Adult Basic Education

Mathematics

Mathematics 3104C

Probability, Circles and Ellipses

Curriculum Guide

Prerequisites: Math 1104A, Math 1104B and Math 1104C
Math 2104A, Math 2104B and Math 2104C
Math 3104A

Credit Value: 1

<u>**Required Mathematics Courses**</u>   **[Degree and Technical Profile]**
Mathematics 1104A
Mathematics 1104B
Mathematics 1104C
Mathematics 2104A
Mathematics 2104B
Mathematics 2104C
Mathematics 3104A
Mathematics 3104B
Mathematics 3104C

Table of Contents

To the Instructor

I.   Introduction to Mathematics 3104C

This course has two distinct topics. The first is a continuation of the study of probability that was begun in previous courses. The second topic is an introduction to circles and ellipses as conic shapes, and a development of the circle as a trigonometric function. The second topic, in particular, is an essential component of all post-secondary Math courses.

II.   Prerequisites

Students taking this course should be able to calculate simple probabilities and create tree diagrams. They should be able to evaluate expressions that involve fractions and simplify fractional expressions that have common factors in the numerator and denominator. They should be able to use the translation rules that stretch or translate graphs and equations. Students should be familiar with right triangle trigonometry and be able to apply the sine and cosine definitions in terms of opposite adjacent and hypotenuse.

There are exercises (with solutions) for each **Section** of the textbook in **Prerequisites** in the *Teacher's Resource Book*. The instructor may use these problems to determine the student's ability. In the Study Guide, students are often directed to see the instructor for **Prerequisite** exercises. The instructor, however, must ensure that the student has mastered these concepts before beginning a particular unit.

III. Textbook

The Study Guide for this course has an Appendix A, which contains pages photocopied from *Mathematics 11*.

Most of the concepts are introduced, developed and explained in the **Examples**. The instructor must insist that the student carefully studies and understands each **Example** before moving on to the **Exercises**. In the Study Guide, the student is directed to see the instructor if there are any difficulties.

There are four basic categories included in each section of the textbook which require the student to complete questions:
1.   Investigate
2.   Discussing the Ideas
3.   Exercises
4.   Communicating the Ideas

To the Instructor

Investigate: This section looks at the thinking behind new concepts. The answers to its questions are found in the back of the text.

Discussing the Ideas: This section requires the student to write a response which clarifies
and demonstrates understanding of the concepts introduced. The answers to these questions are not in the student text but are in the *Teacher's Resource Book*. Therefore, in the Study Guide, the student is directed to see the instructor for correction. This will offer the instructor some perspective on the extent of the student's understanding. If necessary, reinforcement or remedial work can be introduced. Students should not be given the answer key for this section as the opportunity to assess the student's understanding is then lost.

Exercises: This section helps the student reinforce understanding of the concepts introduced. There are three levels of **Exercises**:
   A: direct application of concepts introduced;
   B: multi-step problem solving and some real-life situations;
   C: problems of a more challenging nature.

The answers to the **Exercises** questions are found in the back of the textbook.

Communicating the Ideas: This section helps confirm the student's understanding of a particular lesson by requiring a clearly written explanation. The answers to **Communicating the Ideas** are not in the student text, but are in the *Teacher's Resource Book*. In the Study Guide students are asked to see the instructor for correction.

IV. <u>Technology</u>

It is important that students have a **scientific** calculator for their individual use. Ensure that the calculators used have the word "scientific" on it, as there are calculators designed for business or statistics which would not have the functions needed for this course.

A graphing calculator should be **available** to the students since the text provides many opportunities for its use. The *Teacher's Resource Book* suggests many opportunities to use a graphing calculator. These suggestions are outlined where there is the heading *Integrating Technology*. In the Study Guide, students are directed to see the instructor when a graphing calculator is required. The *Teacher's Resource Book* contains a module called **Graphing Calculator Handbook** which will help the instructor and student get acquainted with some of the main features of the TI-83 Plus graphing calculator.

To the Instructor

Graphing software such as *Graphmatica* or *Winplot* can also be used it the students don't have access to a graphing calculator but do have access to a computer. The textbook doesn't offer the same guidance for graphing with these tools as it does for a graphing calculator, but each software program does have a HELP feature to answer questions.

V.   Curriculum Guides

Each new ABE Mathematics course has a Curriculum Guide for the instructor and a Study Guide for the student. The Curriculum Guide includes the specific curriculum outcomes for the course. Suggestions for teaching, learning, and assessment are provided to support student achievement of the outcomes. Each course is divided into units. Each unit comprises a **two-page layout of four columns** as illustrated in the figure below. In some cases the four-column spread continues to the next two-page layout.

