Adult Basic Education

Mathematics

Mathematics 2104C

Trigonometry

Study Guide

Prerequisites: Mathematics 1104A, 1104B, 1104C
Mathematics 2104A, 2104B

Credit Value: 1

Text: *Mathematics 12,* Alexander and Kelly; Addison - Wesley, 1999.

Required Mathematics Courses
[Degree and Technical Profile/Business-Related College Profile]

Mathematics 1104A
Mathematics 1104B
Mathematics 1104C
Mathematics 2104A
Mathematics 2104B
Mathematics 2104C
Mathematics 3104A
Mathematics 3104B
Mathematics 3104C

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To the Student

I. <u>Introduction to Mathematics 2104C</u>

Trigonometry is the only topic studied in Mathematics 2104C. At first, you will work with radians and degrees and do conversions between them. You will apply the unit circle definitions for the sine and cosine of an angle in standard position. Using these same definitions, you will determine the exact values of the sine and cosine of the special angles and their multiples will be determined. The unit circle definition for the tangent function is introduced as well as the definitions of the reciprocal functions in terms of the primary trigonometry functions.

You will draw and analyze the sine and cosine graphs by plotting points as well as by using a graphing calculator. You will then graph sine and cosine functions that have been transformed by a change in amplitude, by a phrase shift, by a vertical translation and by a change in period.

Finally, you will solve some trigonometric equations by graphing and other trigonometric equations by finding the exact solutions.

This unit also introduces trigonometric identities which are verified numerically, graphically and algebraically.

II. <u>Resources</u>

You will require the following:

- *Addison Wesley Mathematics 12*, Western Canadian edition Textbook
- Scientific calculator
- graph paper
- Access to a TI-83 Plus graphing calculator (see your instructor) and/or *Graphmatica* or *Winplot* graphing software

To the Student

Notes concerning the textbook:

Glossary: Knowledge of mathematical terms is essential to understand concepts and correctly interpret questions. Written explanations will be part of the work you submit for evaluation, and appropriate use of vocabulary will be required.

Your text for this course includes a Glossary where definitions for mathematical terms are found. Be sure you understand such definitions and can explain them in your own words. Where appropriate, you should include examples or sketches to support your definitions.

Examples: You are instructed to study carefully the **Examples** in each section and see your instructor if you have any questions. These **Examples** provide full solutions to problems that can be of great use when answering assigned **Exercises**.

Notes concerning technology:

It is important that you have a **scientific** calculator for your individual use. Ensure that the calculator used has the word “scientific” on it as there are calculators designed for calculation in other areas such as business or statistics which would not have the functions needed for study in this area. Scientific calculators are sold everywhere and are fairly inexpensive. You should have access to the manual for any calculator that you use. It is a tool that can greatly assist the study of mathematics but, as with any tool, the more efficient its use, the better the progress.

You will require access to some sort of technology in order to meet some of the outcomes in this course. Since technology has become a significant tool in the study of Mathematics, your textbook encourages you to become proficient in its use by providing you with step-by-step exercises that will teach you about the useful functions of the TI-83 Plus Graphing calculator. **See your instructor concerning this**. Please note that a graphing calculator is not essential for success in this course but it is useful.

While graphing calculators and graphing software (*Graphmatica* or *Winplot*) are useful tools, they cannot provide the same understanding that comes from working paper and pencil exercises.

III. Study Guide

This Study Guide is required at all times. It will guide you through the course and you should take care to complete each unit of study in the order given in this Guide. Often, at the beginning of each unit, you will be instructed to see your instructor for **Prerequisites** exercises. Please do not skip this step! It should only take a few minutes for you and your instructor to discover what, if any, prerequisite skills need review.

To the Student

To be successful, you should read the **References and Notes** first and then, when indicated by the ▶▶ symbols, complete the **Work to Submit** problems. Many times you will be directed to see your instructor, and this is vital, especially in a Mathematics course. If you have only a hazy idea about what you just completed, nothing will be gained by continuing on to the next set of problems.

