

Adult Basic Education

Mathematics

Mathematics 3104A Quadratic Functions, Graphs and Equations

Study Guide

Prerequisite: Math 1104A, Math 1104B, Math 1104C
Math 2104A, Math 2104B, Math 2104C

Credit Value: 1

Text: *Mathematics 11*. Alexander and Kelly; Addison-Wesley, 1998.

Required Mathematics Courses [General College Profile]

Math 1104A

Math 1104B

Math 1104C

Math 2104A

Math 2104B

Math 2104C

Math 3104A

Math 3104B

Math 3104C

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To the Student

I. Introduction to Mathematics 3104A

This course introduces you to quadratic functions, equations and graphs. Quadratics, in particular, is an essential and fundamental component of post-secondary Math courses.

You will learn to work with quadratics in many different forms:

- functions
- graphs
- tables of values
- written description
- ordered pairs

You will learn to translate between these different ways of presentation.

Quadratic functions can be used to represent real-life situations such as maximizing area, profit or height. You will also learn different ways to solve quadratic equations.

II. Resources

You will require the following:

- *Addison Wesley Mathematics 11*, Western Canadian edition Textbook
- Scientific calculator
- graph paper
- Access to a TI-83 Plus graphing calculator (see your instructor) and/or *Graphmatica* or *Winplot* graphing software

Notes concerning the textbook:

Glossary: Knowledge of mathematical terms is essential to understand concepts and correctly interpret questions. Written explanations will be part of the work you submit for evaluation, and appropriate use of vocabulary will be required.

Your text for this course includes a Glossary where definitions for mathematical terms are found. Be sure you understand such definitions and can explain them in your own words. Where appropriate, you should include examples or sketches to support your definitions.

Examples: You are instructed to study carefully the **Examples** in each section and see your instructor if you have any questions. These **Examples** provide full solutions to problems that can be of great use when answering assigned **Exercises**.

To the Student

Notes concerning technology:

It is important that you have a **scientific** calculator for your individual use. Ensure that the calculators used have the word “scientific” on it as there are calculators designed for calculation in other areas such as business or statistics which would not have the functions needed for study in this area. Scientific calculators are sold everywhere and are fairly inexpensive. You should have access to the manual for any calculator that you use. It is a tool that can greatly assist the study of mathematics but, as with any tool, the more efficient its use, the better the progress.

You will require access to some sort of technology in order to meet some of the outcomes in this course. Since technology has become a significant tool in the study of Mathematics, your textbook encourages you to become proficient in its use by providing you with step-by-step exercises that will teach you about the useful functions of the TI-83 Plus graphing calculator. **See your instructor concerning this.** Please note that a graphing calculator is not essential for success in this course but it is useful.

While graphing calculators and graphing software (*Graphmatica* or *Winplot*) are useful tools, they cannot provide the same understanding that comes from working paper and pencil exercises.

III. Study Guide

This Study Guide is required at all times. It will guide you through the course and you should take care to complete each unit of study in the order given in this Guide. Often, at the beginning of each unit, you will be instructed to see your instructor for **Prerequisite** exercises. Please do not skip this step! It should only take a few minutes for you and your instructor to discover what, if any, prerequisite skills need review.

To be successful, you should read the **References and Notes** first and then, when indicated by the ☐☐ symbols, complete the **Work to Submit** problems. Many times you will be directed to see your instructor, and this is vital, especially in a Mathematics course. If you only have a hazy idea about what you just completed, nothing will be gained by continuing on to the next set of problems.

To the Student



Reading for this Unit:

In this box, you will find the name of the text, and the chapters, sections and pages used to cover the material for this unit. As a preliminary step, skim the referenced section, looking at the name of the section, and noting each category. Once you have completed this overview, you are ready to begin.

References and Notes

This left hand column guides you through the material to read from the text.

It will also refer to specific **Examples** found in each section. You are directed to study these **Examples** carefully and see your instructor if you have any questions. The **Examples** are important in that they not only explain and demonstrate a concept, but also provide techniques or strategies that can be used in the assigned questions.

The symbols   direct you to the column on the right which contains the work to complete and submit to your instructor. You will be evaluated on this material.

Since the answers to **Discussing the Ideas** and **Communicating the Ideas** are not found in the back of the student text, you **must** have these sections corrected by your instructor **before** going on to the next question.

