Guide for the student. The Curriculum Guide includes the specific curriculum outcomes for the course. Suggestions for teaching, learning, and assessment are provided to support student achievement of the outcomes. Each course is divided into units. Each unit comprises a **two-page layout of four columns** as illustrated in the figure below. In some cases the four-column spread continues to the next two-page layout.

Curriculum Guide Organization:
The Two-Page, Four-Column Spread



VI. Study Guides

The Study Guide provides the student with the name of the text(s) required for the course and specifies the sections and pages that the student will need to refer to in order to complete the required work for the course. It guides the student through the course by assigning relevant reading and providing questions and/or assigning questions from the text or some other resource. Sometimes it also provides important points for students to note. (See the *To the Student* section of the Study Guide for a more detailed explanation of the use of the Study Guides.) The Study Guides are designed to give students some degree of independence in their work. Instructors should note, however, that there is much material in the Curriculum Guides in the *Notes for Teaching and Learning* and *Suggestions for Assessment* columns that is not included in the Study Guide and instructors will need to review this information and decide how to include it.

VII. Resources

Essential Resources

Addison Wesley Mathematics 10 (Western Canadian edition)
ISBN:0-201-34619-2

Addison Wesley Mathematics 12 (Western Canadian edition)
ISBN:0-201-34629-X

Mathematics 10 Teacher's Resource Book (Western Canadian edition)
ISBN: 0-201-34621-4

Mathematics 12 Teacher's Resource Book (Western Canadian edition)
ISBN: 0-201-34631-1

Math 2104B Study Guide

Recommended Resources

*Mathematics 10 Independent Study Guide (*Western Canadian edition)
ISBN: 0-201-34620-6

*Mathematics 12 Independent Study Guide (*Western Canadian edition)
ISBN: 0-201-34630-3

Center for Distance Learning and Innovation: http://www.cdli.ca

Winplot: http://math.exeter.edu/rparris/winplot.html
(Free graphing software)

Graphmatica (Evaluation software available on CD-ROM contained in *Teacher's Resource Book*)

CD Rom accompanying *Teacher's Resource Book*
This CD contains selected solutions from the text and self test solutions from the *Independent Study Guide*.

Other Resources

Math Links: http://mathforum.org

http://www.purplemath.com

http://www.sosmath.com/index.html

http://www.math.com/

http://spot.pcc.edu/~ssimonds/winplot
(Free videos concerning Winplot)

http://www.pearsoned.ca/school/math/math/

VIII. Recommended Evaluation

Written Notes	10%
Assignments	10%
Test(s)	30%
Final Exam *(entire course)*	50%
	100%

The overall pass mark for the course is 50%.

Rational Expressions, Trigonometric Applications, and Functions

Unit 1 - Rational Expressions

Outcomes

1.1 Demonstrate an understanding of the relationship between operations on fractions and rational algebraic expressions.

1.1.1 Evaluate and simplify rational expressions and state restrictions on values for variables.

1.1.2 Simplify products and quotients of rational expressions.

1.1.3 Add and subtract rational expressions with monomial denominators.

1.1.4 Add and subtract rational expressions with binomial and trinomial denominators.

Notes for Teaching and Learning

Note: Assign review questions from **Prerequisites**, *Teacher's Resource Book*, Chapter 7, pages 4, 6, 9, 11 and 14.

Since many trigonometric identities involve working with fractional form, it is important that students have strong skills with combining, multiplying, dividing and simplifying rational expressions. If students have a TI-83 graphing calculator, the instructor should use Masters 7.1 and 7.3 in the *Teacher's Resource Book* to explore nonpermissible values of x in a rational expression.

Students should see that simplifying rational expressions is like simplifying fractions. Again, if students have a TI-83 graphing calculator, they could use it to verify that an expression was simplified correctly. The original and simplified expressions must be entered as Y1 and Y2. The table of values displayed for each should have the same y-value for each x-value. Similarly, if the graphs of the two functions are displayed, they should be identical. If there is a hole in one of the graphs, it will occur when the value of the variable is nonpermissible.