Reading for this Unit: In this box, you will find the name of the text, and the chapters, sections and pages used to cover the material for this unit. As a preliminary step, skim the referenced section, looking at the name of the section, and noting each category. Once you have completed this overview, you are ready to begin.

References and Notes	Work to Submit
This left hand column guides you through the material to read from the text. It will also refer to specific **Examples** found in each section. You are directed to study these **Examples** carefully and see your instructor if you have any questions. The **Examples** are important in that they not only explain and demonstrate a concept, but also provide techniques or strategies that can be used in the assigned questions. The symbols ▶▶ direct you to the column on the right which contains the work to complete and submit to your instructor. You will be evaluated on this material. Since the answers to **Discussing the Ideas** and **Communicating the Ideas** are not found in the back of the student text, you **must** have these sections corrected by your instructor **before** going on to the next question. This column will also contain general **Notes** which are intended to give extra information and are not usually specific to any one question.	There are four basic categories included in this column that correspond to the same categories in the sections of the text. They are **Investigate, Discussing the Ideas**, **Exercises**, and **Communicating the Ideas**. **Investigate:** This section looks at the thinking behind new concepts. The answers to its questions are found in the back of the text. **Discussing the Ideas**: This section requires you to write a response which clarifies and demonstrates your understanding of the concepts introduced. The answers to these questions are not in the student text and will be provided when you see your instructor. **Exercises**: This section helps to reinforce your understanding of the concepts introduced. There are three levels of **Exercises**: **A:** direct application of concepts introduced **B:** multi-step problem solving and some real-life situations **C:** problems of a more challenging nature The answers to the **Exercises** questions are found in the back of the text. **Communicating the Ideas:** This section helps confirm your understanding of the lesson of the section. If you can write a response, and explain it clearly to someone else, this means that you have understood the topic. The answers to these questions are not in the student text and will be provided when you see your instructor This column will also contain **Notes** which give information about specific questions.

IV. Recommended Evaluation

Written Notes	10%
Assignments	10%
Test(s)	30%
Final Exam *(entire course)*	50%
	100%

The overall pass mark for the course is 50%.

Unit 1 - Trigonometric Functions of Angles

To meet the objectives of this unit, students should complete the following:

Reading for this unit:	*Mathematics 12*	
	Chapter 3:	Section 3.1: pages 156 - 162
		Section 3.2: pages 163 - 168
		Section 3.3: pages 170 - 175
		Section 3.4: pages 176 - 181
		Section 3.5: pages 184 - 191
		Section 3.6: pages 193 - 198
		Section 3.7: pages 205 - 210
		Section 3.9: pages 216 and 218

References and Notes	Work to Submit
Read **Section 3.1**	
The "time of sunset" graph on page 158 uses the 24-hour clock.	
To convert from 24-hour clock to AM/PM: if the time is 1300 or later, subtract 12 hours and add PM.	
To convert from AM/PM to 24-hour clock: add 12 hours to all times after midday.	
Answer the following questions. ▶▶	1.1 **Discussing the Ideas**, page 160 Answer questions 1, 2, 3 and 4.
See your instructor with your completed answers to **Discussing the Ideas** before moving on to the **Exercises**.	1.2 **Exercises**, pages 161 and 162 Answer questions 1, 2, 3, 4, 6 and 8.
	1.3 **Communicating the Ideas**, page 162

Unit 1 - Trigonometric Functions of Angles

References and Notes

Read **Section 3.2**.

Although angles measured in degrees are common in many applications, radian measure is used extensively in scientific fields as well as in mathematics.

Answer the following questions.
▶▶

The TI-83 calculator has only radian and degree mode, but other scientific calculators may have a “grad” mode. One right angle is equal to 100 grads.

If the circle is a unit circle, (*radius* is 1), the radian measure gives the arc length directly. One rotation around the unit circle is 2π radians. The arc length or circumference of the unit circle is 2π.

So, the formula for arc length, $a = r\theta$, becomes $a = \theta$ when the circle is a <u>unit</u> circle.

Since π radians is equal to $180°$, when you multiply by $\frac{\pi}{180°}$ radians or $\frac{180°}{\pi}$, you are simply multiplying by 1.