This column will also contain general **Notes** which are intended to give extra information and are not usually specific to any one question.

Work to Submit

There are four basic categories included in this column that correspond to the same categories in the sections of the text. They are **Investigate**, **Discussing the Ideas**, **Exercises**, and **Communicating the Ideas**.

Investigate: This section looks at the thinking behind new concepts. The answers to its questions are found in the back of the text.

Discussing the Ideas: This section requires you to write a response which clarifies and demonstrates your understanding of the concepts introduced. The answers to these questions are not in the student text and will be provided when you see your instructor.

Exercises: This section helps to reinforce your understanding of the concepts introduced. There are three levels of **Exercises**:
A: direct application of concepts introduced
B: multi-step problem solving and some real-life situations
C: problems of a more challenging nature
The answers to the **Exercises** questions are found in the back of the text.

Communicating the Ideas: This section helps confirm your understanding of the lesson of the section. If you can write a response, and explain it clearly to someone else, this means that you have understood the topic. The answers to these questions are not in the student text and will be provided when you see your instructor

This column will also contain **Notes** which give information about specific questions.

To the Student

IV. Recommended Evaluation

Written Notes	10%
Assignments	10%
Test(s)	30%
Final Exam (<i>entire course</i>)	<u>50%</u>
	100%

The overall pass mark for the course is 50%.

Unit 1 - Quadratic Functions and Graphs

To meet the objectives of this unit, students should complete the following:

Reading for this unit:	<i>Mathematics 11</i>
Chapter 2:	Linking Ideas: pages 84 - 87
	Section 2.1: pages 88 - 97
	Section 2.2: pages 101 - 107
	Section 2.3: pages 109 - 119
	Section 2.4: pages 122 - 126
	Section 2.5: pages 127 - 131

References and Notes

Carefully read **Graphing a Function** on pages 84 to 87. Using a graphing calculator, follow the **Graphing a Function** exercise and do the following questions. ▶▶

Note: If you have a graphing calculator, pages 84 to 87 will guide you through the series of steps to graph quadratic functions and obtain significant information from the graph. Take the time to go through the exercises to learn about or review the capabilities of your calculator which is a powerful tool when used knowledgeably. See your instructor if you have any questions.

If you don't have a graphing calculator, move on to the next Study Guide question.

Work to Submit

- 1.1 See your instructor for **Prerequisites** review questions before beginning this section.
- 1.2 **Graphing a Function**, page 84-87
Answer questions 1 - 9.

Unit 1 - Quadratic Functions and Graphs

References and Notes

Read **Section 2.1**, on pages 88 to 97. Study and work through each step in **Examples 1 to 4**. See your instructor if you have any questions. Do the following questions. ▶▶

Note: The glossary at the end of the text can be used as an additional resource when looking for definitions. If the text definition is not sufficiently clear, write a definition in your own words and have your instructor check it for accuracy. If appropriate, use an example or diagram to support your definition.

The answers to **Discussing the Ideas** questions will not be in the back of the text. Have your answers checked by your instructor before proceeding to the next question in the Study Guide.

Work to Submit

- 1.3 Define the following terms and, where appropriate, give an example or sketch:
- i) quadratic function
 - ii) parabola
 - iii) axis of symmetry
 - iv) vertex
 - v) quadratic equation
 - vi) roots of the quadratic equation
 - vii) zeros of the quadratic equation
 - viii) x - intercepts
 - ix) domain
 - x) range

Note: Look at the table of values that accompanies **Example 1** on pages 89 and 90. See if you can identify the intercepts from the coordinates. The vertex can be estimated from the graph that accompanies the table of values or found by using the **TRACE** or **CALC** features of your graphing calculator. For supplementary material concerning the use of these functions refer to pages 19 to 23 of the **Graphing Calculator Handbook** which is included in the *Teacher's Resource Book*.

- 1.4 **Discussing the Ideas**, page 94
Answer questions 1 and 2.
(See note below on question 2.)

Answer questions 3 - 5.

Note: In question 2, recall that x-intercepts have a y value of 0. Therefore you can set the function equal to zero, factor and solve.