Students may need to be reminded that when working a problem like $\frac{a}{b} \div \frac{c}{d}$, a zero (0) in either the numerator or the denominator of the divisor will result in an undefined value. In this example, b and d cannot be 0 because they are denominators. c cannot be 0 because that would mean that $\frac{a}{b}$ would be divided by $\frac{0}{d}$, or 0. The instructor should impress upon the students that, when adding or subtracting rational expressions, two basic concepts should be used: 1) any number or expression can be multiplied by 1; and 2) 1 can be written in many ways. Students need to understand that, when adding and subtracting rational expressions, they need to expand and simplify the numerator, but not the denominator, in the last couple of steps of the problem.

Unit 1 - Rational Expressions

Suggestions for Assessment

Study Guide questions 1.1 to 1.13 will meet the objectives of Outcome 1.1.

Each section in the *Teacher's Resource Book* contains extra problems in **Supplementary Examples** and **Assessing the Outcome** which could be used for practice or homework.

The Written Test and Multiple Choice Test in Masters 7.5 to 7.8 could be used for an assessment of this topic.

The *Independent Study Guide* has many extra problems.

Resources

Mathematics 10,
Sections 7.1 - 7.5,
pages 400 - 449

Mathematics 10,
Teacher's Resource Book,
Chapter 7, pages 4 - 15

Masters 7.1, 7.2 and 7.5 - 7.8

Independent Study Guide,
Chapter 7, pages 81- 84

Unit 2 - Trigonometry and Its Application

Outcomes

2.1 Find the area of a triangle when given two sides and the included angle.

Notes for Teaching and Learning

The first application in this section involves finding the area of a triangle which is not a right triangle. The textbook does not cover this topic, but Appendix A helps students develop the formula for the area of a triangle ABC; Area = ½ *ab* sin*C*.

The instructor should ensure that students have mastered this topic and give extra guidance and practice problems if necessary.

Unit 2 - Trigonometry and Its Applications

Suggestions for Assessment

Study Guide questions 2.1 and 2.2 will meet the objectives of Outcome 2.1.

Resources

Appendix, pages 27 and 28
www.cdli.ca, Unit 06, Section 01, Lesson 02

Unit 2 - Trigonometry and Its Application

Outcomes

2.2 Develop the definition for the sine and cosine ratios for angles from 0° to 180°.

Notes for Teaching and Learning

Linking Ideas on page 489 of *Mathematics 10,* focuses on patterns in the sine and cosine of obtuse and acute angles. The formal definition is developed in **Section 8.5** which follows.

The trigonometric ratios of acute angles were previously defined as ratios of the sides of a right triangle. These definitions do not work when looking at obtuse angles.

Section 8.5 develops definitions for the sine and cosine ratio in terms of the *x* and *y*-coordinates on the unit circle.

Suggestions for Assessment

Study Guide questions 2.3 to 2.7 will meet the objectives of Outcome 2.2.

Resources

Mathematics 10, Section 8.5, Defining the Sine and Cosine of an Obtuse Angle, pages 489 - 497

Mathematics 10, Teacher's Resource Book, Chapter 8, pages 17 - 19

Mathematics 10, Independent Study Guide, Chapter 8, pages 85 - 94

Unit 2 - Trigonometry and Its Applications

Outcomes

2.3 Use trigonometric ratios to solve triangles that are not right-angled.

Notes on Teaching and Learning

In this section, students will use trigonometric ratios to solve triangles that are not right-angled. Emphasis is placed on visualizing the 'givens' and planning a problem-solving strategy. Students should be told to draw a sketch for every problem, even the seemingly easy problems. The '**Think**' paragraphs in the blue boxes in each of the **Examples** provide excellent problem-solving strategies for students to follow.

Suggestions for Assessment

Study Guide questions 2.8 to 2.10 will meet the objectives of Outcome 2.3.