Work to Submit

1.4 **Investigate**, page 163
Answer questions 1, 2, 3 and 4.

Unit 1 - Trigonometric Functions of Angles

References and Notes

Answer the following questions. ▶▶

Work to Submit

1.5 Using a sketch, define the term *radian*.

1.6 What is the formula for arc length? Draw a sketch.

1.7 **Exercises**, pages 167 - 168
Answer questions 1 and 2.
(*See note below on questions 1 and 2.*)

Answer questions 4, 5, 6, 7 and 8.
(*See note below on question 4.*)

Answer questions 10, 11 and 14.

Questions 1 and 2: Special angles are often expressed in terms of π. Quadrantal angles 0°, 90°, 180°, 270° (or $\frac{\pi}{2}$, π, $\frac{3\pi}{2}$) as well as angles 30°, 60°, 45° (or $\frac{\pi}{6}$, $\frac{\pi}{3}$, $\frac{\pi}{4}$) and all multiples are considered special angles.

Question 4: You may be able to use shortcuts here. If you know that $\frac{\pi}{4}$ = 45°, it should be clear that $\frac{5\pi}{4}$ is 5 times 45° or 225°.

Unit 1 - Trigonometric Functions of Angles

References and Notes	Work to Submit
In **Section 3.3**, carefully read pages 170 and 171 and study **Examples 1 - 3**. You should notice that the initial arm is always on the positive x-axis and it does not move. When indicating the measure of an angle in standard position, always use an arrow which starts at the initial arm. The quadrants are numbered counter-clockwise with quadrant 1 being the upper right hand quadrant.	1.8 See your instructor for **Prerequisites** exercises on **Section 3.3**.
Angles such as 3π and $\frac{\pi}{2}$ are called quadrantal angles, since the terminal arm is on an axis. Answer the following questions.	1.9 Define the following terms and draw a sketch where appropriate: i) standard position of an angle ii) coterminal angles iii) initial arm iv) terminal arm v) clockwise rotation vi) counterclockwise rotation 1.10 **Discussing the Ideas**, page 173 Answer questions 1 and 2. 1.11 **Exercises**, page 173 - 175 Answer questions 1 - 9.

Unit 1 - Trigonometric Functions of Angles

References and Notes

Carefully study **Section 3.4**.

Recall that the unit circle is a circle that has a *radius* of 1 unit.

The definitions of sine and cosine functions will be given in terms of the unit circle.

Answer the following questions.
▶▶

Work to Submit

1.12 **Communicating the Ideas**, page 175

1.13 **Discussing the Ideas**, page 179
Answer questions 1 - 5.

1.14 **Exercises**, pages 179 - 181
Answer questions 1 - 3.
(*See note below on question 2 and 3.*)

Answer question 4.
(*See note below on question 4.*)

Answer questions 5 - 10 and 16, 17 and 18.
(*See note below on question 16.*)

Questions 2 and 3: When completing questions 2 and 3, recall that an angle in degrees must have a degree symbol, but an angle in radians does not have a unit.

Question 4: In question 4, it will be helpful if you choose increments of $30°$ for θ, $(0°, 30°, 60°, 90°...)$ and record the values of $\cos\theta$ and $\sin\theta$ in a table.

Question 16: In question 16, use Pythagorean Theorem.

References and Notes

Read **Section 3.5**.

The special angles mentioned here are ones which you will use frequently. You should learn the cosine and sine of each of these special angles: $\frac{\pi}{4}, \frac{\pi}{6}, \frac{\pi}{3}$.

Using these angles as reference angles, you should be able to determine the cosine and sine of any multiple of these angles.

Answer the following questions.

Work to Submit

1.15 Define the term *reference angle*. Use a sketch.

1.16 **Discussing the Ideas**, page 187
Answer questions 1 - 3.

1.17 **Exercises**, pages 188 and 189
Answer questions 1 - 6.
(*See note below on Question 3.)*

Question 3: When finding exact value in question 3, you should draw the angle to determine the sign of the trigonometric function, then find the reference angle.

