Adult Basic Education

**Mathematics**

# Mathematics 2104B

## Rational Expressions, Trigonometric Applications, and Functions

# Study Guide

**Prerequisites:** Mathematics 1104A, 1104B, 1104C
Mathematics 2104A

**Credit Value**: 1

**Texts:** *Mathematics 10.* Alexander and Kelly; Addison - Wesley, 1998.
*Mathematics 12*. Alexander and Kelly; Addison - Wesley, 1999.

**Required Mathematics Courses**
**[Degree and Technical Profile/Business-Related College Profile]**

Mathematics 1104A
Mathematics 1104B
Mathematics 1104C
Mathematics 2104A
**Mathematics 2104B**
Mathematics 2104C
Mathematics 3104A
Mathematics 3104B
Mathematics 3104C

## Table of Contents

# To the Student

## I. Introduction to Mathematics 2104B

In the first unit, *Rational Expressions*, you will evaluate and simplify rational expressions, and identify restrictions on values for variables. You will multiply, divide, add and subtract rational expressions which have monomial, binomial or trinomial denominators.

In the second unit, the definitions for the sine and cosine ratios are developed in terms of the coordinates ($x$, $y$). This definition is more general than the right-triangle definition developed in *Mathematics 1104B* (Sections 8.1 - 8.3, *Mathematics 10*). You will apply the Sine Law and Cosine Law to solve problems involving unknown sides and angles of triangles.

The final unit covers the concepts of transformations of functions which are revisited throughout this course and all remaining courses in Academic Math. You will study vertical and horizontal translations and stretches of graphs of functions. Graphs which are reflected in the $x$-axis and $y$-axis are introduced. Finally, you will examine what happens when the various transformations are combined.

## II. Resources

You will require the following:

- *Addison Wesley Mathematics 10*, Western Canadian edition Textbook
- *Addison Wesley Mathematics 12*, Western Canadian edition Textbook
- Scientific calculator
- graph paper
- Access to a TI-83 Plus graphing calculator (see your instructor) and/or *Graphmatica* or *Winplot* graphing software

**Notes concerning the textbook:**

**Glossary**: Knowledge of mathematical terms is essential to understand concepts and correctly interpret questions. Written explanations will be part of the work you submit for evaluation, and appropriate use of vocabulary will be required.

Your text for this course includes a Glossary where definitions for mathematical terms are found. Be sure you understand such definitions and can explain them in your own words. Where appropriate, you should include examples or sketches to support your definitions.

## To the Student

**Examples**: You are instructed to study carefully the **Examples** in each section and see your instructor if you have any questions. These **Examples** provide full solutions to problems that can be of great use when answering assigned **Exercises**.

**Notes concerning technology**:

It is important that you have a **scientific** calculator for your individual use. Ensure that the calculator used has the word "scientific" on it as there are calculators designed for calculation in other areas such as business or statistics which would not have the functions needed for study in this area. Scientific calculators are sold everywhere and are fairly inexpensive. You should have access to the manual for any calculator that you use. It is a tool that can greatly assist the study of mathematics but, as with any tool, the more efficient its use, the better the progress.

You will require access to some sort of technology in order to meet some of the outcomes in this course. Since technology has become a significant tool in the study of Mathematics, your textbook encourages you to become proficient in its use by providing you with step-by-step exercises that will teach you about the useful functions of the TI-83 Plus Graphing calculator. **See your instructor concerning this**. Please note that a graphing calculator is not essential for success in this course but it is useful.

While graphing calculators and graphing software (*Graphmatica* or *Winplot*) are useful tools, they cannot provide the same understanding that comes from working paper and pencil exercises.

## III. Study Guide

**This Study Guide is required at all times.** It will guide you through the course and you should take care to complete each unit of study in the order given in this Guide. Often, at the beginning of each unit, you will be instructed to see your instructor for **Prerequisites** exercises. Please do not skip this step! It should take only a few minutes for you and your instructor to discover what, if any, prerequisite skills need review.

