Adult Basic Education

# Mathematics

# Mathematics 2104C

# Trigonometry

# Curriculum Guide

**Prerequisites:** Mathematics 1104A, 1104B, 1104C
Mathematics 2104A, 2104B

**Credit Value**: 1

**Required Mathematics Courses**
**[Degree and Technical Profile/ Business-Related College Profile]**

Mathematics 1104A
Mathematics 1104B
Mathematics 1104C
Mathematics 2104A
Mathematics 2104B
**Mathematics 2104C**
Mathematics 3104A
Mathematics 3104B
Mathematics 3104C

## Table of Contents

# To the Instructor

## I. Introduction to Mathematics 2104C

Trigonometry is the only topic studied in Mathematics 2104C. At first, students will work with radians and degrees and do conversions between them. Students will apply the unit circle definitions for the sine and cosine of an angle in standard position. Using these same definitions, students will determine the exact values of the sine and cosine of the special angles and their multiples. The unit circle definition for the tangent function is introduced as well as the definitions of the reciprocal functions in terms of the primary trigonometry functions. Students will also draw and analyze the sine and cosine graphs by plotting points as well as by using a graphing calculator. They will then graph sine and cosine functions that have been transformed by a change in amplitude, by a phrase shift, by a vertical translation and/or by a change in period.

Finally, students will solve some trigonometric equations by graphing and other trigonometric equations by finding the exact solutions.

This unit also introduces trigonometric identities which are verified numerically, graphically and algebraically.

## II. Prerequisites

Students should be able to apply the sine and cosine definitions in terms of opposite, adjacent and hypotenuse. Students need to be familiar with the concepts of radian measure and arc length. The transformations which they learned in Chapter 1 of *Mathematics 12* should be well understood.

## III. Textbook

Most of the concepts are introduced, developed and explained in the **Examples**. The instructor must insist that students carefully study and understand each **Example** before moving on to the **Exercises**. In the Study Guide, students are directed to see the instructor if there are any difficulties.

## To the Instructor

There are four basic categories included in each section of the textbook which require the student to complete questions:

1. Investigate
2. Discussing the Ideas
3. Exercises
4. Communicating the Ideas

**Investigate:** This section looks at the thinking behind new concepts. The answers to its questions are found in the back of the text.

**Discussing the Ideas**: This section requires the student to write a response which clarifies and demonstrates understanding of the concepts introduced. The answers to these questions are not in the student text but are in the *Teacher's Resource Book*. Therefore, in the Study Guide, the student is directed to see the instructor for correction. This will offer the instructor some perspective on the extent of the student's understanding. If necessary, reinforcement or remedial work can be introduced. Students should not be given the answer key for this section as the opportunity to assess the student's understanding is then lost.

**Exercises**: This section helps the student reinforce understanding of the concepts introduced. There are three levels of **Exercises**:

**A:** direct application of concepts introduced;
**B:** multi-step problem solving and some real-life situations;
**C:** problems of a more challenging nature.

The answers to the **Exercises** questions are found in the back of the text.

**Communicating the Ideas**: This section helps confirm the student's understanding of a particular lesson by requiring a clearly written explanation. The answers to **Communicating the Ideas** are not in the student text, but are in the *Teacher's Resource Book*. In the Study Guide students are asked to see the instructor for correction.

## IV. Technology

It is important that students have a **scientific** calculator and its manual for their individual use. Ensure that the calculator used has the word "scientific" on it as there are calculators designed for calculation in other areas such as business or statistics which would not have the functions needed for study in this area.

## To the Instructor

A graphing calculator should be **available** to the students since the text provides many opportunities for its use. The *Teacher's Resource Book* suggests many occasions to utilize a graphing calculator. These suggestions are outlined where there is the heading *Integrating Technology*. In the Study Guide, students are directed to see the instructor when a graphing calculator is required. The *Teacher's Resource Book* contains a module called **Graphing Calculator Handbook** which will help the instructor and student get acquainted with some of the main features of the TI-83 Plus graphing calculator.