References and Notes	Work to Submit
Read **Exploring with a Graphing Calculator** on page 191. Using a TI-83 calculator, you will graph the sine and cosine functions using different window settings in both degrees and radians. Answer the following questions. ▶▶	1.18 **Exploring with a Graphing Calculator**, pages 191 and 192 Answer questions 1, 2, 3 and 5. (*See note below on question 1.*) Answer questions 6, 7, 8 and 10. (*See notes below on questions 6 and 7.)* **Question 1:** Although the text refers to the function $y = \sin\theta$, the calculator will refer to $y = \sin x$. When setting window dimensions, you should let $X_{scl} = 90$. **Question 6:** Don't forget to change your calculator to radian mode. The value, π, should be entered directly when setting the window. The numerical approximation will appear on the screen once [ENTER] is pressed. You should set X_{scl} to $\frac{\pi}{2}$. **Question 7:** When an angle such as $\frac{\pi}{3}$ is entered on the screen, its approximate decimal equivalent will appear. You should become familiar with the approximate values for special angles, since the calculator cannot give exact answers.

References and Notes	Work to Submit
Read **Section 3.6**. See your instructor for **Prerequisites** exercises before you begin this section. Although you have used a graphing calculator to produce sine and cosine graphs, it is important that you are able to draw these graphs by plotting points as indicated in the text. Carefully read **Graphing the Function *y* = sin *θ*** and **Graphing the function *y* = cos *θ***. Make sure that you understand how the graphs are generated. Since $\sin\frac{\pi}{6} = \frac{1}{2}$, it is convenient to use $\frac{\pi}{6}$ and angles that are multiples of $\frac{\pi}{6}$ to plot the graph. When drawing the scale on the θ-axis, (x-axis) divide the distance from 0 to π into 6 intervals, because $\frac{\pi}{6}$ is $\frac{1}{6}$ of the way from 0 to π.	

References and Notes	Work to Submit
Answer the following questions. ▶▶	1.19 See your instructor for **Prerequisites** exercises on this section. 1.20 Using a diagram, define the terms: i) cycle ii) period 1.21 **Exercises**, pages 196 - 198 Answer questions 1, 2 and 3. (*See note below on question 2.*) Answer questions 4, 5, 6, 7 and 8. (*See notes below on questions 4 and 5.*) Answer questions 9, 10, 11 and 12. (*See notes below on questions 9 and 11.)* **Question 2:** Since $\cos\frac{\pi}{3} = \frac{1}{2}$, $\frac{\pi}{3}$, and angles that have $\frac{\pi}{3}$ as a reference angle, are used to graph $\cos\theta$. Angles that are multiples of $\frac{\pi}{2}$ are also used to graph $\cos\theta$. The distance from 0 to π along the θ-axis should be divided into 6 intervals. [Recall that $2\left(\frac{\pi}{6}\right) = \frac{\pi}{3}$.] **Question 4:** You can draw a sketch to show that 140° has a reference angle of 40°. Add or subtract 360° from 40° and 140° to find coterminal angles.

References and Notes	Work to Submit
	Question 5: Any angle coterminal with $40°$ or $^{-}40°$ is a correct answer.
	Questions 9 and 11: Don't make a table of values; but choose some critical points to plot. For $y = \sin\theta$, the strategic points are $(0, 0)$, $(\frac{\pi}{2}, 1)$, $(\pi, 0)$, $(\frac{3\pi}{2}, -1)$ and $(2\pi, 0)$. For $y = \cos\theta$, the strategic points are $(0, 1)$, $(\frac{\pi}{2}, 0)$, $(\pi, -1)$, $(\frac{3\pi}{2}, 0)$ and $(2\pi, 1)$.
Read **Section 3.7**. Work through all calculations given. The definition of the tangent function in the text should explain why the name "tangent" is given to this function. There are two other definitions for the tangent function. One is $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\cos\theta \neq 0$ and the other is $\tan\theta = \frac{y}{x}$, where the terminal arm of angle θ in standard position intersects the unit circle at (x, y).	