## To the Student

To be successful, you should read the **References and Notes** first and then, when indicated by the ⏯⏯ symbols, complete the **Work to Submit** problems. Many times you will be directed to see your instructor, and this is vital, especially in a Mathematics course. If you have only a hazy idea about what you just completed, nothing will be gained by continuing on to the next set of problems.

**Reading for this Unit:** In this box, you will find the name of the text, and the chapters, sections and pages used to cover the material for this unit. As a preliminary step, skim the referenced section, looking at the name of the section, and noting each category. Once you have completed this overview, you are ready to begin.

| **References and Notes** | **Work to Submit** |
|---|---|
| This left hand column guides you through the material to read from the text.<br><br>It will also refer to specific **Examples** found in each section. You are directed to study these **Examples** carefully and see your instructor if you have any questions. The **Examples** are important in that they not only explain and demonstrate a concept, but also provide techniques or strategies that can be used in the assigned questions.<br><br>The symbols ⏯⏯ direct you to the column on the right which contains the work to complete and submit to your instructor. You will be evaluated on this material.<br><br>Since the answers to **Discussing the Ideas** and **Communicating the Ideas** are not found in the back of the student text, you **must** have these sections corrected by your instructor **before** going on to the next question.<br><br>This column will also contain general **Notes** which are intended to give extra information and are not usually specific to any one question. | There are four basic categories included in this column that correspond to the same categories in the sections of the text. They are **Investigate, Discussing the Ideas**, **Exercises**, and **Communicating the Ideas**.<br><br>**Investigate:** This section looks at the thinking behind new concepts. The answers to its questions are found in the back of the text.<br><br>**Discussing the Ideas**: This section requires you to write a response which clarifies and demonstrates your understanding of the concepts introduced. The answers to these questions are not in the student text and will be provided when you see your instructor.<br><br>**Exercises**: This section helps to reinforce your understanding of the concepts introduced. There are three levels of **Exercises**:<br>**A:** direct application of concepts introduced<br>**B:** multi-step problem solving and some real-life situations<br>**C:** problems of a more challenging nature<br>The answers to the **Exercises** questions are found in the back of the text.<br><br>**Communicating the Ideas:** This section helps confirm your understanding of the lesson of the section. If you can write a response, and explain it clearly to someone else, this means that you have understood the topic. The answers to these questions are not in the student text and will be provided when you see your instructor<br><br>This column will also contain **Notes** which give information about specific questions. |

## IV. Recommended Evaluation

| | |
|---|---|
| Written Notes | 10% |
| Assignments | 10% |
| Test(s) | 30% |
| Final Exam *(entire course)* | 50% |
| | 100% |

The overall pass mark for the course is 50%.

## Unit 1 - Rational Expressions

**To fulfill the objectives of this unit, students should complete the following:**

**Reading for this unit**: *Mathematics 10*

| | | |
|---|---|---|
| Chapter 7: | Section 7.1: | pages 400 - 404 |
| | Section 7.2: | pages 405 - 410 |
| | Section 7.3: | pages 411 - 415 |
| | Section 7.4: | pages 418 - 423 |
| | Section 7.5: | pages 426 - 430 |

| References and Notes | Work to Submit |
|---|---|
| Read **Section 7.1**. | |
| Study **Examples 1** and **2**. Make sure that you understand how the nonpermissible values were determined in each problem in **Example 2**. | |
| Answer the following questions. ▶▶ | 1.1 Define the term *rational expression*. Give an example.<br>1.2 When is a *rational expression* not defined?<br>1.3 **Discussing the Ideas**, page 402<br>1.4 **Exercises**, pages 402 - 404<br>Answer questions 1 - 7 and 11 - 16. |
| Read **Section 7.2**. | |
| Answer the following questions. ▶▶ | 1.5 **Investigate,** page 405<br>Answer questions 1 - 5. |
| Carefully read and work the problems in **Examples 1** and **2** before completing **Discussing the Ideas**. | 1.6 **Discussing the Ideas**, page 407<br>Answer questions 1 - 5. |