Resources

Mathematics 10, Section 8.6, Solving Triangles That Are Not Right-Angled, pages 499 - 504

Mathematics 10, Teacher's Resource Book, Chapter 8, pages 20 and 21

Mathematics 10, Independent Study Guide, Chapter 8, pages 85 - 94

Unit 2 - Trigonometry and Its Applications

Outcomes

2.4 Apply the Sine Law to solve problems involving unknown sides and angles.

Notes for Teaching and Learning

One example from **Section 8.6** is examined and then a general rule, the Sine Law, is formulated which gives a shortcut for the process. The textbook does not provide a derivation for the Sine Law, but if some students are interested, it would be an easy derivation for them to work on.

The instructor could suggest the following to the students. Given Δ ABC, the area can be written as:

- Area = ½ bc sin A
- Area = ½ ac sin B
- Area = ½ ab sin C

Use these three relationships to derive the Sine Laws.

The instructor should point out to students that it doesn't matter whether the sines are in the numerator or denominator.

Students should know the two situations in which the Sine Law applies. Basically, you must know one side and its opposite angle and one other side or angle.

The *ambiguous case* might need more explanation since the textbook just mentions it. The ambiguous case can only occur when solving a triangle in which two sides and the non-included angle are known. In that case, two different triangles satisfy the given conditions. Students must be reminded to check the two angles found to determine when both make sense or when one answer has to be discarded.

Unit 2 - Trigonometry and Its Applications

Suggestions for Assessment

Study Guide questions 2.11 to 2.15 will meet the objectives of Outcome 2.4.

Resources

Mathematics 10, Section 8.7, The Sine Law, pages 505 - 512

Mathematics 10, Teacher's Resource Book, Chapter 8, pages 22 and 23

Mathematics 10, Independent Study Guide, Chapter 8, pages 85 - 94

www.cdli.ca, Unit 06, Section 02, Lesson 01, Lesson 03

Unit 2 - Trigonometry and Its Applications

Outcomes

2.5 Apply the Cosine Law to solve problems involving unknown sides and angles.

Notes for Teaching and Learning

This section develops the Cosine Law in a manner similar to **Section 8.7** where the Sine Law was developed. Again, the textbook uses the example from a previous section, looks at the steps in the solution, and tries to formulate a general rule. Students should understand that this is not a proof or derivation of the Cosine Law.

The instructor should ensure that students know to use the Cosine Law if
a) two sides and the included angle are known or
b) all three sides are known.

Once the Cosine Law is used to find another angle or side, it is easier to use the Sine Law to solve for the remainder of the triangle.

Suggestions for Assessment

Study Guide questions 2.16 to 2.18 will meet the objectives of Outcome 2.5.

Questions for assessment can be chosen from the Written Test and Multiple Choice Test from Masters 8.3 to 8.6 in the *Teacher's Resource Book*.

Resources

Mathematics 10, Section 8.8, The Cosine Law, pages 513 - 521

Mathematics 10, Teacher's Resource Book, Chapter 8, pages 24 and 25
Masters 8.3 to 8.6

Mathematics 10, Independent Study Guide, Chapter 8, pages 85 - 94

www.cdli.ca, Unit 06, Section 02, Lesson 02

Unit 3 - A Functions Toolkit

Outcomes

3.1 Recognize and graph many different types of functions and consider real situations that can be modeled by these functions.

Notes for Teaching and Learning

The concepts of transformations covered in this chapter will be used throughout this course.

This chapter, however, covers more transformations than are required in Math 2104. Particular sections have been carefully selected to meet the required objectives. The instructor may have to give students extra guidance throughout this unit.

In particular, **Examples** and **Exercises** which include the following are to be omitted:
i) reflection in *y*-axis
ii) reflection in the line $y = x$

A graphing tool would be very advantageous in this unit. If a TI-83 graphing calculator is not available to students, the instructor should install WINPLOT on the classroom computer. The software for WINPLOT is on the CD-ROM which is in the *Teacher's Resource Book.*

Note: The instructor should assign **Prerequisites** exercises on page 5 in *Teacher's Resource Book*.

Examples 1 and **2,** on pages 8 and 9 of *Mathematics 12*, introduce the semicircle and absolute value functions. Students should be familiar with linear, quadratic and square root functions.