References and Notes	Work to Submit
Answer the following questions. ▶▶	1.22 See your instructor for **Prerequisites** exercises. 1.23 **Exercises**, pages 209 and 210 Answer questions 1 - 5. (*See note below on question 5.*) Answer questions 7 - 10. (*See note below on questions 8 - 10.*) **Question 5:** Other angles with the same tangent as 1.92 radians can be generated by adding or subtracting multiples of π. **Questions 8 - 10:** Use the definition $\tan\theta = \frac{y}{x}$ which was developed in question 7.
Read page 216 of **Section 3.9**. Study **Example 1**. Use your scientific calculator to work through the problems in **Example 1.** The definitions of the reciprocal functions are given in terms of the primary trigonometric functions. On the TI-83, the reciprocal key is [x^{-1}]; on a scientific calculator, the reciprocal key is [$1/x$]. Sin^{-1} is <u>not</u> the reciprocal of the sine function.	

References and Notes	Work to Submit
In **Example 1**, csc 65° could be written as $\frac{1}{\sin 65°}$ and then this expression could be directly keyed into your calculator. Answer the following questions. ▶▶	1.24 **Exercises**, page 218 Answer questions 2 and 3. (*See note below on these questions*.) **Questions 2 and 3:** Make sure that you change the settings on your calculator to degrees or radians as required. When finding sec 1.75, you can enter $\frac{1}{\cos 1.75}$ directly <u>or</u> enter cos 1.75 and then press [x^{-1}] or [$1/x$] to determine the secant. 1.25 Complete the table on special angles found in the Appendix.

Unit 2 - Trigonometric Functions of Real Numbers

To meet the objectives of this unit, students should complete the following:

Reading for this unit: *Mathematics 12*

Chapter 4:
- Section 4.1: pages 226 - 236
- Section 4.2: pages 237 - 247
- Section 4.3: pages 249 - 256

References and Notes	Work to Submit
Read **Section 4.1**.	
In this chapter, each of the sine and cosine functions is defined as a function of a real number. This real number is associated with an arc length on the unit circle. The arc length on the unit circle is equal to the radian measure of the central angle.	2.1 See your instructor for **Prerequisites** exercises before continuing.
Answer the following questions. ▶▶	2.2 **Investigate**, page 226 Ask your instructor for a copy of the large unit circle. Answer questions 1 - 5.
	2.3 **Discussing the Ideas**, page 232 Answer questions 1 - 3.

References and Notes

Before you complete **Investigate** on page 237, do the following.

Work to Submit

2.4 **Exercises**, pages 232 and 233
Answer questions 1 - 6.
(*See notes below on questions 5 and 6.*)

Question 5: You should use a calculator to complete these questions, but use a large unit circle to verify two or three of these problems.

Question 6: Graph these functions using a TI-83 and also using a pencil and grid paper.

2.5 **Exploring with a Graphing Calculator**, page 236
Answer questions 1 - 4.

Note: Make sure that your calculator is in radian mode.

2.6 See your instructor for **Prerequisites** exercises on **Section 4.2**.

2.7 Graph the functions $y = \sin x$, $y = 2 \sin x$, $y = {}^{-}1 \sin x$, and $y = .5 \sin x$ using the window settings given on page 237. What happens to the graph of $y = a \sin x$ when a is changed?
Repeat this exercise with cosine functions.

2.8 Graph the functions $y = \sin x$, $y = \sin x + 2$, $y = \sin x - .5$, and $y = \sin x - 2$. What happens to the graph of $y = \sin x + d$ when d is changed?
Repeat this exercise with cosine functions.

References and Notes

Study **Section 4.2**.

Answer the following questions. ▶▶

This section contains a lot of new material. Take your time and make sure that you understand each type of problem <u>before</u> moving on to the next type. Much of the material reinforces the transformations that you studied in Chapter 1.

When working through this section, you should sketch the graphs that you see on your TI-83 screen. Use grid paper or obtain copies of 'Graphing Calculator Screen Template' from your instructor. The 'Grid Templates' (from your instructor) should also prove helpful.

You may need some extra explanations from your instructor while completing this section.

Work to Submit

2.9 **Investigate**, pages 237 and 238
Answer questions 1 - 4.
(*See note below on question 1.*)

Answer questions 5 - 8.
(*See note below on these questions*.)