## Unit 1 - Rational Expressions

| References and Notes | Work to Submit |
|---|---|
| | 1.7 **Exercises**, pages 407 - 409<br>Answer questions 1 - 4.<br>(*See note below on question 4.*)<br><br>Answer questions 7, 8, 10, 11 and 12.<br><br>**Question 4**: Make sure that you identify any non-permissible values before you eliminate common factors. |
| Read **Section 7.3**.<br><br>Answer the following questions. ▶▶<br><br>Carefully read and work the problems in **Examples 1, 2** and **3** before completing **Discussing the Ideas**.<br><br>Notice, in **Example 3b**, that if the numerator or the denominator of the divisor is 0, the result will be undefined. | 1.8 **Investigate,** page 411<br>Answer questions 1 - 4.<br><br>1.9 **Discussing the Ideas**, page 413<br>Answer questions 1 - 3.<br><br>1.10 **Exercises**, pages 413 - 415<br>Answer questions 1 - 5.<br>(*See note below on questions 3 and 5.*)<br><br>Answer questions 7, 10, 11, 12 and 15.<br><br>**Question 3 and 5**: Don't forget to look for non-permissible values in the numerator in the divisor.<br><br>For example: 3a) $\frac{5}{8} \div \frac{3b}{4a}$ can be written $\frac{5/8}{3b/4a}$.<br><br>Therefore, the denominator becomes 0 when $b = 0$ and becomes undefined when $a = 0$. |

## Unit 1 - Rational Expressions

| References and Notes | Work to Submit |
|---|---|
| Read **Section 7.4**.<br><br>Answer the following questions. ▶▶<br><br>Carefully study **Examples 1, 2** and **3** and work through the calculations.<br><br>When studying this section you should keep the following two basic concepts in mind:<br><br>1) Any number or expression can be multiplied by 1 without changing its value.<br><br>2) 1 can be written in different ways.<br><br>Note that to simplify a complex fraction, the numerator and denominator should be multiplied by the common denominator of the individual fractions.<br><br>Answer the following questions. ▶▶ | 1.11 **Investigate**, page 418<br>Answer questions 1 - 4.<br><br>1.12 **Exercises**, pages 421 - 423<br>Answer questions 1 - 7, 11 - 14 and 18 - 20. |

## Unit 1 - Rational Expressions

| References and Notes | Work to Submit |
|---|---|
| Read **Section 7.5**.<br><br>Study **Examples 1**, **2**, and **3**. Work through each calculation on your own.<br><br>Answer the following questions.<br>▶▶ | 1.13 **Exercises**, pages 428 - 430<br>Answer questions 1 - 4, 6, 7, 13, 14, 16 and 17. |

## Unit 2 - Trigonometry and Its Application

**To fulfill the objectives of this unit, students should complete the following:**

| | | | |
|---|---|---|---|
| **Reading for this unit**: | *Appendix*: | Mathematics 2104B Study Guide: pages 23 and 24 | |
| | *Mathematics 10* | | |
| | Chapter 8: | Section 8.5: | pages 490 - 497 |
| | | Section 8.6: | pages 499 - 504 |
| | | Section 8.7: | pages 505 - 512 |
| | | Section 8.8: | pages 513 - 520 |

| **References and Notes** | **Work to Submit** |
|---|---|
| Read Appendix: **Area of a Triangle**.<br><br>Answer the following questions.<br>▶▶ | 2.1 Find the area of $\triangle ABC$ given AB = 40 cm, BC = 65 cm and $\angle B = 36°$.<br><br>2.2 Find the area of $\triangle PQR$ given $\angle P = 18.4°$, QP = 5.4 m and PR = 9.3 m. |
| Read **Linking Ideas**, page 289.<br><br>Read **Section 8.5**.<br><br>This section develops the definitions for the sine and cosine ratio in terms of the coordinates $(x, y)$.<br>Answer the following questions.<br>▶▶ | 2.3 See your instructor for **Prerequisites** exercises before beginning this section.<br><br>2.4 **Investigate**, pages 490 - 492<br>Answer questions 1 - 10. |
| After completing **Investigate**, you should know that, given a unit circle, $\cos A = x$ and $\sin A = y$. | |