Graphing software such as *Graphmatica* or *Winplot* can also be used if the students don't have access to a graphing calculator but do have access to a computer. The textbook doesn't offer the same guidance for graphing with these tools as it does for a graphing calculator but each software program does have a HELP feature to answer questions.

# V. Curriculum Guides

Each new ABE Mathematics course has a Curriculum Guide for the instructor and a Study Guide for the student. The Curriculum Guide includes the specific curriculum outcomes for the course. Suggestions for teaching, learning, and assessment are provided to support student achievement of the outcomes. Each course is divided into units. Each unit comprises a **two-page layout of four columns** as illustrated in the figure below. In some cases the four-column spread continues to the next two-page layout.

**Curriculum Guide Organization:**
**The Two-Page, Four-Column Spread**

| **Unit Number - Unit Title** | | **Unit Number - Unit Title** | |
|---|---|---|---|
| **Outcomes**<br><br>Specific curriculum outcomes for the unit. | **Notes for Teaching and Learning**<br><br>Suggested activities, elaboration of outcomes, and background information. | **Suggestions for Assessment**<br><br>Suggestions for assessing students' achievement of outcomes. | **Resources**<br><br>Authorized and recommended resources that address outcomes. |

## VI. Study Guides

The Study Guide provides the student with the name of the text(s) required for the course and specifies the sections and pages that the student will need to refer to in order to complete the required work for the course. It guides the student through the course by assigning relevant reading and providing questions and/or assigning questions from the text or some other resource. Sometimes it also provides important points for students to note. (See the *To the Student* section of the Study Guide for a more detailed explanation of the use of the Study Guides.) The Study Guides are designed to give students some degree of independence in their work. Instructors should note, however, that there is much material in the Curriculum Guides in the *Notes for Teaching and Learning* and *Suggestions for Assessment* columns that is not included in the Study Guide and instructors will need to review this information and decide how to include it.

## VII. Resources

### *Essential Resources*

*Addison Wesley Mathematics 12* (Western Canadian edition)
ISBN:0-201-34629-X

*Mathematics 12 Teacher's Resource Book* (Western Canadian edition)
ISBN: 0-201-34631-1

Math 2104C Study Guide

### *Recommended Resources*

*Mathematics 12 Independent Study Guide (*Western Canadian edition) ISBN: 0-201-34630-3

*Center for Distance Learning and Innovation*: http://www.cdli.ca
*Winplot*: http://math.exeter.edu/rparris/winplot.html
(Free graphing software)
*Graphmatica* (Evaluation software available on CD-ROM contained in *Teacher's Resource Book*)
CD Rom accompanying *Teacher's Resource Book*
This CD contains selected solutions from the text and self test solutions from the *Independent Study Guide*.

## To the Instructor

***Other Resources***

Math Links: http://mathforum.org

http://www.purplemath.com

http://www.sosmath.com/index.html

http://www.math.com/

http://spot.pcc.edu/~ssimonds/winplot
(Free videos concerning Winplot)

http://www.pearsoned.ca/school/math/math/

## VIII. Recommended Evaluation

| | |
|---|---|
| Written Notes | 10% |
| Assignments | 10% |
| Test(s) | 30% |
| Final Exam *(entire course)* | 50% |
| | 100% |

**The overall pass mark for the course is 50%**.

# Trigonometry

## Unit 1 - Trigonometric Functions of Angles

### Outcomes

1.1 Identify periodic relations and describe their characteristics.

1.1.1 Demonstrate an understanding of real-world relationships by translating between graphs, tables and written descriptions.

### Notes for Teaching and Learning

This unit deals with a fundamental topic: Trigonometric Functions of Angles. Most of the angle measurements are in radians. Students may prefer degree measure, but radian measure is most often used in scientific and mathematical fields.

This unit also introduces the exact value of the sine and cosine functions of special angles, $\frac{\pi}{4}$ , $\frac{\pi}{3}$ and $\frac{\pi}{6}$.

Students will also graph the sine and cosine functions. This course does not include the tangent graph, although it is covered in this chapter of the textbook.