Question 1: When entering $y = 2 \sin x + 1$ on your graphing calculator, be sure to close the bracket after the x, $y = 2 \sin (x) + 1$.

Since the function $y = 2 \sin x + 1$ has amplitude 2 and a vertical shift 1, the maximum value is 3, therefore the window setting should be changed to $^{-}1 \le y \le 3$.

When sketching the graph that you see on the screen, the following steps may be useful:

1) Draw the vertical shift line in red and mark this 'new' axis with the same scale as the x-axis.

2) Mark the five strategic points that define one period.

The strategic points for $y = \sin x$ are $(0,0)$, $(\frac{\pi}{2}, 1)$, $(\pi, 0)$, $(\frac{3\pi}{2}, -1)$ and $(2\pi, 0)$.

The strategic points for $y = \cos x$ are $(0,1)$, $(\frac{\pi}{2}, 0)$, $(\pi, {}^{-}1)$, $(\frac{3\pi}{2}, 0)$, $(2\pi, 1)$.

3) Draw the graph.

Questions 1 - 4 deal with amplitude change and vertical displacement.

Questions 5 - 8: These questions work with horizontal translation (or phase shift).

Unit 2 - Trigonometric Functions of Real Numbers

References and Notes	Work to Submit
Carefully read pages 238 and 239, which describe the vertical displacement and the amplitude of a sinusoidal function. Study **Example 1**, page 240. One way to graph this type of function is to graph $y = \sin x$ first, then expand it vertically and finally incorporate the vertical translation. Read page 241 which explains the phase shift (or horizontal translation) of a sinusoidal function. Study the graphs of $y = \sin (x - c)$ and $y = \cos (x - c)$ in **Visualizing** on page 242. Study **Example 2**, page 242. It may be easier to expand vertically then translate vertically, and lastly, shift the graph horizontally. Study **Example 3**. Notice that, depending on the phase shift, this graph could be a cosine or sine function.	

Unit 2 - Trigonometric Functions of Real Numbers

References and Notes	Work to Submit
Answer the following questions. ▶▶	2.10 **Discussing the Ideas**, page 244 Answer questions 1 - 9.
See your instructor to have your answers to **Discussing the Ideas** corrected before moving on to the **Exercises**.	2.11 **Exercises**, pages 245 - 247 Answer questions 1 - 3. (*See note below on questions 1 and 3.*) Answer questions 10 - 15 and 19. (*See notes below on questions 10 and 19.*)
	Questions 1 and 3: Each constant in the equation has significance for the graph. See page 238 in *Mathematics 12* and write out a summary of the transformation for the general formulas: $y = a \sin (x - c) + d$ and $y = a \cos (x - c) + d$.
	Questions 10: Note that the phase shifts in parts b and d are <u>not</u> multiples of π which you have been used to seeing. If you approximate π as 3.14, then you can see where the values 1.5 and 4 are located.
	Question 19: Obtain copies of the 'Grid Template' from your instructor before completing this question.
Study **Section 4.3**. Answer the following questions. ▶▶	2.12 See your instructor for **Prerequisites** exercises.

References and Notes	Work to Submit
After completing **Investigate**, you should have discovered that the period of the function $y = \sin bx$ is $\frac{2\pi}{b}$.	2.13 **Investigate**, page 249 Answer questions 1 - 6. (*See note below on question 6.*) **Question 6:** You may experience difficulty sketching these functions and scaling the horizontal axis. If you look ahead to the purple box on page 250 as well as **Example 1** on page 252, you will find a method which should make sketching easier.
Study pages 250 and 251. Work through **Examples 1** and **2**, following the steps given in the solution. These two examples provide two methods for drawing sinusoidal curves. When combining a change in period with a phase shift, it is very important that you find the period <u>first</u> and apply the phase shift <u>second</u>.	
Answer the following questions. ▶▶	2.14 **Discussing the Ideas**, page 253 Answer questions 1 - 5.
See your instructor to have your answers for **Discussing the Ideas** corrected before moving on to the **Exercises**.	