## Unit 2 - Trigonometry and Its Application

| References and Notes | Work to Submit |
|---|---|
| Study **Visualizing** and work through the solutions given for **Examples 1**, **2** and **3**. | |
| Answer the following questions. ▶▶. | 2.5 **Discussing the Ideas**, page 496<br>Answer questions 1 - 4. |
| See your instructor for correction of **Discussing the Ideas** before moving on to the **Exercises**. | |
| | 2.6 **Exercises**, pages 496 and 497<br>Answer questions 1 - 9. |
| | 2.7 **Communicating the Ideas**, page 497 |
| Read **Section 8.6.** | |
| This section emphasizes *visualizing* and *sketching* what is given in a problem. This is used to plan a strategy which applies trigonometric ratios to solve triangles that are not right-angled. The examples and exercises given will help develop the Sine Law and Cosine Law which are introduced in the next two sections. | 2.8 See your instructor for **Prerequisites** exercises before beginning this section. |

## Unit 2 - Trigonometry and Its Application

| References and Notes | Work to Submit |
|---|---|
| Study **Examples 1** and **2**. You should work through all of the calculations given in the solutions.<br><br>Study **Example 3**. Again, work through the given solution.<br><br>In **Examples 1** and **2**, two angles and one side were given; whereas in **Example 3**, two sides and the angle between them are given.<br><br>Answer the following questions. ▶▶<br><br>See your instructor for correction of **Discussing the Ideas** before moving on to the **Exercises**. | 2.9 **Discussing the Ideas**, page 503<br>Answer questions 1 - 5.<br><br>2.10 **Exercises**, pages 503 and 504<br>Answer questions 1, 2, 4, 5 and 6. |

## Unit 2 - Trigonometry and Its Application

| References and Notes | Work to Submit |
|---|---|
| Read **Section 8.7**.<br><br>On page 505, an example from **Section 8.6** is examined, and then a general rule, the Sine Law, is formulated.<br><br>Carefully study and work through **Example 1** and **Example 2**.<br><br>There are only two situations in which the Sine Law applies. You must know one angle and its opposite side. You must have <u>one</u> other piece of information, and there are two possibilities:<br><br>1) if you know a second <u>angle</u>, you can solve for the side opposite it.<br><br>2) if you know a second <u>side</u>, you can solve for the angle opposite it. | 2.11 See your instructor for **Prerequisites** exercises for review. |
| Answer the following questions. ▶▶ | 2.12 **Exercises**, pages 509 - 512<br>Answer questions 1 - 7. |

| **References and Notes** | **Work to Submit** |
|---|---|
| The case when you are given two sides and a non-included angle, is called the *ambiguous case*. When solving the ambiguous case, you will get an equation of the form $\sin A = k$, where $k$ is a positive number. This means that angle $A$ could be between $0°$ and $90°$, or it could be between $90°$ and $180°$. Whether both angles are possible in a given triangle can only be determined by considering the values given and checking both possibilities to see that they both make sense.<br><br>To give an example: find $\sin 30°$ and $\sin 150°$ on your calculator. You found that $\sin 30° = .5$ and $\sin 150° = .5$.<br><br>Now, if you were given $\sin A = .5$, there are *two* possible values for angle $A$, $30°$and $150°$, both less than $180°$; hence, the *ambiguous case*. | |

## Unit 2 - Trigonometry and Its Application

| References and Notes | Work to Submit |
|---|---|
| Answer the following questions. ▶▶ | 2.13 Given triangle ABC, AB = 38, AC = 49 and ∠C = 41°. Draw a sketch. Find the measure of ∠B. Check whether both measures are possible in this triangle.<br><br>2.14 Given triangle ABC, AC = 2.9, CB = 3.9 and ∠A = 38°. Draw a sketch. Find the measure of ∠B. Again, check whether both measures make sense in this triangle.<br><br>2.15 **Exercises**, page 512<br>Answer question 20. |
| Read **Section 8.8**.<br><br>A general rule, called the Cosine Law, is developed by looking at the steps in the solution of **Example 3** on page 501.<br><br>Carefully study and work through the solutions given for **Examples 1** and **2**.<br>You will need the Cosine Law when<br>a) you know the length of two sides and the measure of the included angle **or**<br>b) when you know the lengths of all three sides. | 2.16 See your instructor for **Prerequisites** problems before beginning this section. |