**Section 3.1** exposes students to a number of examples of periodic behavior in real life. These examples should provide some incentive for learning about the graphs of sinusoidal curves.

The "time of sunset" example on page 158 and questions 1 and 2 in the **Exercises**, use the 24-hour clock. Students may need some review on how to change from 24-hour clock to AM/PM.

## Unit 1 - Trigonometric Functions of Angles

### Suggestions for Assessment

Study Guide questions 1.1 to 1.3 will meet the objectives of Outcome 1.1.

### Resources

*Mathematics 12,*
Section 3.1, Introduction to Periodic Functions, pages 156 - 162

*Mathematics 12*, *Teacher's Resource Book*, Chapter 3, pages 4 and 5

*Mathematics 12*, *Independent Study Guide*, Chapter 3, page 47

## Unit 1 - Trigonometric Functions of Angles

### Outcomes

1.2 Convert degrees to radians and radians to degrees.

1.3 Determine the angle or the arc length using the formula $a = r\theta$.

### Notes for Teaching and Learning

Since students have used only degree measure in previous years, they may be reluctant to change to radian measure. After completing this section, students will have discovered that some formulas involving angles become easier to work with when radians are used instead of degrees.

**Note**: Assign the **Prerequisites** exercises on page 6 of the *Teacher's Resource Book*.

The instructor should ensure that students understand that a central angle of measure 1 radian subtends an arc whose length is equal to the radius of the circle.

When completing angle conversions, similar to the problems in **Example 1,** students should be reminded that when using the conversion factors $\frac{\pi}{180}$ radians or $\frac{180^\circ}{\pi}$, they are really multiplying by 1.

## Unit 1 - Trigonometric Functions of Angles

### Suggestions for Assessment

Study Guide questions 1.4 to 1.7 will meet the objectives of Outcomes 1.2 and 1.3.

### Resources

*Mathematics 12,*
Section 3.2, Radian Measure, pages 163 - 168

*Mathematics 12*,
*Teacher's Resource Book*,
Chapter 3, pages 6 - 8

*Mathematics 12*,
*Independent Study Guide*,
Chapter 03, page 48

www.cdli.ca, Unit 04, Section 03, Lesson 01 (**Radian Measure and Arc Length**).

## Unit 1 - Trigonometric Functions of Angles

### Outcomes

1.4 Describe the sine and cosine functions as circular functions with reference to the unit circle and an angle in standard position.

1.4.1 Draw an angle in standard position and determine the measures of coterminal angles.

1.4.2 Determine the exact values of sine and cosine functions for any multiples of $30°$, $45°$ and $90°$, and $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{2}$.

### Notes for Teaching and Learning

Assign **Prerequisites** exercises from Chapter 3, *Teacher's Resource Book*, page 13. This section extends the definitions of sine and cosine in terms of a right triangle to the definitions in terms of a unit circle. A unit circle is a circle of radius 1 unit.

The instructor should point out to students that the answers in **Example 1** could vary. Usually the smallest positive and negative coterminal angles are given as answers.

Instructors should remind students how quadrants are numbered and that angles such a $\frac{3\pi}{2}$ or $270°$ are quadrantal angles since the terminal arm is on an axis.

**Visualizing** on page 177 relates the right triangle definitions to the unit circle definitions.

Assign **Prerequisites** exercises from Chapter 3, *Teacher's Resource Book*, page 18.

The instructor should work with students who have difficulty understanding the difference between an exact result and an approximate result.

Students should be encouraged to memorize the sides of the special triangles and how they were constructed, rather than memorizing the trigonometric values. Students should think in radians; they should <u>not</u> convert radians to degrees.

## Unit 1 - Trigonometric Functions of Angles

### Suggestions for Assessment

Study Guide questions 1.8 to 1.17 will meet the objectives of Outcome 1.4.

The *Teacher's Resource Book, Mathematics 12* has extra problems in **Supplementary Examples** and **Assessing the Outcome** which can be used for extra practice, homework or assessment.

