Adult Basic Education

Mathematics

Mathematics 3104B

Exponents and Logarithms

Study Guide

Prerequisites: Math 1104A, Math 1104B, Math 1104C,
Math 2104A, Math 2104B, Math 2104C
Math 3104A

Credit Value: 1

Text: *Mathematics 12.* Alexander and Kelly; Addison Wesley, 1999.

<u>**Required Mathematics Courses**</u> **[Degree and Technical Profile]**
Mathematics 1104A
Mathematics 1104B
Mathematics 1104C
Mathematics 2104A
Mathematics 2104B
Mathematics 2104C
Mathematics 3104A
Mathematics 3104B
Mathematics 3104C

Table of Contents

To the Student

I. Introduction to Mathematics 3104B

This course is an introduction to exponential and logarithmic functions, equations and graphs. You will interpret information from equations, graphs and written descriptions and to learn translate between these different ways of presentation. Other topics involve a variety of strategies for simplifying or solving exponential and logarithmic equations. Real-life situations demonstrating exponential or logarithmic behavior include population growth (human or bacteria), radioactive decay, earthquake and sound intensity and PH levels.

II. Resources

You will require the following:

- *Addison Wesley Mathematics 12*, Western Canadian edition Textbook
- Scientific calculator
- graph paper
- Access to a TI-83 Plus graphing calculator (see your instructor) and/or *Graphmatica* or *Winplot* graphing software

Notes concerning the textbook:

Glossary: Knowledge of mathematical terms is essential to understand concepts and correctly interpret questions. Written explanations will be part of the work you submit for evaluation, and appropriate use of vocabulary will be required.

Your text for this course includes a Glossary where definitions for mathematical terms are found. Be sure you understand such definitions and can explain them in your own words. Where appropriate, you should include examples or sketches to support your definitions.

Examples: You are instructed to study carefully the **Examples** in each section and see your instructor if you have any questions. These **Examples** provide full solutions to problems that can be of great use when answering assigned **Exercises**.

To the Student

<u>Notes concerning technology</u>:

It is important that you have a **scientific** calculator for your individual use. Ensure that the calculator used has the word "scientific" on it as there are calculators designed for calculation in other areas such as business or statistics which would not have the functions needed for study in this area. Scientific calculators are sold everywhere and are fairly inexpensive. You should have access to the manual for any calculator that you use. It is a tool that can greatly assist the study of mathematics but, as with any tool, the more efficient its use, the better the progress.

You will require access to some sort of technology in order to meet some of the outcomes in this course. Since technology has become a significant tool in the study of Mathematics, your textbook encourages you to become proficient in its use by providing you with step-by-step exercises that will teach you about the useful functions of the TI-83 Plus graphing calculator. **See your instructor concerning this**. Please note that a graphing calculator is not essential for success in this course, but it is useful.

While graphing calculators and graphing software (*Graphmatica* or *Winplot*) are useful tools, they cannot provide the same understanding that comes from working paper and pencil exercises.

III. <u>Study Guide</u>

This Study Guide is required at all times. It will guide you through the course and you should take care to complete each unit of study in the order given in this Guide. Often, at the beginning of each unit, you will be instructed to see your instructor for **Prerequisite** exercises. Please do not skip this step! It should only take a few minutes for you and your instructor to discover what, if any, prerequisite skills need review.

To be successful, you should read first the **References and Notes** and then, when indicated by the ⏵⏵ symbols, complete the **Work to Submit** problems. Many times you will be directed to see your instructor, and this is vital, especially in a Mathematics course. If you only have a hazy idea about what you just completed, nothing will be gained by continuing on to the next set of problems.

To the Student

Reading for this Unit:
In this box, you will find the name of the text, and the chapters, sections and pages used to cover the material for this unit. As a preliminary step, skim the referenced section, looking at the name of the section, and noting each category. Once you have completed this overview, you are ready to begin.

References and Notes	Work to Submit
This left hand column guides you through the material to read from the text. It will also refer to specific **Examples** found in each section. You are directed to study these **Examples** carefully and see your instructor if you have any questions. The **Examples** are important in that they not only explain and demonstrate a concept, but also provide techniques or strategies that can be used in the assigned questions. The symbols ▶▶ direct you to the column on the right which contains the work to complete and submit to your instructor. You will be evaluated on this material. Since the answers to **Discussing the Ideas** and **Communicating the Ideas** are not found in the back of the student text, you **must** have these sections corrected by your instructor **before** going on to the next question. This column will also contain general **Notes** which are intended to give extra information and are not usually specific to any one question.	There are four basic categories included in this column that correspond to the same categories in the sections of the text. They are **Investigate, Discussing the Ideas**, **Exercises**, and **Communicating the Ideas**. **Investigate:** This section looks at the thinking behind new concepts. The answers to its questions are found in the back of the text. **Discussing the Ideas**: This section requires you to write a response which clarifies and demonstrates your understanding of the concepts introduced. The answers to these questions are not in the student text and will be provided when you see your instructor. **Exercises**: This section helps to reinforce your understanding of the concepts introduced. There are three levels of **Exercises**: **A:** direct application of concepts introduced **B:** multi-step problem solving and some real-life situations **C:** problems of a more challenging nature The answers to the **Exercises** questions are found in the back of the text. **Communicating the Ideas:** This section helps confirm your understanding of the lesson of the section. If you can write a response, and explain it clearly to someone else, this means that you have understood the topic. The answers to these questions are not in the student text and will be provided when you see your instructor This column will also contain **Notes** which give information about specific questions.

IV. Recommended Evaluation

Written Notes	10%
Assignments	10%
Test(s)	30%
Final Exam *(entire course)*	50%
	100%

The overall pass mark for the course is 50%.

Unit 1: Rational Exponents

To meet the objectives of this unit, students should complete the following:

Reading for this unit:	*Appendix A* pages 13 - 32

References and Notes	Work to Submit
Read pages 15-19 in **Appendix A**. Carefully study **Examples 1- 4** and answer these questions. ▶▶	1.1 **Exercises**, pages 18 and 19 Answer questions 1-14. (*See notes below on questions 13 and 14.*) **Note:** In question 13, recall that the order of operations, (BEDMAS) dictates that the exponent must be evaluated before *multiply* or *divide*. **Note:** In question 13(d), $\left(x^{-1}y^{2}\right)$ must be raised to the power of -3, and x^2 raised to the power of -1 before they are multiplied. **Note:** In question 14, complete some of these exercises in two ways: substitute before multiplying and dividing; then simplify before substituting. Which method is more efficient?
Read pages 20-25 in **Appendix A**. Carefully study **Examples 1-4** and answer the following questions. ▶▶ The answers to the **Exercises** found in **Appendix A** are in *Mathematics 10*. Refer to: Section 1.5, pages 32-36 Section 2.1, pages 68-73 Section 2.2, pages 76-83	1.2 **Exercises**, pages 24 and 25 Answer questions 1- 8. (*See note below on question 8.*) Answer questions 10 and 11. **Note:** In question 8, you have to work backwards. To write 5 in square root form, first square 5 and then write it as a radical, $\sqrt{25}$.

Unit 1: Rational Exponents

References and Notes	Work to Submit
Read pages 26-32 in **Appendix A.**	
Answer the following questions. ▶▶	1.3 **Investigate**, page 26 Answer questions 1-7.
Carefully study **Examples 1-3**. See your instructor if you have any questions. Answer the following questions. ▶▶	1.4 **Exercises**, pages 30-32 Answer questions 1-8. (*See note below on questions 1 and 8.*) Answer questions 9-14 and 16. (*See note below on question 16.*)
	Note: For question 1, remember that a power with a rational exponent can be written in two different ways. For example in 1c), $8^{2/3} = (8^{1/3})^2 = (\sqrt[3]{8})^2$ and $8^{2/3} = (8^2)^{1/3} = \sqrt[3]{8^2}$.
	Note: In question 8, you need to work backwards. To write 3 as a power with an exponent of $\frac{1}{2}$, square 3 and then write $9^{\frac{1}{2}}$.
	Note: In question 16, when a variable doesn't have an exponent indicated, the exponent is 1.