## Unit 2 - Trigonometry and Its Application

| References and Notes | Work to Submit |
|---|---|
| Answer the following questions. ▶▶ | 2.17 **Discussing the Ideas**, page 517<br>Answer questions 1 - 4. |
| See your instructor to have **Discussing the Ideas** corrected before completing **Exercises**. | 2.18 **Exercises**, pages 517 - 520<br>Answer questions 1 - 3, 5 - 10, 12, 13, 20, 21 and 22.<br><br>**Note**: You should draw a sketch for each of these problems. |

## Unit 3 - A Functions Toolkit

**To fulfill the objectives of this unit, students should complete the following:**

**Reading for this unit**: *Mathematics 12*

| | | |
|---|---|---|
| Chapter 1: | Section 1.1: | pages 2 - 12 |
| | Section 1.2: | pages 16 - 22 |
| | Section 1.3: | pages 25 - 33 |
| | Section 1.4: | pages 35 - 43 |
| | Section 1.5: | pages 45 - 52 |

### References and Notes

Read **Section 1.1**.

Draw the graphs in **Examples 1**, **2** and **3** on pages 4 and 5. Use grid paper if you do not have access to a TI-83 or computer program such as *Graphmatica*.

Study **Examples 1** and **2** on pages 8 and 9. Sketch the graphs.

Answer the following questions. ▶▶

### Work to Submit

3.1 **Discussing the Ideas,** page 10
Answer questions 1, 2 and 5.

3.2 **Exercises**, pages 10 - 12
Answer questions 1, 4, 5, 7 and 8
(*See notes below on questions 1 and 5.*)

**Question 1**: In 1d), recall that when the object hits the ground, the height, *h*, is 0. Now you have to solve the equation for time, *t*.

**Question 5**: If you are using a TI-83, instead of entering every function, you can use the set bracket and all four functions will be graphed at the same time. Try the following: $Y1 = \sqrt{\left(\{16,9,4,1\} - x^2\right)}$ .

## Unit 3 - A Functions Toolkit

| References and Notes | Work to Submit |
|---|---|
| Read **Section 1.2**. | |
| Use a TI - 83 graphing calculator or graphing software to complete **Investigate**. | |
| Answer the following questions. ▶▶ | 3.3 **Investigate**, pages 16 and 17<br>Answer questions 1 - 10. |
| After completing **Investigate**, you should conclude that when comparing the graph of $y = f(x - k)$ with $y = f(x)$, it is translated **horizontally** by $k$ units. | |
| Similarity, when the graph of $y = f(x)$ is compared with $y - k = f(x)$, it is translated **vertically** by $k$ units. | |
| Carefully read page 18 and become very familiar with Vertical Translation Tool and the Horizontal Translation Tool in the purple box on page 19. | |
| Study **Examples 1** and **2**. Reproduce the graphs of these functions on your graphing calculator or computer. You will notice in **Example 1a**) that the "translation tools" are used to graph the absolute value function. First the horizontal tool is used and then the vertical tool is applied. | |

## Unit 3 - A Functions Toolkit

### References and Notes

Answer the following questions.
▶▶

When sketching several graphs on a single grid, be sure to label each one.

Read **Section 1.3**, Reflecting Graphs of Functions.

This section deals with **three** different reflections. For this course, however, we only are interested in one type of reflection; reflection in the $x$ axis. This reflection occurs when $y$ is replaced with $-y$ in the equation $y = f(x)$.

In other words, the graphs of the functions $y = f(x)$ and $-y = f(x)$ are reflections in the $x$ - axis (**vertical** reflections).

### Work to Submit

3.4 **Exercises**, pages 20 - 24
Answer questions 1, 2 and 3.
(*See note below on questions 1 - 3.*)

Answer questions 4, 5, 6a), 6b), 9, 11, 12, 13 and 14.
(*See note below on question 9.*)

**Questions 1 - 3**: If necessary, use the graph of the function $y = x^2$ to help you describe what happens to the function.