There is also a Written Test and Multiple Choice Test in the *Teacher's Resource Book* which can be used for further assessment.

### Resources

*Mathematics 12,*
Sections 3.3 - 3.5, pages 170 - 192

*Mathematics 12*, *Teacher's Resource Book*, Chapter 3, pages 9 - 21

*Mathematics 12*, *Independent Study Guide*, pages 48 and 49

www.cdli.ca, Unit 04, Section 01, Lessons 01 and 02 (**Special Rotations**)

## Unit 1 - Trigonometric Functions of Angles

### Outcomes

1.5 Draw (with and without technology) and analyze the graphs of the sine and cosine functions of an angle.

### Notes for Teaching and Learning

The *Teacher's Resource Book*, Chapter 3, page 21 contains useful tips for the instructor and student on how to complete **Exploring with a Graphing Calculator** on page 191 of *Mathematics 12*. The **Graphing Calculator Handbook**, *Teacher's Resource Book* for *Mathematics 10* or *Mathematics 11* contains much essential information on the TI-83.

Students should be assigned **Prerequisites** exercises from the *Teacher's Resource Book*, Chapter 3, page 23. Although students may be able to use a TI-83 to produce the sine and cosine graphs, they should be able to generate the graphs by plotting points.

## Suggestions for Assessment

Study Guide questions 1.18 to 1.21 will meet the objectives of Outcome 1.5.

## Resources

*Mathematics 12,* Section 3.6, Graphing the Sine and Cosine Functions of an Angle, pages 193 - 198

*Mathematics 12*, *Teacher's Resource Book*, Chapter 3, pages 23 - 25

*Mathematics 12*, *Independent Study Guide*, page 49

www.cdli.ca, Unit 03, Section 01, Lessons 01 and 02, Unit 03, Section 02, Lesson 01

## Unit 1 - Trigonometric Functions of Angles

### Outcomes

1.6 Determine the unit circle definition for the tangent function of an angle in standard position.

### Notes for Teaching and Learning

Students should be assigned **Prerequisites** exercises from the *Teacher's Resource Book*, Chapter 3, page 28.

Students should understand why the tangent function is undefined at $\frac{\pi}{2}$ and $\frac{3\pi}{2}$.

The instructor should ensure that the students also understand two more definitions for $\tan\theta$:

1) $\tan\theta = \frac{y}{x}$ where the terminal arm of the angle $\theta$ in standard position intersects the unit circle at $(x, y)$

2) $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cos\theta \neq 0$.

## Unit 1 - Trigonometric Functions of Angles

### Suggestions for Assessment

Study Guide questions 1.22 and 1.23 will meet the objectives of Outcome 1.6.

### Resources

*Mathematics 12,*
Section 3.7, The Tangent Function of an Angle, pages 205 - 210

*Mathematics 12*, *Teacher's Resource Book*, Chapter 3, pages 28 - 31

*Mathematics 12*, *Independent Study Guide*, page 49 and 50

## Unit 1 - Trigonometric Functions of Angles

### Outcomes

1.7 Determine the values of reciprocal trigonometric functions.

### Notes for Teaching and Learning

At this time, students are <u>not</u> responsible for the graphs of tan $\theta$ (**Section 3.8**) and the reciprocal trigonometric functions. (**Sections 3.8 and 3.9**)

Students have been assigned the first page (page 216) of **Section 3.9** in order to learn the definitions of the reciprocal functions and how to use a calculator to determine the values of reciprocal trigonometric functions.

## Suggestions for Assessment

Study Guide questions 1.24 and 1.25 will meet the objectives of Outcome 1.7.

Each section in the *Teacher's Resources Book* has extra problems in **Supplementary Examples** and **Assessing the Outcome.**

There is a Multiple Choice Test and Written Test contained in Masters 3.3 - 3.6.

The *Independent Study Guide* has a Self-Test and Multiple Choice Test for each chapter.

The CD-ROM in the *Teacher's Resource Book* contains Model Solutions to each **Self -Test**.