Curriculum Guide Organization:
The Two-Page, Four-Column Spread

Unit Number - Unit Title			Unit Number - Unit Title	

Outcomes	Notes for Teaching and Learning		Suggestions for Assessment	Resources
Specific curriculum outcomes for the unit.	Suggested activities, elaboration of outcomes, and background information.		Suggestions for assessing students' achievement of outcomes.	Authorized and recommended resources that address outcomes.

VI.   <u>Study Guides</u>

The Study Guide provides the student with the name of the text(s) required for the course and specifies the sections and pages that the student will need to refer to in order to complete the required work for the course. It guides the student through the course by assigning relevant reading and providing questions and/or assigning questions from the text or some other resource. Sometimes it also provides important points for students to note. (See the *To the Student* section of the Study Guide for a more detailed explanation of the use of the Study Guides.) The Study Guides are designed to give students some degree of independence in their work. Instructors should note, however, that there is much material in the Curriculum Guides in the *Notes for Teaching and Learning* and *Suggestions for Assessment* columns that is not included in the Study Guide and instructors will need to review this information and decide how to include it.

VII.   <u>Resources</u>

Essential Resources

Addison Wesley Mathematics 12 (Western Canadian edition) Textbook
ISBN:0-201-34624-9

Mathematics 12 Teacher's Resource Book (Western Canadian edition)
ISBN: 0-201-34626-5

Math 3104C Study Guide

Recommended Resources

*Mathematics 12 Independent Study Guide (*Western Canadian edition)
ISBN: 0-201-34625-7

Center for Distance Learning and Innovation: http://www.cdli.ca
Winplot: http://math.exeter.edu/rparris/winplot.html
          (Free graphing software)
Graphmatica   (Evaluation software available on CD-ROM contained in
          Teacher's Resource Book)
CD Rom accompanying *Teacher's Resource Book*
     This CD contains selected solutions from the text and self test
     solutions from the *Independent Study Guide*.

To the Instructor

Other Resources

Math Links: http://mathforum.org

http://spot.pcc.edu/~ssimonds/winplot
(Free videos concerning Winplot)

http://www.pearsoned.ca/school/math/math/

http://www.ed.gov.nl.ca/edu/sp/mathlist.htm
(Math Companion 3204 Supporting Document)

VIII. Recommended Evaluation

Written Notes	10%
Assignments	10%
Test(s)	30%
Final Exam *(entire course)*	<u>50%</u>
	100%

The overall pass mark for the course is 50%.

Probability, Circles and Ellipses

Unit 1 - Permutations and Combinations

Outcomes

1.1   Use the Fundamental Counting Principle to solve problems.

   1.1.1   State the Fundamental Counting Principle.

   1.1.2   Use the Fundamental Counting Principle to determine the total number of outcomes for a situation**.**

Notes for Teaching and Learning

Note: Assign review questions 1 and 2 from **Prerequisites**, page 4, Chapter 6, *Teacher's Resource Book*.

The Fundamental Counting Principle will give the total number of outcomes but will not describe them. Have students give an example to support their definition.

The students are directed to do **Investigate Counting without Counting** (page 354, *Mathematics 12*). Initially, a tree diagram can be used to describe and list the number of outcomes for the situation described and thereby determine the total number of outcomes. The answer found by using the Fundamental Counting Principle should agree.

Tree diagrams are not in the text but can be used to help clarify thinking and list choices. For small numbers of choices, it can be useful but this method becomes cumbersome as the number increases. Go through some examples with the students to demonstrate. Each branch represents another type of choice.

The **Suggestions for Assessment** refers to a worksheet which deals with a comparison of tree diagrams and the Fundamental Counting Principle. These questions can be used to show students the strengths and deficiencies of both methods.

As a general rule, students should begin by examining the restrictions of a problem. The two key questions to be considered are: how many choices are to be made? how many ways can the choice be made?

Unit 1 - Permutations and Combinations

Suggestions for Assessment

Study Guide questions 1.1 to 1.5 will meet the objectives of Outcome 1.1.

See Appendix B, pages 41 and 42, for a worksheet on Tree Diagrams and the Fundamental Counting Principle.

See *Teacher's Resource Book*, Chapter 6, pages 4 and 6, for questions that can be used for assessment or reinforcement.