Unit 2: Exponential Functions

To meet the objectives of this unit, students should complete the following:

Reading for this unit:	*Mathematics 12* Chapter 2: Section 2.1: pages 66-73

References and Notes	Work to Submit
Read **Section 2.1** on page 66 - 71. Study pages 66 and 67 and carefully read the **Example** on page 68. See your instructor if you have any questions.	2.1 See your instructor for **Prerequisites** exercises before beginning this unit.
Answer the following questions. ▶▶	2.2 Define in your own words, using an example, if appropriate: i) exponential function ii) exponential growth iii) exponential decay
See your instructor for correction of **Discussing the Ideas** before going on to the next set of questions.	2.3 **Discussing the Ideas,** page 68 Answer questions 1 and 2. 2.4 **Exercises,** pages 69-71 Answer questions 1, 2, 3 and 7. *(See note below on question 7.)* Answer questions 8-11. *(See note below on question 10.)* **Note**: In 7(d) , refer to the Compound Interest graph on page 66 in *Mathematics 12*. **Note**: When completing question 10, remember that, for exponential growth, $b>0$.
If you have access to a graphing calculator, answer the following question. ▶▶	2.5 Read and work through the steps in **Exploring with a Calculator** on page 72. Answer questions 1 and 2.

Unit 3: Logarithmic Functions and Their Properties

To meet the objectives of this unit, students should complete the following:

Reading for this unit:	*Mathematics 12*	
	Chapter 2:	Section 2.2: pages 74-78
		Section 2.3: pages 79-85

References and Notes	**Work to Submit**
One reason for studying logarithms is to develop a method for solving the exponential equations that were encountered in the previous section.	3.1 See your instructor for **Prerequisite** exercises on **Sections 2.2** and **2.3** before continuing this unit.
Read **Section 2.2** on pages 74 - 78. Work through **Investigate the Log Key on a Calculator**. Carefully study **Examples 1**, **2**, and **3**. See your instructor if you have any questions. Answer the following questions.▶▶	3.2 Define in your own words, using an example, when appropriate. i) logarithm ii) $\log_a x$
Note: In **Example 1** on page 75, the symbol , ≐ , means that the answer is approximate.	3.3 Identify the parts of the following equation: $\log_a x = y$ What is a ? What is y ? What is x?
Note: Every logarithm has a base which is written smaller and to the lower right of the word "log". In $\log_2 x$, the base is 2.	3.4 **Discussing the Ideas,** page 77 Answer questions 1 and 2.
See your instructor for correction of **Discussing the Ideas** before going on to the next set of questions.	**Note:** An example may be the best way to answer these two questions.

Unit 3: Logarithmic Functions and Their Properties

References and Notes	Work to Submit
The "log" button on your calculator always assumes a base of 10. This base is called a *common log*. When no base is specified, written log x, base 10 is implied. Although the base is not shown, its value must be remembered for some calculations.	3.5 **Exercises,** pages 77 and 78 Answer questions 1 and 2. *(See note below for question 2.)* Answer questions 3 - 7. **Note:** In question 2, you should remember that a number raised to a negative exponent is not made negative. 3.6 **Communicating the Ideas,** page 78
Read **Section 2.3** on pages 79 - 86. Carefully study **Examples 1 - 4**. See your instructor if you have any questions. Answer the following questions. ▶▶	3.7 **Discussing the Ideas,** page 83 Answer questions 1 - 3. 3.8 **Exercises,** pages 83 - 85 Answer questions 6 - 12. Answer questions 17 and 18. (*See note below for question 17.)* **Note:** Question 17 is similar to question 6. The only difference is that there are 3 terms instead of 2.
Note: If possible, go through the derivations of the laws of logarithms for powers and multiplication with your instructor. It is not necessary to be able to do them yourself but understanding the reason for each step will help develop problem solving skills. Note: $\frac{\log x}{\log y} \neq \log\left(\frac{x}{y}\right)$ and $\log x \times \log y \neq \log(xy)$ See your instructor for correction of **Discussing the Ideas** and **Communicating the Ideas** before going on to the next set of questions.	3.9 **Communicating the Ideas,** page 85

Unit 4: Real Situations Involving Exponential and Logarithmic Functions

To meet the objectives of this unit, students should complete the following:

Reading for this unit: *Mathematics 12*
Chapter 2: Section 2.4: pages 86-94
Section 2.5: pages 95-101

References and Notes	**Work to Submit**
Read **Section 2.4** on pages 86-94. Carefully study the **Examples**. See your instructor if you have any questions. Answer the following questions. ▶▶	4.1 See your instructor for **Prerequisite** exercises on **Sections 2.4** and **2.5** before beginning this unit.
You will recall that in **Section 2.1** on page 67 of the textbook, you saw that an exponential function has an equation that can be written as $y = Ab^x$, where $b > 0$. If $b > 1$, the function models growth. If $0 < b < 1$, the function models decay.	4.2 Read **Investigate** on page 86 and work through exercises 1-6. When using your graphing calculator in exercises 1-3, the following window settings are recommended: Xmin = 0, Xmax = 47, Ymin = 0, Ymax = 30000 In exercises 4-6, try the following settings: Xmin = 0, Xmax = 47, Ymin = 0, Ymax = 100
See your instructor for correction of **Discussing the Ideas** before going on to the next set of questions.	4.3 **Discussing the Ideas**, page 92 Answer questions 1-3. 4.4 **Exercises**, pages 92-94 Answer questions 1, 3, 4, 9, 10.

Unit 4: Real Situations Involving Exponential and Logarithmic Functions

References and Notes	Work to Submit
Read **Section 2.5** on pages 95-101. Carefully study **Examples 1** and **2**. See your instructor if you have any questions. Answer the following questions. ▶▶	4.5 Express in your own words: i) logarithmic scale 4.6 **Exercises**, pages 98-101 Answer question 2. *(See note below on question 2.)* Answer questions 4 and 7. *(See note below on question 7.)* Answer questions 10, 11, 13, 15, and 18. *(See notes below on questions 13, 15 and 18.)* **Note**: In question 2, since the insects multiply fivefold in 4 weeks, the equation you need to use is $P = P_0 \times 5^{\frac{n}{4}}$ **Note:** In question 7 use the equation on top of page 97 in *Mathematics 12*. **Note:** In questions 13 and 15, the decibel scale is different from the Richter scale in that an increase of 10 decibels represents a tenfold increase in sound intensity. Therefore an increase of 20 decibels corresponds to a 10^2 or a hundredfold increase (two tenfold increases). **Note:** In question 18, the pH scale is similar to the Richter scale in that a change of 1 on the pH scale corresponds to a tenfold increase. However, since a pH of 7 is neutral, each 1-unit increase above 7 corresponds to a tenfold increase in *alkalinity*. Similarly, each 1-unit decrease below 7 represents a tenfold increase in *acidity*.

Unit 5: Exponential Functions and Their Graphs

To meet the objectives of this unit, students should complete the following:

Reading for this unit: *Mathematics 12*
Chapter 2: Section 2.6: pages 104-112

References and Notes	Work to Submit
Read **Section 2.6** on pages 104-112. Carefully study **Examples 1-3**. When studying **Example 1**, make a table of values for each of a), b) and c). Look at each sketch. Identify the *domain*, *range*, *intercepts* and *asymptotes*. You should notice that in the exponential function $f(x) = b^x$, If $b > 1$, the graph rises to the right. If b is between 0 and 1, the graph goes down to the right. When graphing the three different functions in **Example 3**, make sure that you use brackets when entering $Y_2 = 10^{\wedge(X/3)}$. See your instructor if you have any questions. Answer the following questions. ▶▶	5.1 Define the following terms in your own words: i) asymptote ii) vertical intercept iii) horizontal intercept 5.2 Sketch the graph of the exponential function of the form $f(x) = b^x$, $b > 0$. Identify the following properties: i) increasing or decreasing ii) vertical intercept iii) horizontal intercept iv) asymptote v) domain and range 5.3 **Exercises**, pages 109-112 Answer questions 1, 2a), and 3a). *(See note below on questions 2a and 3a.)* Answer questions 9, 14 and 15. **Note**: Use a graphing calculator to sketch the graphs in 2a and 3a..