Unit 1 - Quadratic Functions and Graphs

References and Notes	Work to Submit
<p>Read Section 2.2, on pages 101 to 107. Study Examples 1 and 2 carefully and see your instructor if you have any questions. Do the following questions.▶▶</p> <p>Bring your completed questions in Discussing the Ideas to your instructor for correction before proceeding to the next question in the Study Guide.</p>	<p>1.5 Exercises, pages 94 - 97 Answer questions 1 - 7. <i>(See note below on question 7.)</i></p> <p>Answer questions 9a, 12 - 15. <i>(See note below on questions 12 and 14.)</i></p> <p>Note: Answer question 7 using a table of values as well as using a graphing calculator.</p> <p>Note: Question 12 says “solve the equation” while question 14 says “determine the zeros of each quadratic function”. Both are asking for the same thing. The equation is always set equal to 0 while the function is equal to $f(t)$ or y. When solving a function, the first step will be to set $f(t) = 0$ and then proceed in exactly the same fashion as the “solve the equation” question.</p> <p>1.6 Discussing the Ideas, page 105 Answer questions 1 - 6.</p>

Unit 1 - Quadratic Functions and Graphs

References and Notes

Note: When reading a word problem, underline key words such as “perimeter, area, rectangle”, etc. If you don’t remember the significance of these words, look up their meanings because this type of vocabulary provides necessary information for problem solving.

Diagrams can be useful when you are in the initial stages of reading a word problem. See **Example 1** on page 101 of your textbook.

Read **Section 2.3**, on pages 109 to 119.

Study **Examples 1, 2 and 3**. See your instructor if you have any questions. Do the following questions. ▶▶

Work to Submit

1.7 **Exercises**, pages 105 - 107
Answer questions 1 - 4.
(See note below on question 4.)

Answer questions 8 and 9.
(See note below on question 8.)

Note: For question 4, write the expression in words before translating into mathematical terms.

Note: For question 8, notice that you are asked for profit, not revenue. Profit is calculated as revenue – expenses.

1.8 Define the term *congruent parabolas* in your own words.

1.9 **Investigate : Transforming Quadratic Graphs**, pages 109 - 110
Answer questions 1 - 6.

Unit 1 - Quadratic Functions and Graphs

References and Notes

Note: In the **Investigate** section on pages 109 and 110, you are asked to compare graphs of related quadratic functions. Whether you are using a graphing calculator or table of values, you are looking for the choice of x -values that will give you the characteristic upward facing or downward facing U - shape of a parabola. In the table of values, this would be represented in the y -values by the unique value of the vertex with repeated y -values on either side. You may have to adjust the x -values you are choosing in order to see this type of pattern in the y -values.

Read **Section 2.4**, on pages 122 and 123.

Carefully study **Examples 1 and 2** and **Visualizing**. See your instructor if you have any difficulties. Answer the following questions. ▶▶

Bring your completed questions in **Discussing the Ideas** to your instructor for correction before proceeding to the next question in the Study Guide.

Work to Submit

1.10 **Discussing the Ideas**, page 115
Answer questions 1 and 2.

1.11 **Exercises**, pages 115 - 119
Answer questions 1- 5, 7 - 9, 12, 14 and 15.
(See note below on question 14.)

Note: In 14a), substitute the coordinates of the vertex $(0,2)$ for (p, q) and $(-3,11)$ for (x, y) in the equation

$$y = a(x - p)^2 + q$$

and then solve for a .

1.12 **Discussing the Ideas**, page 124
Answer questions 1, 3 and 4.

Unit 1 - Quadratic Functions and Graphs

References and Notes

Note: Questions 11 and 12 on page 124 are to be done with a graphing calculator in order to compare related quadratic functions. If you don't have a graphing calculator, compare a variety of functions from the question using tables of values. Do a sufficient number so you can answer parts b) and c) for each question.

Read **Section 2.5**, pages 127 to 129.

Follow **Examples 1** and **2**. See your instructor if you have any questions. Answer the following questions. ▶▶

Bring your completed questions in **Discussing the Ideas** to your instructor for correction before proceeding to the next question in the Study Guide.

Work to Submit

1.13 **Exercises**, pages 124 - 126
Answer questions 1-13.
(See note below on questions 1, 5, 7 and 8.)

Note: In questions 1, 5, 7 and 8, if the x^2 coefficient is not 1, be sure to begin by removing that coefficient as a common factor from the x^2 and x terms before completing the square.

Taking one-half of the x -coefficient may result in a fraction. If this is the case, leave it as a fraction rather than changing it to a decimal. It will ultimately be easier to factor if it is in a fraction form.