When students are completing question 5 in the **Exercises**, they can use the set bracket on the TI-83 instead of entering each function. Students should enter the following:

$Y1 = \sqrt{\left(\{16, 9, 4, 1\} - x^2\right)}$.

Unit 3 - A Functions Toolkit

Suggestions for Assessment

Study Guide questions 3.1 and 3.2 will meet the objectives of Outcome 3.1.

Resources

Mathematics 12, Section 1.1, Some Functions and Their Graphs, pages 2 - 12

Mathematics 12, Teacher's Resource Book, Chapter 1, pages 5 - 7

Mathematics 12, Independent Study Guide, Chapter 1, pages 20- 24

Outcomes

3.2 Relate changes to the equation of a function to vertical and horizontal translations of its graph.

Notes for Teaching and Learning

This section deals with vertical and horizontal translations of graphs of functions.

Note: The questions from **Prerequisites** on page 9 should be assigned to the students.

Since this section builds on work done in *Mathematics 11* with translations of the quadratic function, the instructor should assign some questions from **Section 2.3** in *Mathematics 11*.

After completing **Investigate**, and exploring how $y = f(x - k)$ is related to $y = f(x)$, students should have discovered that when k is subtracted from x, there is a *horizontal* translation.

Similarly, when looking at how $y - k = f(x)$ is related to $f(x)$, they should see that when k is subtracted from y, there is a *vertical* translation.

Suggestions for Assessment

Study Guide questions 3.3 and 3.4 will meet the objectives of Outcome 3.2.

Resources

Mathematics 12,
Section 1.2, Translating Graphs of Functions, pages 16 - 22

Mathematics 12, Teacher's Resource Book, Chapter 1, pages 9 - 12

Mathematics 12, Independent Study Guide, Chapter 1, pages 20- 24

Unit 3 - A Functions Toolkit

Outcomes

3.3 Relate changes to the equation of a function to reflections of its graph in the *x*-axis.

Notes for Teaching and Learning

This section deals with three reflections of graphs of functions. However, for this course, students should study only the material and exercises related the reflection in the *x*-axis (vertical reflections).

The three types of reflection are all together in this section. Therefore, students may need some extra guidance in what problems and material are relevant to this course.

Note: Assign **Prerequisites** exercises on page 12 of the *Teacher's Resource Book.*

Suggestions for Assessment

Study Guide questions 3.5 and 3.6 will meet the objectives of Outcome 3.3.

Resources

Mathematics 12, Section 1.3, Reflecting Graphs of Functions, pages 25 - 33

Mathematics 12, Teacher's Resource Book, Chapter 1, pages 12 - 16

Mathematics 12, Independent Study Guide, Chapter 1, pages 20- 24

Unit 3 - A Functions Toolkit

Outcomes

3.4 Relate changes to the equation of a function to vertical and horizontal stretches of its graph.

Notes for Teaching and Learning

This section introduces vertical and horizontal stretches.

Students need a graphing tool to complete the **Investigate** on page 35. If a TI-83 graphing calculator is not available, *Graphmatica* software on the classroom computer will work nicely.

Suggestions for Assessment

Study Guide questions 3.7 and 3.8 will meet the objectives of Outcome 3.4.

Resources

Mathematics 12,
Section 1.4, Stretching Graphs of Functions,
pages 35 - 42

Mathematics 12,
Teacher's Resource Book,
Chapter 1, pages 16 - 19

Mathematics 12,
Independent Study Guide,
Chapter 1, pages 20- 24

Outcomes

3.5 Relate several changes to the equation of a function to translations and vertical and horizontal expansions or compressions.

Notes for Teaching and Learning

The instructor should remind students that when translations are combined with expansions and compressions, the order in which they are applied gives different results. Unless otherwise stated, always apply the expansions or compressions before the translations.

Suggestions for Assessment

Study Guide questions 3.9 and 3.10 will meet the objectives of Outcome 3.5.

In the *Teacher's Resource Book*, each section contains extra problems (with answers) in **Supplementary Examples** and **Assessing the Outcome**.