Unit 6: Logarithms and Their Graphs

To meet the objectives of this unit, students should complete the following:

Reading for this unit: *Mathematics 12*
Chapter 2: Section 2.10: pages 133-140

References and Notes	Work to Submit
Read **Section 2.10** on pages 133-140. Carefully study **Examples 1-3.** See your instructor if you have any questions.	6.1 See your instructor for **Prerequisites** review exercises on **Section 2.10** before beginning this unit.
Answer the following questions. ▶▶	6.2 Define in your own words: i) inverse of a function
	6.3 Sketch the graph of the logarithmic function $f(x) = \log_b x$ ($b > 0$, $b \neq 1$). Identify the following properties: i) increasing or decreasing ii) vertical intercept iii) horizontal intercept iv) asymptote v) domain and range
	6.4 Consider the function $f(x) = 10^x$. a) Find the inverse of $f(x)$ algebraically. b) Graph $f(x)$. c) Find the inverse of $f(x)$ graphically.
See your instructor for correction of **Discussing the Ideas** before going on to the next set of questions.	6.5 **Discussing the Ideas**, page 137 Answer question 1. 6.6 **Exercises**, pages 138-140 Answer questions 1-3, 7, 8, 11-13, 15, 16.

Unit 7: Exponential and Logarithmic Equations and Identities

To meet the objectives of the unit, students should complete the following:

Reading for this unit:	*Mathematics 12*	
	Chapter 2:	Section 2.11: pages 146-149
		Section 2.12: pages 150-153

References and Notes	Work to Submit
Read **Section 2.11** on pages 146 - 149.	7.1 See your instructor for **Prerequisite** review exercises on **Sections 2.11** and **2.12**.
Read **Investigate** and answer the following questions.▶▶ **Note:** See your instructor for correction and discussion of **Investigate Solving Exponential Equations**.	7.2 Read **Investigate Solving Exponential Equations** on page 146. Answer questions 1-6. **Note:** Not <u>all</u> 5 methods can be used to solve the equation in question 6.
Carefully study **Examples 1** and **2**. In **Example 1**, notice that both sides of the equation are being expressed as powers of 3 in Solution 2, but in Solution 3, both sides are being expressed as a power of 10. In **Example 2**, the equation <u>cannot</u> be expressed with the same base unless logarithms are used. The answer is in *exact* form. If log 3 and log 0.5 are evaluated, the answer becomes approximate.	7.3 **Exercises**, page 149 Answer questions 1-3 and 5-8. *(See note below on question 8)*. **Note**: Question 8 requires that the answers be given in *exact* form. (see **Example 2** on page 148.)

Unit 7: Exponential and Logarithmic Equations and Identities

References and Notes	Work to Submit
Read **Section 2.12** on pages 150-153. Carefully study **Examples 1** and **2**. Read the notes below and answer the following questions ▶▶ **Note:** In **Example 1a)**, $x = -2$ is an extraneous root because when this value is substituted into the first term it becomes $\log_5(-1)$, which is undefined. **Note:** In **Example 2a)**, the logarithmic identity is verified for a given value of *x* and *a*. . This does *not* prove the identity. In **2b**), the identity is defined only when $1/x$ is a positive number, ie when $1/x > 0$, which means that $x > 0$. **Note:** When solving log equations, always check the solutions in the *original* equation to ensure that each solution is valid. See your instructor for correction of **Discussing the Ideas** before going on to the next set of questions.	7.4 Define in your own words: i) identity ii) extraneous root 7.5 **Discussing the Ideas**, page 152 Answer questions 1-3. 7.6 **Exercises**, pages 152-153 Answer question 1. *(See note below on question 1.)* Answer questions 2-6. *(See note below on questions 2-6.)* Answer questions 7 and 8. *(See note below on question 8.)* **Note**: In question 1, it may be necessary to rewrite the equation in exponential form. **Note:** In questions 2-6, all solutions to log equations must be verified. **Note:** In question 8 it doesn't matter which side is simplified, although usually it is better to simplify the most complicated side.

Appendix A
Exponents

A geometric sequence involves repeated multiplication by the same number, the common ratio. When the first term and the common ratio of a geometric sequence are equal, the terms of the sequence are powers. For example, the terms of the geometric sequence with first term 2 and common ratio 2 are powers of 2.

Numeral form:	2	4	8	16	32	...
Power form:	2^1	2^2	2^3	2^4	2^5	...

Recall that the definition of a power depends on whether the exponent is a positive integer, zero, or a negative integer.

Positive Integral Exponent

$$a^n = \underbrace{a \bullet a \bullet a \bullet \cdots \bullet a}_{n \text{ factors}}$$

Zero Exponent

a^0 is defined to be equal to 1.

$$a^0 = 1, \quad (a \neq 0)$$

Negative Integral Exponent

a^{-n} is defined to be the reciprocal of a^n.

$$a^{-n} = \frac{1}{a^n}, \quad (a \neq 0)$$

These definitions permit us to extend the sequence of powers of 2 above to the left.

Multiplying each term by 2 increases the exponent by 1.

$\rightarrow$

...	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8	16	...
...	2^{-4}	2^{-3}	2^{-2}	2^{-1}	2^0	2^1	2^2	2^3	2^4	...

$\leftarrow$

Dividing each term by 2 decreases the exponent by 1.

We use these definitions to evaluate a power with any integral exponent.

Example 1

Simplify each power.

a) 4^3 b) 3^{-2} c) $(-2)^{-3}$ d) $\left(\frac{1}{2}\right)^0$

Solution

Use mental math.

a) $4^3 = 4 \times 4 \times 4$
$= 64$

b) $3^{-2} = \frac{1}{3^2}$
$= \frac{1}{9}$

c) $(-2)^{-3} = \frac{1}{(-2)^3}$
$= -\frac{1}{8}$

d) $\left(\frac{1}{2}\right)^0 = 1$

The definitions of integral exponents lead to some basic laws for working with exponents. These examples will help you recall the laws they illustrate.

Expression	Exponent Law
$x^3 \bullet x^2 = x \bullet x \bullet x \bullet x \bullet x$ $= x^{3+2}$, or x^5	$x^m \bullet x^n = x^{m+n}$
$x^5 \div x^3 = \frac{x \bullet x \bullet x \bullet x \bullet x}{x \bullet x \bullet x}$ $= x^{5-3}$, or x^2	$x^m \div x^n = x^{m-n}$ $(x \neq 0)$
$(x^3)^2 = (x \bullet x \bullet x)(x \bullet x \bullet x)$ $= x^{3 \times 2}$, or x^6	$(x^m)^n = x^{mn}$
$(xy)^3 = xy \bullet xy \bullet xy$ $= x \bullet x \bullet x \bullet y \bullet y \bullet y$ $= x^3y^3$	$(xy)^n = x^ny^n$
$\left(\frac{x}{y}\right)^2 = \frac{x}{y} \bullet \frac{x}{y}$ $= \frac{x^2}{y^2}$	$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ $(y \neq 0)$

We use the exponent laws to simplify products and quotients involving powers.