References and Notes	Work to Submit
	2.15 **Exercises**, pages 254 - 256 Answer questions 1 and 2**.** (*See note below on question 1.*) Answer questions 6, 7 and 8. (*See note below on question 8.)* Answer questions 11, 14, 15 and 18. (*See note below on questions 14 and 15.*) **Question 1:** Determine the period by looking at the graph. Make the period equal to $\frac{2\pi}{b}$ and solve for b. Each of these graphs can be defined as a sine equation or cosine equation. (See page 243 in *Mathematics 12*). If you write a cosine equation, there will have to be a phase shift. **Question 8:** You can write a sine or cosine equation to represent each of these functions. **Questions 14 and 15:** These questions are similar to **Example 2** on page 252. You may use the method of **Example 1** or **Example 2** to do these questions.

Unit 3 - Trigonometric Equations and Identities

To meet the objectives of this unit, students should complete the following:

Reading for this unit:	*Mathematics 12*		
	Chapter 5:	Section 5.1:	pages 298, 299 and 302
		Section 5.2:	pages 308 - 314
		Section 5.3:	pages 317 - 320
		Section 5.4:	pages 322 - 327

References and Notes	Work to Submit
You will <u>not</u> be studying all of **Section 5.1**. Read **Example 1** on pages 298 and 299. Use your graphing calculator to work through the solution given in the text. (Omit **Examples 2** and **3**). Answer the following questions. ▶▶	3.1 See your instructor for **Prerequisites** exercises before completing **Exercises**. 3.2 **Exercises**, page 302 Answer questions 1a), 1d), 2, 3a), 3e) and 4. (*See notes below on these questions*.) **Note:** You should draw a sketch of the function to determine the number of solutions, before you use your TI-83. **Question 3:** Don't forget to change the domain to $-\pi$ to π.

References and Notes	**Work to Submit**
Section 5.2 uses a different method to solve trigonometric equations than normally taught in Math textbooks. You can read this section, and, if you understand and are comfortable with the method used in the **Examples**, you can work through the **Exercises** using that method. Otherwise, you should refer back to **Section 3.5** in *Mathematics 12* and use the concept of reference angles to solve trigonometric equations which have exact answers. Look at **Example 1**. Since we know the sine and cosine of special angles, we should realize that if $\sin x = \frac{-1}{2}$ for $0 \le x \le 2\pi$, then the *reference* angle must be $\frac{\pi}{6}$, since $\sin \frac{\pi}{6} = \frac{1}{2}$. Look at the sketch on page 185 of *Mathematics 12*. The point P on the unit circle has coordinates $(\cos \frac{\pi}{6}, \sin \frac{\pi}{6})$ or $(\frac{\sqrt{3}}{2}, \frac{1}{2})$. Sine has a negative value in quadrants 3 and 4. (We want $\sin x = \frac{-1}{2}$). Since the reference angle is $\frac{\pi}{6}$, the two angles we want are $\pi + \frac{\pi}{6} = \frac{7\pi}{6}$ and $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$.	

References and Notes	**Work to Submit**
You can use your calculator to verify that $\sin\frac{7\pi}{6} = \frac{-1}{2}$ <u>and</u> $\sin\frac{11\pi}{6} = \frac{-1}{2}$. So, for $0 \le x < 2\pi$, the solution is $x = \frac{7\pi}{6}$ and $x = \frac{11\pi}{6}$. When looking at **Example 2**, $\cos 3x = \frac{\sqrt{2}}{2}$ for $0 \le x < 2\pi$. Consider $\cos\theta = \frac{\sqrt{2}}{2}$ where $\theta = 3x$. (Note that the period of $\cos 3x$ is $\frac{2\pi}{3}$). The reference angle is $\frac{\pi}{4}$. (See page 184 for review.) Therefore, since $\cos\theta$ is positive, then $\theta = \frac{\pi}{4}$ and $\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$. Now, find x. $3x = \theta = \frac{\pi}{4}$ $x = \frac{\pi}{12}$ Also, $3x = \theta = \frac{7\pi}{4}$ $x = \frac{7\pi}{12}$	