## Resources

*Mathematics 12,*
Section 3.9, Reciprocal Trigonometric Functions, pages 216 and 218

*Mathematics 12*, *Teacher's Resource Book*, Chapter 3, pages 36 - 38
Masters 3.3 - 3.6

*Mathematics 12*, *Independent Study Guide*, page 50

## Unit 2 - Trigonometric Functions of Real Numbers

### Outcomes

2.1 Define the sine and cosine functions as functions of real numbers.

2.2 Graph the functions $y = \sin x$ and $y = \cos x$.

2.2.1 Use a graphing calculator to graph the sine and cosine functions.

### Notes for Teaching and Learning

In Chapter 4, each of the sine and cosine functions are defined as a function of a real number. This real number is associated with an arc length on the unit circle.

The general forms of the equations of the sine and cosine functions are $y = a \sin b (x - c) + d$ and $y = a \cos b (x - c) + d$, with amplitude, |a|; period, $\frac{2\pi}{b}$ ; phase shift, *c;* and vertical displacement, *d*.

Students will need to use the **Functions Toolkit** which they studied in Chapter 1. The instructor should have copies of Master 4.1 (**Large Unit Circle**), Master 4.2 (**Graphing Calculator Screen Template**) and Master 4.3 (**Grid Template**) to distribute to students when required.

Assign **Prerequisites** exercises, page 4, of *Teacher's Resource Book*, Chapter 4.

If there is more than one student studying this topic, it may be advantageous to work in a group to complete **Investigate**, page 228 in *Mathematics 12*.

The instructor should emphasize that when an angle is in radians, the arc length it subtends in the unit circle is equal to the angle measure in radians. Students should know that, when graphing a trigonometric function, the horizontal axis can be scaled using unit measures or using the number π and its multiples.

Students should be able to graph $y = \sin x$ and $y = \cos x$ using paper and pencil as well as using a TI-83. When using a calculator to complete the exercises, the instructor should ensure that students have their calculators in radian mode.

## Unit 2 - Trigonometric Functions of Real Numbers

### Suggestions for Assessment

Study Guide questions 2.1 to 2.5 will meet the objectives of Outcomes 2.1 and 2.2.

### Resources

*Mathematics 12,*
Section 4.1, The Functions of $y = \sin x$ and $y = \cos x$, pages 226 - 236

*Mathematics 12*,
*Teacher's Resource Book*,
Chapter 4, pages 4 - 8
Master 4.1

*Mathematics 12*,
*Independent Study Guide*,
Chapter 3, pages 55 - 64

## Unit 2 - Trigonometric Functions of Real Numbers

### Outcomes

2.3 Recognize and graph sine and cosine functions that have been transformed by:

i) a change in amplitude
ii) a phase shift
iii) a vertical translation

### Notes for Teaching and Learning

This is a topic which contains much new material. Students should take their time working through each type of problem. Students need to be familiar with the transformations they learned in Chapter 1.

Assign **Prerequisites** exercises, page 9, of *Teacher's Resource Book*, Chapter 4.

**Investigate**, on page 237 of *Mathematics 12*, starts with comparing the graphs of $y = a \sin x + d$ with $y = \sin x$ and $y = a \cos x + d$ with $y = \cos x$. This introduces 2 transformations at once, which may be difficult for some students. The Study Guide has 2 exercises inserted before this **Investigate** which separate these 2 transformations and compares the graphs of $y = a \sin x$ with $y = \sin x$ and then compares $y = \sin x + d$ with $y = \sin x$.

As students graph these functions on a TI-83, they should also sketch them on paper. There are a few steps which should prove useful when sketching the graph on the TI-83:

1) Draw the vertical shift line in red (different colour colour than $x$-axis).
2) Mark this new axis with the same scale as $x$-axis.
3) Mark the 5 strategic points that define one period.
4) Draw the curve.

The strategic points for $y = \sin x$ are $(0,0)$, $(\frac{\pi}{2}, 1)$, $(\pi, 0)$, $(\frac{3\pi}{2}, -1)$, $(2\pi, 0)$.