Example 2

Simplify.

a) $(x^3y^2)(x^2y^4)$ **b)** $\frac{a^5b^3}{a^2b^2}$ **c)** $\left(\frac{x^2}{z^3}\right)^2$

Solution

Use mental math.

a)
$$\begin{aligned}&(x^3y^2)(x^2y^4)\\&= x^3 \bullet y^2 \bullet x^2 \bullet y^4\\&= x^5y^6\end{aligned}$$

b)
$$\begin{aligned}\frac{a^5b^3}{a^2b^2} &= \frac{a^5}{a^2} \bullet \frac{b^3}{b^2}\\&= a^3b\end{aligned}$$

c)
$$\begin{aligned}\left(\frac{x^2}{z^3}\right)^2 &= \frac{x^2}{z^3} \bullet \frac{x^2}{z^3}\\&= \frac{x^4}{z^6}\end{aligned}$$

Example 3

Simplify.

a) $x^{-3} \bullet x^5$ **b)** $m^2 \div m^{-3}$ **c)** $(n^{-2})^{-3}$

Solution

Use mental math.

a)
$$\begin{aligned}x^{-3} \bullet x^5 &= x^{-3+5}\\&= x^2\end{aligned}$$

b)
$$\begin{aligned}m^2 \div m^{-3} &= m^{2-(-3)}\\&= m^5\end{aligned}$$

c)
$$\begin{aligned}(n^{-2})^{-3} &= n^{(-2)\times(-3)}\\&= n^6\end{aligned}$$

Example 4

The number of insects in a colony doubles every month. There are 1000 insects in the colony now. About how many were there in the colony three months ago?

Solution

Let x represent the number of insects 3 months ago. Then, after 3 successive doublings the colony grows to 1000 insects.

$$\begin{aligned}x \times 2^3 &= 1000\\x &= \frac{1000}{2^3}\\&= 125\end{aligned}$$

There were about 125 insects in the colony 3 months ago.

Exercises

1. Simplify.

a) 2^4 b) 5^{-2} c) 3^{-1} d) $\left(\frac{1}{4}\right)^{-1}$

e) $\left(\frac{2}{3}\right)^{-1}$ f) $\left(\frac{3}{4}\right)^{-2}$ g) 0.5^{-1} h) 1.5^0

2. Simplify.

a) 10^0 b) $(-3)^{-2}$ c) $\left(-\frac{1}{2}\right)^3$ d) $\left(-\frac{2}{3}\right)^{-1}$

e) $\left(-\frac{3}{5}\right)^{-2}$ f) $(-1)^{-4}$ g) 0.1^{-4} h) $\frac{1}{2^{-3}}$

3. Choose one part of exercise 1 or 2. Write to explain how you simplified the expression.

4. Simplify.

a) $x^3 \bullet x^4$ b) $a^2 \bullet a^5$ c) $b^3 \bullet b^5 \bullet b$ d) $m^2 \bullet m^3 \bullet m^4$

5. Simplify.

a) $\frac{x^4}{x^2}$ b) $\frac{y^7}{y^3}$ c) $\frac{n^6}{n^5}$ d) $\frac{a^8}{a^5}$

6. Simplify.

a) $(x^3)^3$ b) $(y^2)^3$ c) $(a^2b^2)^3$ d) $(xy^3)^2$

7. Simplify.

a) $\frac{x^2}{x^5}$ b) $\frac{c^3}{c^4}$ c) $\frac{y^2}{y^7}$ d) $\frac{a^2}{a^6}$

8. Choose one part of exercises 4 to 7. Write to explain how you simplified the expression.

9. A colony of 10 000 insects doubles in number every month. How many insects were there at each time?

a) 2 months ago **b)** 5 months ago

10. Simplify.

a) $x^{-3} \bullet x^4$ **b)** $d^{-4} \bullet d^{-1}$ **c)** $a^6 \bullet a^{-2}$ **d)** $y^4 \bullet y^{-4}$

e) $x^{-5} \bullet x^{-1} \bullet x^{-3}$ **f)** $b^4 \bullet b^{-3} \bullet b^2$ **g)** $k^8 \bullet k^{-2} \bullet k^{-6}$ **h)** $p^{-1} \bullet p^7 \bullet p^{-6}$

11. Simplify.

a) $\frac{x^{-5}}{x^2}$ **b)** $\frac{r^3}{r^{-2}}$ **c)** $\frac{s^5}{s^{-5}}$ **d)** $\frac{t^{-4}}{t^{-4}}$

e) $\frac{c^{-1}}{c^{-2}}$ **f)** $\frac{x^{-2}}{x^{-4}}$ **g)** $\frac{b^{-8}}{b^{-3}}$ **h)** $\frac{t^4}{t^{-7}}$

12. Simplify.

a) $(x^{-2})^3$ **b)** $(y^{-1})^{-2}$ **c)** $(m^{-3})^2$ **d)** $(c^3)^{-3}$

e) $(a^4)^{-1}$ **f)** $(x^{-1}y^2)^{-1}$ **g)** $(x^2y^{-3})^2$ **h)** $(a^{-2}b^2)^{-2}$

13. Simplify.

a) $(a^{-2}b^4)(a^2b^{-5})$ **b)** $\frac{(x^{-2})^3}{(x^3)^{-2}}$ **c)** $\frac{x^2y^{-2}}{y^{-1}}$

d) $(x^{-1}y^2)^{-3}(x^2)^{-1}$ **e)** $\frac{(c^{-3}d)^{-1}}{(c^2d)^{-2}}$ **f)** $(m^2n^{-2})(m^{-1}n^2)^{-1}$

14. a) Evaluate each expression for $x = -1$ and $y = 2$.

i) $(x^3y^2)(x^2y^3)$ **ii)** $\frac{x^{-4}y^5}{xy^3}$ **iii)** $(x^3y^2)^3$

iv) $(x^{-1}y^{-2})(x^{-2}y^{-3})$ **v)** $\frac{x^{-3}y^{-2}}{x^2y^{-6}}$ **vi)** $(x^{-4}y^{-3})^{-2}$

b) Choose one expression from part a. Write to explain how you evaluated it.

Visualizing

Gold leaf is so thin that one dollar's worth would cover a square with an approximate area of 3600 cm^2.

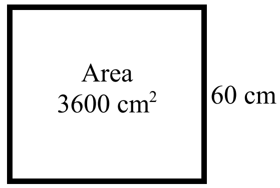
Since $60^2 = 3600$, each side of the square is 60 cm long.

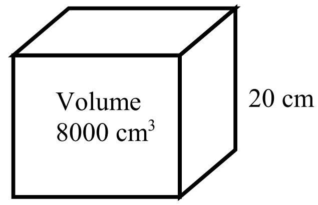
We say that 60 is a square root of 3600, and we write $\sqrt{3600} = 60$.

The gold produced in Canada in one day would almost fill a cube with an approximate volume of 8000 cm^3.

Since $20^3 = 8000$, each edge of the cube is 20 cm long.

We say that 20 is a cube root of 8000, and we write $\sqrt[3]{8000} = 20$.





Expressions such as $\sqrt{3600}$ and $\sqrt[3]{8000}$ are *radicals*. We use radicals when we work with square roots and cube roots of numbers.

Square Roots

A number r is a *square root* of a number x if $r^2 = x$.

A positive number always has two square roots, one positive, the other negative. Since the squares of both positive and negative numbers are positive, it is impossible to obtain a negative number when a number is squared. Hence, the square root of a negative number is not defined.

The *radical sign*, $\sqrt{\ }$, always denotes the positive square root.

$\sqrt{x}$ means the positive square root of x, where $x \geq 0$.

Example 1

Determine each square root, without using a calculator.

a) $\sqrt{1600}$ b) $\sqrt{2.25}$ c) $\sqrt{0.09}$

Solution

a) $\sqrt{1600} = 40$, since $40^2 = 1600$

b) $\sqrt{2.25} = 1.5$, since $1.5^2 = 2.25$

c) $\sqrt{0.09} = 0.3$, since $0.3^2 = 0.09$

In *Example 1*, the square roots were exact. Many numbers do not have exact square roots, but you can use a calculator to determine approximations of them. Since the calculator displays only a fixed number of digits, the number displayed may only be an approximation of the square root. For example, if you use your calculator to determine $\sqrt{7}$, you will obtain 2.645 751 311 (assuming your calculator has 10-digit accuracy). You can obtain different approximations of $\sqrt{7}$ by rounding or by truncating.