References and Notes	Work to Submit
To find the other roots in the interval $0 \le x < 2\pi$, add the period ($\frac{2\pi}{3}$) to each root and repeat, if necessary. The other roots are $\frac{\pi}{12} + \frac{2\pi}{3} = \frac{9\pi}{12}$ and $\frac{9\pi}{12} + \frac{2\pi}{3} = \frac{17\pi}{12}$ and, $\frac{7\pi}{12} + \frac{2\pi}{3} = \frac{15\pi}{12}$ and $\frac{15\pi}{12} + \frac{2\pi}{3} = \frac{23\pi}{12}$. Therefore when $0 \le x < 2\pi$, the solution of the equation is $x = \frac{\pi}{12}, \frac{7\pi}{12}, \frac{9\pi}{12}, \frac{15\pi}{12}, \frac{17\pi}{12}$ and $\frac{23\pi}{12}$. You should use your TI-83 to sketch this curve and see why there are 6 angles. In **Example 3**, factor and solve as shown in the textbook. However, when you find $\cos x = \frac{-1}{2}$ and $\cos x = 1$, use your special angle and quadrantal angles to find the solution. You will need to go over this section with your instructor.	

Unit 3 - Trigonometric Equations and Identities

References and Notes	Work to Submit
Answer the following questions. ▶▶	3.3 **Exercises**, page 313 and 314 Answer questions 1, 2 and 3. (*See note below on question 1.*) Answer questions 5a), 5b), 5d), 5e), 5f), 6a) and 6b). (*See note below on question 5.*) Answer questions 8 and 9. (*See note below on question 8.*) **Question 1:** You should be able to sketch each curve and recognize that a decrease in the period increases the number of roots. (A decrease in the period ($\frac{2\pi}{b}$) means that there are more cycles in a given interval, therefore there are more roots.) **Question 5:** Why does part b have one solution, yet there are 2 solutions for each of the other parts? **Question 8:** 8b) and 8d) have no solution. You should be able to sketch the graph of the equation sin $x = {}^{-}2$ and explain why there is no solution.
In **Section 5.3**, omit pages 315 and 316, read page 317 and study **Example** on page 318. Take note of the **Quotient Identity** and **Pythagorean Identity** on page 317. Answer the following questions. ▶▶	3.4 **Exercises**, page 319 and 320 Answer questions 4, 6a), 6b), 7a), 14a), 14b) and 14c).

References and Notes	Work to Submit
Read **Section 5.4**. For this course, we will mostly consider verifying identities numerically and algebraically. When studying **Example 1** on page 322, you should particularly note parts a) and d). Study **Examples 2**, **3** and **4.**	3.5 See your instructor for **Prerequisites** exercises on **Section 5.4** before moving on to the **Exercises**.
Proving identities requires much practice. You should have all of the identities written on one page for easy reference. These identities should be memorized. When proving identities, you must keep the left side and right side separate. It is okay to work with one side until you reach an expression which is equal to the other side. Sometimes you may work on each side separately until you obtain the same expression. You <u>cannot</u>, however, cross multiply or multiply or divide both sides by a term.	

References and Notes	Work to Submit
Answer the following questions. ▶▶	3.6 **Exercises**, page 326 and 327 Answer questions 1, 3, 4 and 5. (*See note below on question 4.*) Answer questions 7a) and 7c). (*See note below on question 7c.*) Answer questions 12 and 13. **Questions 4c) and 4d)**: Write tan x and cot x in terms of sin x and cos x. Get a common denominator, combine and then simplify. **Question 7c**: You can algebraically prove this identity by using a method similar to **Example 4a**.

Appendix

Draw and label the sides of a 30-60-90 trangle and a 45-45-90 triangle. Use the sketches to complete the table.
Leave your answers in exact form. (Don't use a calculator!)

θ	sin θ	cos θ	tan θ	csc θ	sec θ	cot θ
0°	0	1	0	Undefined		Undefined
30°				2	$\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$	
45°			1			1
60°			$\sqrt{3}$		2	$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$
90°	1	0	Undefined	1		
120°		$-\frac{1}{2}$				
135°	$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$					
150°						
180°						