The strategic points for $y = \cos x$ are $(0,1)$, $(\frac{\pi}{2}, 0)$, $(\pi, -1)$, $(\frac{3\pi}{2}, 0)$, $(2\pi, 1)$.

## Suggestions for Assessment

Study Guide questions 2.6 to 2.11 will meet the objectives of Outcome 2.3.

## Resources

*Mathematics 12,*
Section 4.2, Graphing
$y = a \sin (x - c) + d$ and
$y = a \cos (x - c) + d$, pages
237 - 248

*Mathematics 12*,
*Teacher's Resource Book*,
Chapter 4, pages 9 - 14
Masters 4.2 and 4.3

*Mathematics 12*,
*Independent Study Guide*,
pages 55 - 64

## Unit 2 - Trigonometric Functions of Real Numbers

### Outcomes

2.3 Recognize and graph sine and cosine functions that have been transformed by:

i) a change in amplitude
ii) a phase shift
iii) a vertical translation

### Notes for Teaching and Learning

Students may experience difficulty when drawing graphs of functions which have a phase shift. Master 4.3 (**Grid Template**) may prove useful.

The instructor could help students who have a weakness in this area by providing the method for drawing these graphs which is in the *Teacher's Resource Book*, Chapter 4, page 10.

Students need to be reminded to enter $y = \sin x + 1$ (vertical displacement = 1) as $y = \sin (x) + 1$ on their TI-83, otherwise the calculator will read it as a phase shift $y = \sin (x + 1)$.

Students should be able to write a summary of the corresponding transformations for the general formula: $y = a \sin (x - c) + d$.

**Note**: **Prerequisites** exercises on page 9 of *Teacher's Resource Book*, Chapter 4 should be assigned and discussed with the students before the chapter is assigned.

## Suggestions for Assessment

Study Guide questions 2.6 to 2.11 will meet the objectives of Outcome 2.3.

## Resources

*Mathematics 12,*
Section 4.2, Graphing
$y = a \sin (x - c) + d$ and
$y = a \cos (x - c) + d$, pages
237 - 248

*Mathematics 12*,
*Teacher's Resource Book*,
Chapter 4, pages 9 - 14
Masters 4.2 and 4.3

*Mathematics 12*,
*Independent Study Guide*,
pages 55 - 64

## Unit 2 - Trigonometric Functions of Real Numbers

### Outcomes

2.4 Recognize and graph the sine and cosine functions that have been transformed by a change in period.

### Notes for Teaching and Learning

**Note**: Students should be able to use the horizontal stretching tool they studied in Chapter 1 of *Mathematics 12*.

Assign **Prerequisites** exercises on page 15 of the *Teacher's Resource Book*, Chapter 4. The instructor should ensure that students discover the formula for finding the period. The period of $y = \sin bx$ is $\frac{2\pi}{b}$.

When completing the exercises, students may have difficulty drawing the graph and scaling the horizontal axis. The instructor should give plenty of guidance to the students while they are working through **Section 4.3.** Extra practice problems may need to be assigned as well. Students can use graphing calculators, but they must be able to graph these sinusoidal functions using paper, pencil and applying the transformational rules learned in Chapter 1 of *Mathematics 12*.

When combining change in period with a phase shift, the students should be instructed to find the period <u>first</u> and apply the phase shift <u>second</u>.

Given the sinusoidal functions $y = a \sin b\,(x - c) + d$ or $y = a \cos b\,(x - c) + d$, students should know how $a$, $b$, $c$ and d affect the graph of $y = \sin x$ or $y = \cos x$.

**Examples 1** and **2** give two different methods to graph a sinusoidal function. Students can study both and decide which method they prefer.

## Suggestions for Assessment

Study Guide questions 2.12 to 2.15 will meet the objectives of Outcome 2.4.

The CDLI site has some interactive lessons on graphing using transformations. The method used, mapping notation, is different from *Mathematics 12*.

The horizontal axis is scaled in degrees and not in terms of π. Instructors, therefore, should use wisdom in deciding, if and what lessons should be used.