Example 2

Sharon used her calculator to determine $\sqrt{7}$. She obtained 2.645 751 311. Write approximations to 1, 2, and 3 decimal places that are obtained by rounding and by truncating. Use your calculator to check your approximations.

Solution

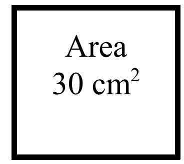
Use a table to record the results.

Number of decimal places	Rounding	Check	Truncating	Check
1	$\sqrt{7} \doteq 2.6$	$2.6 \times 2.6 = 6.76$	$\sqrt{7} \doteq 2.6$	$2.6 \times 2.6 = 6.76$
2	$\sqrt{7} \doteq 2.65$	$2.65 \times 2.65 = 7.0225$	$\sqrt{7} \doteq 2.64$	$2.64 \times 2.64 = 6.9696$
3	$\sqrt{7} \doteq 2.646$	$2.646 \times 2.646 = 7.001\ 316$	$\sqrt{7} \doteq 2.645$	$2.645 \times 2.645 = 6.996\ 025$

In *Example 2*, observe that if you include more decimal places when you estimate the square root, you get closer to 7 when you square the estimate. Furthermore, rounding and truncating provide either the same estimate, or one that differs only by 1 in the final digit.

Example 3

A square has an area of 30 cm². Determine the perimeter of the square in exact form, and in approximate form to 2 decimal places.



Solution

Since $\sqrt{30} \times \sqrt{30} = 30$, the length of each side is $\sqrt{30}$ cm.
Hence, the perimeter is $4 \times \sqrt{30}$ cm, which we write as $4\sqrt{30}$ cm.
In approximate form:

$$4\sqrt{30} \doteq 4 \times 5.4772$$
$$\doteq 21.9089$$

To 2 decimal places, the perimeter of the square is 21.91 cm.

Cube Roots

A number r is a *cube root* of a number x if $r^3 = x$.

The cube root of a positive number is positive and the cube root of a negative number is negative.

$\sqrt[3]{x}$ means the cube root of x.

Example 4

Determine each cube root.

a) $\sqrt[3]{125}$ **b)** $\sqrt[3]{-64}$ **c)** $\sqrt[3]{18}$

Solution

a) Use mental math.
$\sqrt[3]{125} = 5$,
since $5^3 = 125$

b) Use mental math.
$\sqrt[3]{-64} = -4$,
since $(-4)^3 = -64$

c) Use the cube-root function on your calculator. Consult your manual if necessary.
$\sqrt[3]{18} \doteq 2.621$ rounded to 3 decimal places

Higher Roots

Similarly, the *fourth roots* of 16 are 2 and -2, since $2^4 = 16$, and $(-2)^4 = 16$.

We write $\sqrt[4]{16} = 2$ to indicate the positive fourth root of 16.

And the *fifth root* of -32 is -2, since $(-2)^5 = -32$.

We write $\sqrt[5]{-32} = -2$.

An expression of the form $\sqrt[n]{x}$, where n is a natural number, is a *radical*.
If n is even, the expression represents only the positive root.

Exercises

1. Determine the square roots of each number.

   a) 49 b) 81 c) 121 d) 400 e) 529 f) 625

2. Simplify without using a calculator.

   a) $\sqrt{64}$ b) $\sqrt{100}$ c) $\sqrt{144}$ d) $\sqrt{900}$ e) $\sqrt{1600}$

   f) $\sqrt{0.25}$ g) $\sqrt{0.04}$ h) $\sqrt{0.01}$ i) $\sqrt{0.0016}$ j) $\sqrt{0.000\,025}$

3. Use a calculator to determine each square root, to 3 decimal places.

   a) $\sqrt{2}$ b) $\sqrt{3}$ c) $\sqrt{52.3}$ d) $\sqrt{128.5}$ e) $\sqrt{471}$

4. Simplify without using a calculator.

   a) $\sqrt[3]{8}$ b) $\sqrt[3]{-27}$ c) $\sqrt[4]{81}$

   d) $\sqrt[5]{32}$ e) $\sqrt[5]{243}$ f) $\sqrt[3]{0.001}$

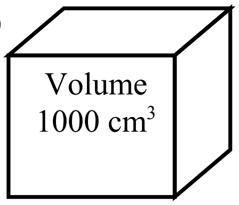
5. Use the area of each square. Determine the length of a side and the perimeter. Give the answer to 1 decimal place, where necessary.

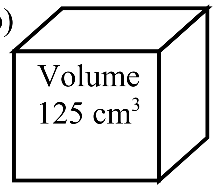
   a) Area 9.61 m^2

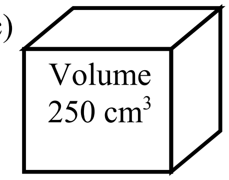
   b) Area 6.4 m^2

   c) Area 8.5 m^2

6. Use the volume of each cube. Determine the length of an edge and the area of a face. Give the
answer to 1 decimal place, where necessary.

a)


b)


c)


7. Use your calculator. Determine each root to 2 decimal places. Use your calculator to check your approximations.

a) $\sqrt{6}$ **b)** $\sqrt{11}$ **c)** $\sqrt[3]{23}$ **d)** $\sqrt{124}$ **e)** $\sqrt[3]{139}$ **f)** $\sqrt[3]{254}$

8. Write each number in square root form.

a) 5 **b)** 3 **c)** 2 **d)** 4 **e)** 7 **f)** 1

9. a) Use your calculator to determine $\sqrt{60}$. Write approximations to 1, 2, and 3 decimal places that are obtained by rounding and by truncating. Use your calculator to check your approximations.

b) Suppose you have two estimates of the square root of a number, one obtained by rounding to a certain number of decimal places and the other by truncating to the same number of decimal places. How can you tell which was obtained by rounding and which by truncating? Use your results from part a to support your answer.

c) Suppose you have only one estimate of the square root of a number. Can you tell if it was obtained by rounding or by truncating? Explain.

10. Simplify the radicals in each list without using a calculator. What patterns can you find in the results? Predict the next line in each list.

a) $\sqrt{9}$
$\sqrt{900}$
$\sqrt{90\,000}$

b) $\sqrt[3]{8}$
$\sqrt[3]{8000}$
$\sqrt[3]{8\,000\,000}$

c) $\sqrt[4]{16}$
$\sqrt[4]{160\,000}$
$\sqrt[4]{1\,600\,000\,000}$

11. Simplify without using a calculator.

a) $\sqrt[3]{64}$ **b)** $\sqrt[3]{125}$ **c)** $\sqrt[4]{16}$ **d)** $\sqrt[5]{-1}$ **e)** $\sqrt[3]{216}$
f) $\sqrt[3]{-1000}$ **g)** $\sqrt[4]{256}$ **h)** $\sqrt[4]{10\,000}$ **i)** $\sqrt[3]{7^3}$ **j)** $\sqrt[5]{10^5}$

Investigate

A power with a natural number exponent is defined using repeated multiplication; for example, 3^4 means $3 \times 3 \times 3 \times 3$.

A power with a rational exponent, such as $3^{\frac{1}{2}}$ or $3^{0.5}$, has no meaning according to this definition. It has been defined in another way. You can use your calculator to discover the definition.

The keystrokes are for the TI-34 calculator. If you use a different calculator, consult your manual for the keystrokes.