**Question 9**: Sketch the graph of $y = \sqrt{x}$ first and then use the vertical and horizontal translation tools.

| References and Notes | Work to Submit |
|---|---|
| As you work through this section we will only be looking at the material that deals with reflections in the $x$ - axis. If you wish, you can read the whole section, but you will not be responsible for the other two types of reflections. | |
| Answer the following questions. ▶▶ | 3.5 **Investigate**, pages 25 and 26<br>Answer questions 5, 7 and 8 in **Comparing the graphs of $y = f(x)$ and $-y = f(x)$**. |
| On page 27, you will see a graph of the function $y = \sqrt{x}$. If $y$ is replaced with $-y$, the graph of $-y = \sqrt{x}$ or $y = -\sqrt{x}$ is a reflection of the graph $y = \sqrt{x}$ in the $x$ - axis.<br><br>If you look at P and R on the graph, you will notice that the $y$ - coordinate of R (which is on the graph of $-y = \sqrt{x}$) is the opposite of the $y$ - coordinate of P (on the graph of $y = \sqrt{x}$).<br><br>On the top of page 28, read the information on ***x* - Axis, Reflection Tool** in the purple box. | |

## References and Notes

Study **Example 2 a**, **b**, **c** and **d**. (In c**,** particularly note the graph of $y = -f(x)$. )

Answer the following questions.

Read **Section 1.4**, Stretching Graphs of Functions.

This section deals with horizontal and vertical stretches.

That is, you will be comparing the graphs of
$y = f(x)$ with $y = f(kx)$ and
$y = f(x)$ with $ky = f(x)$.

Answer the following questions.

## Work to Submit

3.6 **Exercises**, pages 31 - 33
Answer questions 1b), 2a), 4b), 5 and 11.
(*See note below on question 11*.)

**Question 11**: In a) and b), sketch only the first and third equation [or, sketch $y = f(x)$ and $y = -f(x)$].

3.7 **Investigate**, pages 35 and 36
Use a graphing calculator.
**Comparing the graphs of $y = f(x)$ and $y = f(kx)$.**
Answer questions 1, 2, 3, 4 and 5.
(*See notes below on these questions*)

**Comparing the graphs of $y = f(x)$ and $k$y $= f(x)$.**
Answer questions 6, 7, 8 , 9 and 10.
(*See notes below on these questions.*)

**Questions 1 - 5**: These questions introduce **horizontal** compressions and expansions.

**Questions 6 - 10**: These questions introduce **vertical** compressions and expansions.

## Unit 3 - A Functions Toolkit

| References and Notes | Work to Submit |
|---|---|
| Look at the graph on the top right of page 37. You will notice that the graph of $2y = x^2$ is compressed vertically relative to the graph of $y = x^2$.<br><br>Now look at the graph on the bottom of page 37. Here, you will see that replacing $y$ with $2y$ **compresses** the graph vertically; replacing $y$ with $\frac{1}{2}y$ **expands** it vertically.<br><br>In the purple box on page 38, carefully read **Vertical Stretching Tool and Horizontal Stretching Tool**.<br><br>Answer the following questions. ▶▶ | 3.8 **Exercises**, pages 41 and 42<br>Answer questions 1 - 4, 6 - 8, 10, 11 and 14 - 16. |

## References and Notes

Read **Section 1.5**, **Combining Translations and Expansions or Compressions.**

Answer the following questions. ▶▶

## Work to Submit

3.9 **Investigate**, page 45

**Horizontal Transformations**
Answer questions 1, 2, 3 and 4.
(*See note below on question 4a*.)

**Vertical Transformations**
Answer questions 5, 6, 7 and 8.
(*See note below on question 8a*.)

**Question 4a and 8a**: When translations are combined with expansions or compressions, the result depends on the **order** in which the transformations are applied.

Unless otherwise stated, always apply the expansions or compressions **before** applying the translations.

## References and Notes

Carefully study page 47 and **Examples 1** and **2**. Make sure that you work through and understand each step in the solutions.