## Resources

*Mathematics 12,*
Section 4.3, Graphing $y = a \sin b(x - c) + d$ and $y = a \cos b(x - c) + d$, pages 249 - 256.

*Mathematics 12*,
*Teacher's Resource Book*,
Chapter 4, pages 15 - 18

*Mathematics 12*,
*Independent Study Guide*,
pages 55 - 64

## Unit 3 - Trigonometric Equations and Identities

### Outcomes

3.1 Use technology to solve trigonometric equations of the form $\sin x = y$ and $\cos x = y$.

### Notes for Teaching and Learning

In this section, trigonometric equations are solved using graphing technology. Since the emphasis of this course is solving trigonometric equations algebraically, there will be just a cursory look at other methods of solving these trigonometric equations.

Students are not assigned all of **Section 5.1.** If students can understand **Example 1** and use a TI-83 to work through the given solution, they should see the method of how trigonometric equations can be solved graphically.

The instructor should assign **Prerequisites** exercises on page 4 of *Teacher's Resource Book*, Chapter 5.

The problems assigned from **Exercises**, on page 302 of the text should be completed with a graphing calculator.

## Unit 3 - Trigonometric Equations and Identities

### Suggestions for Assessment

Study Guide questions 3.1 and 3.2 will meet the objectives of Outcome 3.1.

### Resources

*Mathematics 12,*
Section 5.1, Solving Trigonometric Equations Using Graphing Technology, pages 298, 299 and 302

*Mathematics 12*, *Teacher's Resource Book*, Chapter 5, pages 4 - 7

*Mathematics 12*, *Independent Study Guide*, pages 69 - 72

## Unit 3 - Trigonometric Equations and Identities

### Outcomes

3.2 Without technology, solve trigonometric equations that have exact solutions.

### Notes for Teaching and Learning

In this section, trigonometric equations are solved without using reference angles. The method used in the text can be confusing for students. The instructor should advise students to have another look at **Section 3.5** in *Mathematics 12* which discusses the sine and cosine of special angles. The instructor should work through **Example 1** and a couple of similar problems using reference angles to find the solution.

In **Example 2**, page 310 of *Mathematics 12*, it may be helpful to the student if the equation was rewritten as $\cos\theta = \frac{\sqrt{2}}{2}$, $\theta = 3x$.

Using reference angles to solve for $\theta$, $\theta = \frac{\pi}{4}$ or $\frac{7\pi}{4}$.

Since $3x = \theta$, we can say $3x = \frac{\pi}{4}$, therefore $x = \frac{\pi}{12}$ and $3x = \frac{7\pi}{4}$, so, $x = \frac{7\pi}{12}$.

Solving for $0 \le x < 2\pi$, two solutions are: $x = \frac{\pi}{12}, \frac{7\pi}{12}$.

Add 1 period, $\frac{2\pi}{3}$, to these solutions to find the remainder;

$x = \frac{\pi}{12} + \frac{2\pi}{3} = \frac{9\pi}{12}$ and $\frac{9\pi}{12} + \frac{2\pi}{3} = \frac{17\pi}{12}$.

Add 1 period, $\frac{2\pi}{3}$, to $\frac{7\pi}{12}$ to get $\frac{15\pi}{12}$ and $\frac{23\pi}{12}$.

Students should be reminded that parts 8d) and 8f) in question 8 require the use of a scientific calculator since some of the solutions are not exact values.