1. a) To determine $3^{\frac{1}{2}}$, press 3 [y^x] .5 [=]. Record the result.
   b) Can you tell how $3^{\frac{1}{2}}$ is defined? If so, use your calculator to confirm this.
2. a) To determine $3^{\frac{1}{3}}$, press 3 [y^x] [(] 1 [÷] 3 [)] [=]. Record the result.
   b) Can you tell how $3^{\frac{1}{3}}$ is defined? If so, use your calculator to confirm this.
3. Copy and complete the tables. Use the results. How do you think $x^{\frac{1}{2}}$ and $x^{\frac{1}{3}}$ are defined?
4. Use the results of exercise 3. How would you define $x^{\frac{1}{n}}$?

x	$x^{\frac{1}{2}}$
1	
2	
3	
4	
9	
16	
25	

x	$x^{\frac{1}{3}}$
1	
2	
3	
8	
27	
64	
125	

5. Predict how $x^{-\frac{1}{2}}$ is defined. Copy the first table. Use your calculator to complete it. Did the results agree with your prediction?
6. Copy the second table. Use your calculator to complete it. Use the results. How do you think $x^{\frac{2}{3}}$ is defined?

x	$x^{-\frac{1}{2}}$
1	
2	
3	
4	
9	
16	
25	

x	$x^{\frac{2}{3}}$
1	
2	
3	
8	
27	
64	
125	

7. Use the results of exercises 5 and 6. How would you define $x^{-\frac{1}{n}}$ and $x^{\frac{m}{n}}$?

To give meaning to powers such as $3^{\frac{1}{2}}$ and $3^{-\frac{1}{2}}$, we *extend* the exponent law $x^m \times x^n = x^{m+n}$ so that it applies when m and n are rational numbers.

By extending the law:

$$3^{\frac{1}{2}} \times 3^{\frac{1}{2}} = 3^{\frac{1}{2}+\frac{1}{2}} = 3^1 = 3$$

But: $\sqrt{3} \times \sqrt{3} = 3$

Therefore, $3^{\frac{1}{2}} = \sqrt{3}$

By extending the law:

$$3^{-\frac{1}{2}} \times 3^{\frac{1}{2}} = 3^{-\frac{1}{2}+\frac{1}{2}} = 3^0 = 1$$

Therefore, $3^{-\frac{1}{2}}$ and $3^{\frac{1}{2}}$ are reciprocals.

$$3^{-\frac{1}{2}} = \frac{1}{3^{\frac{1}{2}}} = \frac{1}{\sqrt{3}}$$

These examples and the results of *Investigate* suggest that $x^{\frac{1}{n}}$ should be defined as the nth root of x, and $x^{-\frac{1}{n}}$ as its reciprocal.

$x^{\frac{1}{n}} = \sqrt[n]{x}$ — n is a natural number, $x \geq 0$ if n is even.

$x^{-\frac{1}{n}} = \frac{1}{\sqrt[n]{x}}$ — n is a natural number, $x \neq 0$, $x > 0$ if n is even.

Example 1

Determine each exact value without using a calculator.

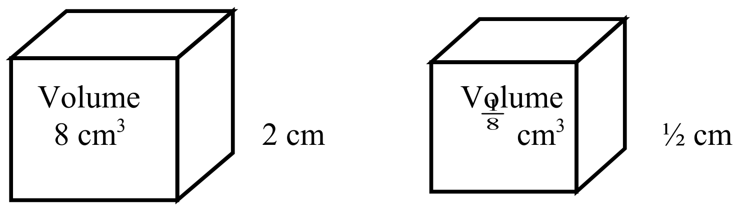
a) $27^{\frac{1}{3}}$ **b)** $\left(\frac{9}{16}\right)^{-\frac{1}{2}}$

Solution

a) $27^{\frac{1}{3}} = \sqrt[3]{27} = 3$

b) $\left(\frac{9}{16}\right)^{-\frac{1}{2}} = \frac{1}{\left(\frac{9}{16}\right)^{\frac{1}{2}}} = \frac{1}{\sqrt{\frac{9}{16}}} = \frac{1}{\frac{3}{4}} = \frac{4}{3}$

Visualizing



$8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

$8^{-\frac{1}{3}} = \frac{1}{\sqrt[3]{8}} = \frac{1}{2}$

To give meaning to a power such as $3^{\frac{2}{3}}$ we extend the exponent law $(x^m)^n = x^{mn}$ so that it applies when m and n are rational numbers.

By extending the law:

$$3^{\frac{2}{3}} = (3^{\frac{1}{3}})^2 = (\sqrt[3]{3})^2 \quad \text{or} \quad 3^{\frac{2}{3}} = (3^2)^{\frac{1}{3}} = \sqrt[3]{3^2}$$

These examples and the results of *Investigate* suggest the following definitions for $x^{\frac{m}{n}}$.

$$x^{\frac{m}{n}} = (\sqrt[n]{x})^m = \sqrt[n]{x^m}$$

n is a natural number, $x \geq 0$ if n is even.

To give meaning to a power such as $3^{-\frac{2}{3}}$, we use the law $x^m \bullet x^n = x^{m+n}$, which we extended to rational exponents previously.

$$3^{-\frac{2}{3}} \times 3^{\frac{2}{3}} = 3^{-\frac{2}{3}+\frac{2}{3}} = 3^0 = 1$$

Therefore, $3^{-\frac{2}{3}}$ and $3^{\frac{2}{3}}$ are reciprocals.

$$3^{-\frac{2}{3}} = \frac{1}{3^{\frac{2}{3}}} = \frac{1}{(\sqrt[3]{3})^2} \quad \text{or} \quad \frac{1}{\sqrt[3]{3^2}}$$

This example suggests the following definitions for $x^{-\frac{m}{n}}$.

$$x^{-\frac{m}{n}} = \frac{1}{(\sqrt[n]{x})^m}$$
$$= \frac{1}{\sqrt[n]{x^m}}$$ n is a natural number, $x \neq 0$, $x > 0$ if n is even.

Example 2

Determine each exact value without using a calculator.

a) $27^{\frac{2}{3}}$ b) $\left(\frac{9}{16}\right)^{-\frac{3}{2}}$

Solution

a) $27^{\frac{2}{3}} = (\sqrt[3]{27})^2$
$= 3^2$
$= 9$

b) $\left(\frac{9}{16}\right)^{-\frac{3}{2}} = \frac{1}{\left(\frac{9}{16}\right)^{\frac{3}{2}}}$
$= \frac{1}{\left(\sqrt{\frac{9}{16}}\right)^3}$
$= \frac{1}{\left(\frac{3}{4}\right)^3}$
$= \frac{1}{\frac{27}{64}}$
$= \frac{64}{27}$

Visualizing

$8^{\frac{2}{3}} = (\sqrt[3]{8})^2$ ← means square of the cube root
$= 2^2$
$= 4$

$8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}}$ ← means reciprocal
$= \frac{1}{(\sqrt[3]{8})^2}$
$= \frac{1}{2^2}$
$= \frac{1}{4}$

You can use the laws of exponents to simplify expressions involving radicals and rational exponents.

Example 3

Simplify. Write each expression as a power and as a radical.

a) $\sqrt[6]{x^3}$ **b)** $\sqrt{\sqrt[3]{x^5}}$ **c)** $(\sqrt[3]{x^4})(\sqrt{x^3})$

Solution

a) $\sqrt[6]{x^3} = (x^3)^{\frac{1}{6}}$ Using the law: $\sqrt[n]{x^m} = (x^m)^{\frac{1}{n}}$

$= x^{3 \times \frac{1}{6}}$ Using the law: $(x^m)^n = x^{mn}$

$= x^{\frac{1}{2}}$, or $\sqrt{x}$

b) $\sqrt{\sqrt[3]{x^5}} = (\sqrt[3]{x^5})^{\frac{1}{2}}$ Using the law: $\sqrt[n]{x^m} = (x^m)^{\frac{1}{n}}$

$= ((x^5)^{\frac{1}{3}})^{\frac{1}{2}}$ Using the law: $\sqrt[n]{x^m} = (x^m)^{\frac{1}{n}}$ again

$= x^{5 \times \frac{1}{3} \times \frac{1}{2}}$

$= x^{\frac{5}{6}}$, or $\sqrt[6]{x^5}$

c) $(\sqrt[3]{x^4})(\sqrt{x^3}) = (x^4)^{\frac{1}{3}} \times (x^3)^{\frac{1}{2}}$ Using the law: $\sqrt[n]{x^m} = (x^m)^{\frac{1}{n}}$ twice

$= x^{\frac{4}{3}} \times x^{\frac{3}{2}}$ Using the law: $(x^m)^n = x^{mn}$

$= x^{\frac{8}{6} + \frac{9}{6}}$ Using the law: $x^m \times x^n = x^{m+n}$

$= x^{\frac{17}{6}}$, or $(\sqrt[6]{x})^{17}$

DISCUSSING THE IDEAS

1. Think about the definition of a rational exponent. Why is there a restriction on x if n is even? Use some examples to illustrate your explanation.
2. Determine another way to solve *Example 1b* and *Example 2b* that involves fewer steps.