Answer the following questions.
▶▶

## Work to Submit

3.10 **Exercises**, pages 51 and 52
Answer questions 1 - 6.
(*See note below on question 6.)*

Answer questions 9 and 10.
(*See note below on these questions.*)

**Question 6**: You can simplify the equation $y = \sqrt{4x + 8}$ to $y = 2\sqrt{x + 2}$

**Questions 9 and 10**: Use the function $f(x) = \sqrt{4 - (x - 2)^2}$ when completing these questions. Question 10 reinforces the idea that the **order** in which expansions or compressions and translations are performed **does** make a difference.

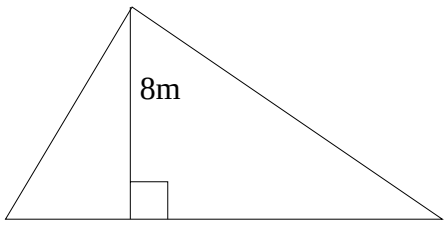
# Appendix

## Area of a Triangle

The formula to find the area, *A*, of a triangle is ½ *bh* where *b* is the base and *h* is the height which is perpendicular to the base.

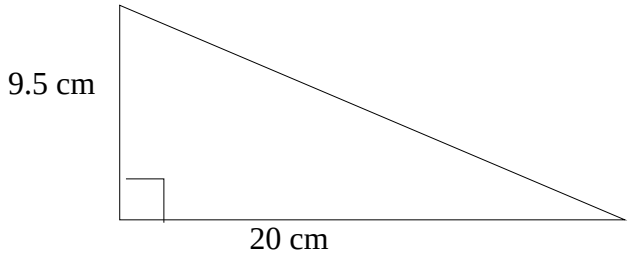
**Examples:**

1.



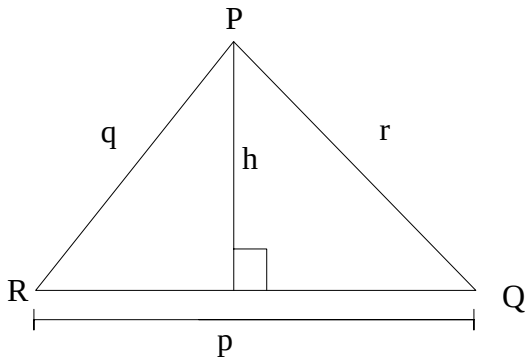

$A = \frac{1}{2} bh$
$A = \frac{1}{2} (10) (8)$
$A = 40 \text{ m}^2$

2.




$A = \frac{1}{2} bh$
$A = \frac{1}{2} (20) (9.5)$
$A = 95 \text{ cm}^2$

3. Given the following triangle, how do you find the area of triangle PQR?




We know that $A = \frac{1}{2} bh$

Since $\sin R = \frac{h}{q}$, solve for $h$ and you will get $h = (\sin R)(q)$.

The base, $b$, is called $p$ in this example.

Substitute in the formula: $A = \frac{1}{2} bh$ and get

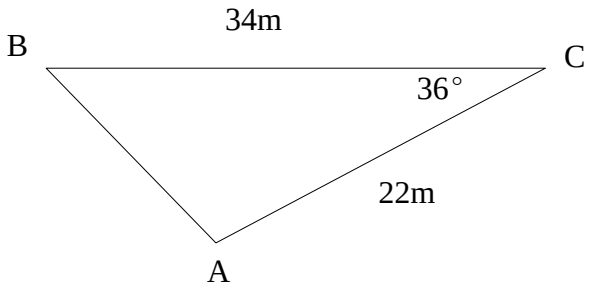
$A = \frac{1}{2} (p)(\sin R)(q) = \frac{1}{2} (p)(q)(\sin R)$

If you are given any two sides and the *included* angle, you can use this formula to find the area.

Given $\Delta$ ABC, the area can be written as:

- Area = $\frac{1}{2} bc \sin A$
- Area = $\frac{1}{2} ac \sin B$
- Area = $\frac{1}{2} ab \sin C$

**Example:**




Area = $\frac{1}{2} ab \sin C$
Area = $\frac{1}{2}$ (34) (22) (sin 36°)
Area = 219.83 $m^2$