## Unit 3 - Trigonometric Equations and Identities

| Suggestions for Assessment | Resources |
|---|---|
| Study Guide question 3.3 will meet the objectives of Outcomes 3.2. | *Mathematics 12,* Section 5.2, Solving Trigonometric Equations without Using Graphing Technology, pages 308 - 314<br><br>*Mathematics 12*, *Teacher's Resource Book*, Chapter 5, pages 10 - 14<br><br>*Mathematics 12*, *Independent Study Guide*, pages 69 - 72 |
| The CDLI site has some helpful interactive demonstrations. The angle measurements however are in degrees and not radians. | www.cdli.ca, Unit 4, Section 01, Lessons 01, 02 and 03 |

## Unit 3 - Trigonometric Equations and Identities

### Outcomes

3.3 Recognize and verify specific trigonometric identities numerically and graphically.

### Notes for Teaching and Learning

This section verifies identities both graphically and numerically . This course looks only at verification numerically . For this reason, students should be reminded to omit pages 315 and 316 and **Example**, part **b**. Encourage students to make a list of the reciprocal identities as well as the **Quotient Identity** and **Pythagorean Identity** found on page 317 of *Mathematics 12*. The instructor should ensure that students can also find the two other Pythagorean Identities.

$\csc x = \frac{1}{\sin x}$ $\quad \tan x = \frac{\sin x}{\cos x}$ $\quad \cot^2 x + 1 = \csc^2 x$

$\sec x = \frac{1}{\cos x}$ $\quad \sin^2 x + \cos^2 x = 1$

$\cot x = \frac{1}{\tan x}$ $\quad \tan^2 x + 1 = \sec^2 x$

All of these identities should be memorized.

## Unit 3 - Trigonometric Equations and Identities

### Suggestions for Assessment

Study Guide question 3.4 will meet the objectives of Outcome 3.3.

### Resources

*Mathematics 12,*
Section 5.3,
Trigonometric Identities,
pages 317 - 320

*Mathematics 12*,
*Teacher's Resource Book*,
Chapter 5, pages 14 - 18

*Mathematics 12*,
*Independent Study Guide*,
pages 69 - 72

## Unit 3 - Trigonometric Equations and Identities

### Outcomes

3.4 Verify trigonometric identities numerically and then prove them algebraically.

3.4.1 Use the fundamental trigonometric identities to prove other identities.

3.4.2 Use the Pythagorean trigonometric identities to prove other identities.

### Notes for Teaching and Learning

Students should be proficient in simplifying rational expressions. (See *Mathematics 10*, Chapter 7.) It will take time and practice before students are comfortable with manipulating trigonometric expressions algebraically.

The textbook does not provide many examples or problems for students to work on. The instructor may have to use some other high school textbook to provide examples for extra practice. There are several basic techniques demonstrated in the four **Examples** given in the textbook.

This textbook sometimes works on both sides of a trigonometric identity. However many textbooks (and instructors) insist that trigonometric identities be proven by working solely on one side. In any case, the instructor should ensure that students realize that when proving identities, they cannot cross multiply or multiply/divide both sides of the equation. They should start with the most complicated side first, simplify as much as possible, then, if necessary work on the other side to obtain the same expression.

Assign **Prerequisites** exercises on page 18 of Chapter 4 of the *Teacher's Resource Book*.

## Unit 3 - Trigonometric Equations and Identities

### Suggestions for Assessment

Study Guide questions 3.5 and 3.6 will meet the objectives of Outcome 3.4.

### Resources

*Mathematics 12,*
Section 5.4, Verifying and Proving Trigonometric Identities, pages 322 - 327

*Mathematics 12*,
*Teacher's Resource Book*,
Chapter 5, pages 18 - 23

*Mathematics 12*,
*Independent Study Guide*,
pages 69 - 72

# Appendix

Draw and label the sides of a 30-60-90 trangle and a 45-45-90 triangle. Use the sketches to complete the table.
Leave your answers in exact form. (Don't use a calculator!)

| θ | sin θ | cos θ | tan θ | csc θ | sec θ | cot θ |
|---|---|---|---|---|---|---|
| 0° | 0 | 1 | 0 | Undefined | | Undefined |
| 30° | | | | 2 | $\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$ | |
| 45° | | | 1 | | | 1 |
| 60° | | | $\sqrt{3}$ | | 2 | $\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ |
| 90° | 1 | 0 | Undefined | 1 | | |
| 120° | | $-\frac{1}{2}$ | | | | |
| 135° | $\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ | | | | | |
| 150° | | | | | | |
| 180° | | | | | | |