Exercises

1. Determine each exact value without using a calculator.

   a) 8^0 **b)** $8^{\frac{1}{3}}$ **c)** $8^{\frac{2}{3}}$ **d)** $8^{\frac{3}{3}}$ **e)** $8^{\frac{4}{3}}$

   f) $8^{-\frac{1}{3}}$ **g)** $8^{-\frac{2}{3}}$ **h)** $8^{-\frac{3}{3}}$ **i)** $8^{-\frac{4}{3}}$ **j)** $8^{-\frac{5}{3}}$

2. Determine each exact value without using a calculator.

   a) $16^{\frac{1}{2}}$ **b)** $36^{\frac{1}{2}}$ **c)** $100^{\frac{1}{2}}$ **d)** $32^{\frac{1}{5}}$ **e)** $64^{\frac{1}{3}}$

   f) $27^{\frac{1}{3}}$ **g)** $(-64)^{\frac{1}{3}}$ **h)** $81^{\frac{1}{4}}$ **i)** $(-27)^{\frac{1}{3}}$ **j)** $(-1000)^{\frac{1}{3}}$

3. Determine each exact value without using a calculator.

   a) $4^{-\frac{1}{2}}$ **b)** $9^{-\frac{1}{2}}$ **c)** $27^{-\frac{1}{3}}$ **d)** $64^{-\frac{1}{3}}$ **e)** $(-64)^{-\frac{1}{3}}$

4. Write each expression using radicals.

a) $4^{\frac{1}{5}}$ b) $4^{\frac{2}{5}}$ c) $4^{\frac{3}{5}}$ d) $4^{\frac{4}{5}}$ e) $4^{\frac{5}{5}}$

f) $4^{-\frac{1}{5}}$ g) $4^{-\frac{2}{5}}$ h) $4^{-\frac{3}{5}}$ i) $4^{-\frac{4}{5}}$ j) $4^{-\frac{5}{5}}$

5. Choose one part of exercise 4. Write to explain how you wrote the power as a radical.

6. Determine each exact value without using a calculator.

a) $9^{\frac{3}{2}}$ b) $27^{\frac{2}{3}}$ c) $4^{\frac{3}{2}}$ d) $25^{\frac{3}{2}}$ e) $32^{\frac{2}{5}}$

f) $(-27)^{\frac{2}{3}}$ g) $36^{\frac{3}{2}}$ h) $(-64)^{\frac{2}{3}}$ i) $100^{\frac{3}{2}}$ j) $(-8000)^{\frac{2}{3}}$

7. Determine each exact value without using a calculator.

a) $27^{-\frac{2}{3}}$ b) $32^{-\frac{3}{5}}$ c) $9^{-\frac{3}{2}}$ d) $16^{-\frac{3}{4}}$ e) $100^{-\frac{3}{2}}$

8. Write each number as a power with an exponent of $\frac{1}{2}$.

a) 3 b) 2 c) 4 d) 1 e) 10 f) 8

9. Choose one part of exercise 8. Write to explain how you wrote the number as a power.

10. Determine each exact value without using a calculator.

a) $27^{-\frac{4}{3}}$ b) $16^{-1.5}$ c) $81^{0.75}$ d) $32^{-0.4}$ e) $49^{\frac{3}{2}}$

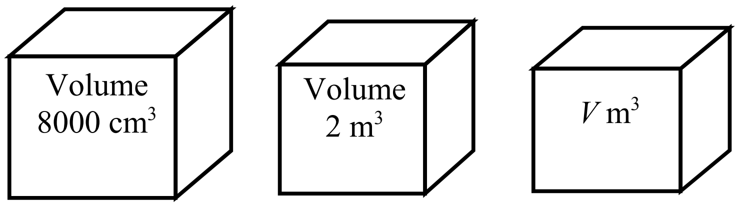
f) $\left(\frac{9}{16}\right)^{\frac{1}{2}}$ g) $\left(\frac{25}{49}\right)^{\frac{3}{2}}$ h) $\left(-\frac{1}{32}\right)^{0.8}$ i) $\left(\frac{8}{27}\right)^{-\frac{2}{3}}$ j) $\left(\frac{81}{16}\right)^{-\frac{3}{4}}$

11. Choose one part of exercise 10. Write to explain how you determined the exact value.

12. Use a calculator. Determine each value to 3 decimal places.

a) $10^{\frac{1}{4}}$ b) $30^{0.7}$ c) $7^{\frac{2}{3}}$ d) $15^{1.4}$ e) $\sqrt[8]{2.17}$

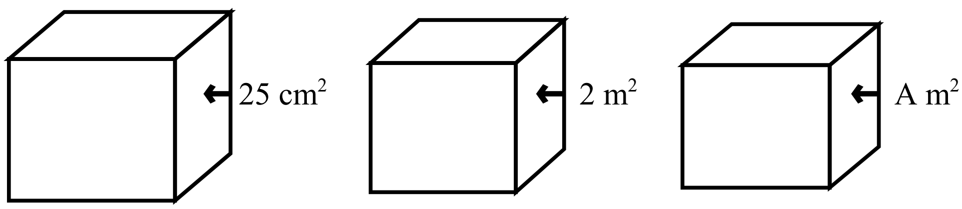
13. a) A cube has a volume of 8000 cm^3. Determine the length of each edge and the area of each face.



b) Repeat part a for a cube with a volume of 2 m^3. Express your answers in radical form.

c) Repeat part a for a cube with a volume of V cubic metres.

14. a) The area of each face of a cube is 25 cm^2. Determine the length of each edge and the volume of the cube.



b) Repeat part a for a cube with each face with area 2 m^2. Express your answers in radical form.

c) Repeat part a for a cube with each face with area A square metres.

15. Write each number as a power with an exponent of $\frac{1}{3}$.

a) 3 **b)** -1 **c)** -2 **d)** -4 **e)** 1 **f)** -3

16. Simplify. Write each expression as a power and as a radical.

a) $x \bullet x^{\frac{1}{2}}$ **b)** $m^{\frac{1}{3}} \bullet m$ **c)** $y^{\frac{3}{2}} \bullet y^{\frac{1}{2}}$ **d)** $b^{\frac{5}{2}} \bullet b^{\frac{3}{2}}$

e) $x \div x^{\frac{1}{2}}$ **f)** $m \div m^{\frac{1}{3}}$ **g)** $d^{\frac{3}{2}} \div d^{\frac{1}{2}}$ **h)** $p^{\frac{3}{5}} \div p^{\frac{1}{5}}$

Appendix B
Worksheets

Rational Exponents

Write in radical form.

1. $2^{\frac{1}{3}}$ 2. $37^{\frac{3}{2}}$ 3. $x^{\frac{1}{2}}$ 4. $a^{\frac{1}{5}}$ 5. $6^{\frac{4}{3}}$ 6. $6^{\frac{3}{4}}$

7. $7^{-\frac{1}{2}}$ 8. $9^{-\frac{1}{5}}$ 9. $x^{-\frac{3}{7}}$ 10. $b^{-\frac{6}{5}}$ 11. $(3x)^{\frac{1}{2}}$ 12. $3x^{\frac{1}{2}}$

Write using exponents.