Questions for assessment can be chosen from the Written Test and Multiple Choice Test on Masters 1.4 to 1.11 in the *Teacher's Resource Book*.

Resources

Mathematics 12,
Section 1.5,
Combining Translations and Expansions or Compressions,
pages 45 - 47

Mathematics 12,
Teacher's Resource Book,
Chapter 1, pages 19 - 22
Masters 1.4 - 1.11

Mathematics 12,
Independent Study Guide,
Chapter 1, pages 20- 24

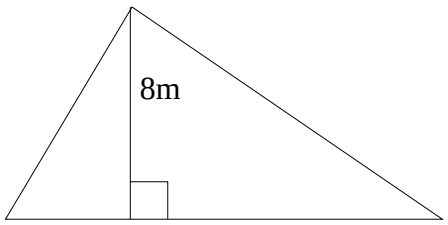
Appendix

Area of a Triangle

The formula to find the area, *A*, of a triangle is ½ *bh* where *b* is the base and *h* is the height which is perpendicular to the base.

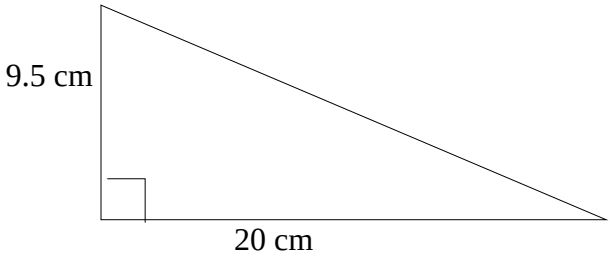
Examples:

1.



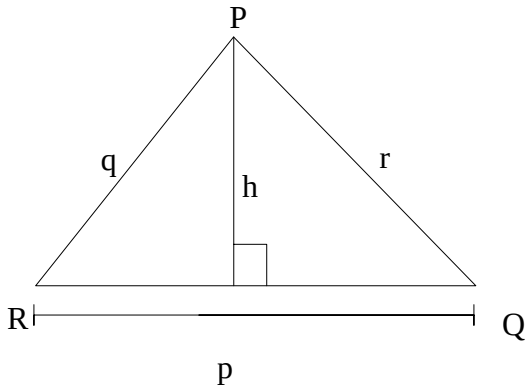
$A = \frac{1}{2} bh$
$A = \frac{1}{2} (10) (8)$
$A = 40 \text{ m}^2$

2.



$A = \frac{1}{2} bh$
$A = \frac{1}{2} (20) (9.5)$
$A = 95 \text{ cm}^2$

3. Given the following triangle, how do you find the area of triangle PQR?



We know that $A = \frac{1}{2} bh$

Since $\sin R = \frac{h}{q}$, solve for h and you will get $h = (\sin R)(q)$.

The base, b, is called p in this example.

Substitute in the formula: $A = \frac{1}{2} bh$ and get

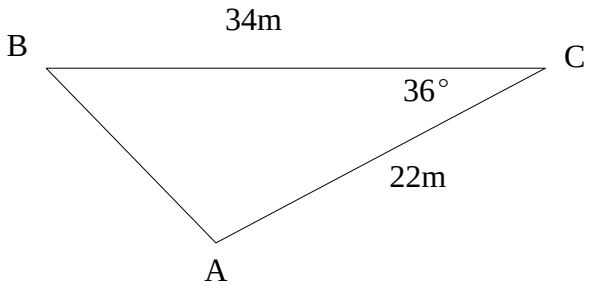
$A = \frac{1}{2} (p)(\sin R)(q) = \frac{1}{2} (p)(q)(\sin R)$

If you are given any two sides and the *included* angle, you can use this formula to find the area.

Given Δ ABC, the area can be written as:

- Area = $\frac{1}{2} bc \sin A$
- Area = $\frac{1}{2} ac \sin B$
- Area = $\frac{1}{2} ab \sin C$

Example:



Area = $\frac{1}{2} ab \sin C$
Area = $\frac{1}{2}$ (34) (22) (sin 36°)
Area = 219.83 m^2