1.14 **Discussing the Ideas**, page 129
Answer question 1.

1.15 **Exercises**, pages 130 - 131
Answer questions 1-3, 5, 6, 8, 9, and 11
(See note below on questions 8 and 9.)

Note: Draw sketches for questions 8 and 9.

Unit 2 : Quadratic Equations

To meet the objectives of this unit, students should complete the following:

Reading for this unit: *Mathematics 11*

Chapter 4: Section 4.1: pages 226 - 233
Mathematics File: pages 234 - 235
Section 4.2: pages 240 - 245
Linking Ideas: pages 246-247

References and Notes

Read **Section 4.1**, on pages 226 to 234.

Answer the following questions.



Note: Investigate on page 226 changes a quadratic equation to a standard form. Question 1a) asks you to solve it in that form. The following is an example for that solution which has not been shown in the text:

$$5(x - 3)^2 - 7 = 0$$

$$5(x - 3)^2 = 7$$

$$(x - 3)^2 = 1.4$$

$$x - 3 = \pm \sqrt{1.4}$$

$$x = 3 \pm \sqrt{1.4}$$

$$x = 3 + \sqrt{1.4}, x = 3 - \sqrt{1.4}$$

Work to Submit

2.1 **Investigate**, page 226
Answer questions 1 and 2.

Note: Question 3 in **Investigate** involves the derivation of the quadratic formula. You can also read **Mathematics File** on page 234 which illustrates this derivation.

Unit 2 : Quadratic Equations

References and Notes

Study **Examples 1, 2** and **3**. See your instructor if you have any questions.

Note: The graphs on page 229 help you visualize the meaning of the algebraic solutions to the functions in **Example 1**. Although you may check your algebraic answers by graphing the function, it is usually done according to the method illustrated in **Example 1**.

The quadratic equation in **Example 2** models the motion of a diver. Previously, you were interested in the vertex of the parabola. Now you are looking at the zeros of the function; where the parabola meets the x -axis.

The Quadratic Formula is the fourth method that can be used to solve a quadratic function. The other methods already discussed, in this or other courses, are factoring, graphing and completing the square.

Bring your completed questions in **Discussing the Ideas** to your instructor for correction before proceeding to the next question in the Study Guide.

Work to Submit

2.2 **Discussing the Ideas**, page 231
Answer questions 1- 4.

Unit 2 : Quadratic Equations

References and Notes

Note: The quadratic formula,

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

will find two solutions.

One solution is

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and the other is

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

The answers to **Communicating the Ideas** questions will not be in the back of the text. Bring your answers to your instructor to check before proceeding to the next question in the Study Guide.

Read **Section 4.2**, on pages 240 to 243.

Study **Examples 1 and 2**. See your instructor if you have any questions.

The graphs on page 240 will help you visualize the relationship between the value of the discriminant and the x -intercepts of the graph of a quadratic function.

Work to Submit

- 2.3 **Exercises**, pages 231 - 233
Answer questions 1 - 3, 5 - 10.
(See notes below on questions 5, 6 and 10.)

Note: In questions 5 and 10, expand and collect all terms on one side when necessary.

Note: Question 6 a) is similar to **Example 2** on page 229. In question 6 b), let $h = 25$ and solve for t . Remember to choose only the answer which will make sense in the context of the problem.

- 2.4 **Communicating the Ideas**, page 233

Unit 2 : Quadratic Equations

References and Notes

The discriminant provides a shortcut to determine the nature of the roots of a quadratic equation, without having to evaluate the complete quadratic formula.

Note: The terms *roots*, *zeros*, and *x-intercepts* refer to the same quantity.

Answer the following questions.



Bring your completed questions in **Discussing the Ideas** to your instructor for correction before proceeding to the next question in the Study Guide.

Bring your answers to the **Communicating the Ideas** questions to your instructor for correction before proceeding to the next question in the Study Guide.

Read **Mathematics & History** on pages 246 and 247. See your instructor if you have any questions.

Answer the following questions.



Work to Submit

2.5 **Discussing the Ideas**, page 243
Answer questions 1-3.

2.6 **Exercises**, pages 244 - 245
Answer questions 1-10.

2.7 **Communicating the Ideas**, page 245

2.8 **Mathematics & History**, page 247