13. $\sqrt{7}$ 14. $\sqrt{34}$ 15. $\sqrt[3]{-11}$ 16. $\sqrt[5]{a^2}$ 17. $\sqrt[3]{6^4}$ 18. $(\sqrt[3]{b})^4$

19. $\frac{1}{\sqrt{x}}$ 20. $\frac{1}{\sqrt[3]{a}}$ 21. $\frac{1}{(\sqrt[5]{x})^4}$ 22. $\sqrt[3]{2b^3}$ 23. $\sqrt{3x^5}$ 24. $\sqrt[4]{5t^3}$

Evaluate without using a calculator..

25. $4^{\frac{1}{2}}$ 26. $125^{\frac{1}{3}}$ 27. $16^{-\frac{1}{4}}$ 28. $(-32)^{\frac{1}{5}}$ 29. $25^{0.5}$ 30. $(-27)^{-\frac{1}{3}}$

31. $(64)^{-\frac{1}{6}}$ 32. $0.04^{\frac{1}{2}}$ 33. $81^{0.25}$ 34. $0.001^{\frac{1}{3}}$ 35. $\left(\frac{4}{9}\right)^{\frac{1}{2}}$ 36. $\left(\frac{-27}{-8}\right)^{\frac{1}{3}}$

37. $8^{\frac{2}{3}}$ 38. $4^{\frac{3}{2}}$ 39. $9^{2.5}$ 40. $81^{\frac{3}{4}}$ 41. $16^{-\frac{3}{4}}$ 42. $(-32)^{\frac{2}{5}}$

43. $(-8)^{-\frac{5}{3}}$ 44. $(-27)^{-\frac{2}{3}}$ 45. $1^{\frac{5}{3}}$ 46. $(-1)^{-\frac{8}{5}}$ 47. $\left(\frac{100}{9}\right)^{\frac{3}{2}}$ 48. $\left(\frac{27}{8}\right)^{-\frac{2}{3}}$

Simplify without using a calculator.

49. a) $9^{\frac{1}{2}}$ b) $9^{-\frac{1}{2}}$ 50. a) $8^{\frac{1}{3}}$ b) $8^{-\frac{1}{3}}$

51. a) $8^{\frac{2}{3}}$ b) $8^{-\frac{2}{3}}$ 52. a) $16^{\frac{1}{4}}$ b) $16^{-\frac{1}{4}}$

53. $27^{-\frac{1}{3}}$ 54. $27^{\frac{2}{3}}$ 55. $4^{\frac{3}{2}}$ 56. $25^{-\frac{1}{2}}$

57. $81^{\frac{1}{2}}$ 58. $49^{-\frac{1}{2}}$ 59. $4^{\frac{3}{2}}$ 60. $16^{\frac{3}{4}}$

61. $(-125)^{-\frac{1}{3}}$ 62. $4^{-0.5}$ 63. $-8^{\frac{2}{3}}$ 64. $(5^{\frac{1}{3}})^{-3}$

65. $(16^{-5})^{\frac{1}{20}}$ 66. $(9^{\frac{1}{2}}+16^{\frac{1}{2}})^{2}$

Equations with Rational Exponents

Give the power to which you would raise both sides of each equation in order to solve the equation.

1. $x^{\frac{1}{2}} = 9$ 2. $x^{\frac{2}{3}} = 4$ 3. $x^{-\frac{1}{3}} = 2$ 4. $x^{-\frac{3}{4}} = 8^{-1}$

Solve each equation.

5. a) $a^{\frac{3}{4}} = 8$ b) $(3x+1)^{\frac{3}{4}} = 8$

6. a) $y^{-\frac{1}{2}} = 6$ b) $(3y)^{-\frac{1}{2}} = 6$

7. a) $2y^{-\frac{1}{2}} = 10$ b) $(2y)^{-\frac{1}{2}} = 10$

8. a) $(9t)^{-\frac{2}{3}} = 4$ b) $9t^{-\frac{2}{3}} = 4$

9. $(8-y)^{\frac{1}{3}} = 4$ 10. $(3n-1)^{-\frac{2}{3}} = \frac{1}{4}$

11. $(x^2+4)^{\frac{2}{3}} = 25$ 12. $(x^2+9)^{\frac{1}{2}} = 5$

Exponential Equations

Solve each of the following equations.

1. $3^x = \frac{1}{9}$
2. $25^x = 125$
3. $4^x = \frac{1}{8}$
4. $36^x = \sqrt{6}$
5. $3^x = \frac{1}{27}$
6. $5^x = \sqrt{125}$
7. $8^{2+x} = 2$
8. $4^{1-x} = 8$
9. $27^{2x-1} = 3$
10. $49^{x-2} = 7\sqrt{7}$
11. $4^{2x+5} = 16^{x+1}$
12. $3^{-(x+5)} = 9^{4x}$
13. $25^{2x} = 5^{x+6}$
14. $6^{x+1} = 36^{x-1}$
15. $10^{x-1} = 100^{4-x}$

Solutions

Rational Exponents

1. $\sqrt[3]{2}$	2. $(\sqrt{37})^3$	3. $\sqrt{x}$	4. $\sqrt[5]{a}$	5. $(\sqrt[3]{6})^4$	6. $(\sqrt[3]{6})^3$
7. $\frac{1}{\sqrt{7}}$	8. $\frac{1}{\sqrt[5]{9}}$	9. $\frac{1}{\sqrt[7]{x^3}}$	10. $\frac{1}{\sqrt[5]{b^6}}$	11. $\sqrt{3x}$	12. $3\sqrt{x}$
13. $7^{\frac{1}{2}}$	14. $34^{\frac{1}{2}}$	15. $(-11)^{\frac{1}{3}}$	16. $a^{\frac{2}{5}}$	17. $6^{\frac{4}{3}}$	18. $b^{\frac{4}{3}}$
19. $x^{-\frac{1}{2}}$	20. $a^{-\frac{1}{3}}$	21. $x^{-\frac{4}{5}}$	22. $2^{\frac{1}{3}}b$	23. $3^{\frac{1}{2}}x^{\frac{5}{2}}$	24. $5^{\frac{1}{4}}t^{\frac{3}{4}}$
25. 2	26. 5	27. $\frac{1}{2}$	28. -2	29. 5	30. $-\frac{1}{3}$
31. $\frac{1}{2}$	32. 0.2	33. 3	34. 0.1	35. $\frac{2}{3}$	36. $\frac{3}{2}$
37. 4	38. 8	39. 243	40. 27	41. $\frac{1}{8}$	42. 4
43. $-\frac{1}{32}$	44. $\frac{1}{9}$	45. 1	46. 1	47. $\frac{1000}{27}$	48. $\frac{4}{9}$

49. a) 3	b) $\frac{1}{3}$	50. a) 2	b) $\frac{1}{2}$	51. a) 4	b) $\frac{1}{4}$
52. a) 2	b) $\frac{1}{2}$	53. $\frac{1}{3}$	54. 9	55. 8	56. $\frac{1}{5}$
57. 9	58. $\frac{1}{7}$	59. 8	60. 8	61. $-\frac{1}{5}$	62. $\frac{1}{2}$
63. -4	64. $\frac{1}{5}$	65. $\frac{1}{2}$	66. 49		

Equations with Rational Exponents

1. 2   2. $\frac{3}{2}$ 3. -3 4. $-\frac{4}{3}$

5. a) 16   b) 5   6. a) $\frac{1}{36}$ b) $\frac{1}{108}$

7. a) $\frac{1}{25}$ b) $\frac{1}{200}$ 8. a) $\frac{1}{72}$ b) $\frac{27}{8}$

9. -56 10. 3   11. ± 11 12. ± 4

Exponential Equations

1. 1   2. $\frac{3}{2}$ 3. $\frac{1}{2}$ 4. $\frac{1}{4}$ 5. -3 6. $\frac{3}{2}$

7. $-\frac{5}{3}$ 8. $-\frac{1}{2}$ 9. $\frac{2}{3}$ 10. $\frac{11}{4}$ 11. No solution

12. $-\frac{5}{9}$ 13. 2   14. 3   15. 3