Resources

Mathematics 12, Section 6.1, The Fundamental Counting Principle, pages 354 - 359

Teacher's Resource Book, *Mathematics 12*, Chapter 6, pages 4 - 6

Independent Study Guide, Mathematics 12, pages 73 - 80

www.cdli.ca
Math 3204, Unit 05, Section 02

Outcomes

1.2   Solve problems using permutations.

  1.2.1   State the definition of a permutation and give an example to support the definition.

  1.2.2   Calculate the number of permutations using the Fundamental Counting Principle.

Notes for Teaching and Learning

Note: Assign review questions 1 to 4, from **Prerequisites** in Chapter 6, in the *Teacher's Resource Book.*

A permutation is an arrangement of objects in which <u>order</u> is important, like position at the finish of a race or numbers in a combination lock. Before the students are given the permutation formula, the number of arrangements will be calculated by using the Fundamental Counting Principle.

If 5 books are to be arranged on a shelf, there are 5 choices for the first position, 4 for the second position, 3 for the third position, 2 for the fourth position and 1 for the last position, therefore there are $5 \times 4 \times 3 \times 2 \times 1 = 120$ different arrangements of the five books.

Investigate Permutations Involving Different Objects (page 360, *Mathematics 12*), involves permutations of 3, 4 and 5 different letters. The students will find listing the arrangements to be easier if a tree diagram is used.

The Fundamental Counting Principle will give the total number of arrangements but not actually describe them; whereas the tree diagram will give the students the opportunity to examine the arrangements to verify that none have been omitted.

Suggestions for Assessment

Study Guide questions 1.6 to 1.12 will meet the objectives of Outcome 1.2.

See the Teacher's Resource Book, Chapter 6, pages 6 to 8, for questions that can be used for assessment or reinforcement. See Appendix B on page 40 for a Worksheet on Factorial Notation.

The following questions can be used to assess the students' understanding.

1. The town of Karsville, which has 32,505 automobiles, is designing its own licence plates for residents to place on their vehicles.
Use the permutation formula to determine the best of the following three options and explain your choice. Check your answers by using the Fundamental Counting Principle:
Note: Digits and letters can be used more than once.

    i) a licence made from using four single-digit numerals from 1 to 9
    ii) a licence made of three single-digit numerals from 1 to 9 and one letter from the alphabet
    iii) a licence made from three single-digit numerals from 1 to 9 and two letters from the alphabet

2. A local pizza restaurant has a special on its four-ingredient 20 cm pizza. If there are 15 ingredients from which to choose, how many different "specials" are possible?

Resources

Mathematics 12, Section 6.2, Permutations Involving Different Objects, pages 360 - 365

Teacher's Resource Book, Mathematics 12, Chapter 6, pages 6 to 8

Independent Study Guide, Mathematics 12, pages 73 - 80

www.cdli.ca
Math 3204, Unit 05, Section 03, Lessons 01 and 02

Outcomes

1.2.3   Use factorial notation (*n!*) to calculate the number of permutations.

1.2.4   Calculate number of permutations using the permutation function of a calculator.

1.2.5   State the permutation formula. Explain what the variables *n* and *r* represent.

Notes for Teaching and Learning

A shorter way of referring to the multiplication of decreasing natural numbers is the factorial notation where $5 \times 4 \times 3 \times 2 \times 1$ is represented as 5!

Factorials will be of particular use when calculating permutations and combinations. Most calculators will have factorial keys as well as shortcut keys for calculations of permutations and combinations. Ensure that students are aware of the capabilities of their calculators.

Note: In factorials that involve a product of factorials in the denominator, it may be necessary to put brackets around the two denominator factorials to ensure the denominator is calculated before it is divided into the numerator.

The representation for a permutation is $_n P_r$ where *n* represents the total number of objects and *r* represents the number of objects taken. Students should be aware of the permutation function of their calculator.

Suggestions for Assessment

Study Guide questions 1.6 to 1.12 will meet the objectives of Outcome 1.2.

See Chapter 6, *Teacher's Resource Book*, pages 6 to 8, for questions that can be used for assessment or reinforcement.

See Appendix B on page 40 for a Worksheet on Factorial Notation.

See **Review**, page 397, questions 1 to 6, for additional questions that can be used for assessment or reinforcement.

Resources

Mathematics 12, Section 6.2, Permutations Involving Different Objects, pages 360 - 365

Teacher's Resource Book, Mathematics 12, Chapter 6, pages 6 to 8

Independent Study Guide, Mathematics 12, pages 73 - 80

www.cdli.ca
Math 3204, Unit 05, Section 03, Lesson 03

Unit 1 - Permutations and Combinations

Outcomes

1.3   Define and evaluate combinations.

   1.3.1   Define combinations.

   1.3.2   State the meaning of *n* and *r* in the combination formula $_n C_r$.

   1.3.3   Evaluate combinations:

      i) using the combination formula and
      ii) using the combination function of the calculator.

Notes for Teaching and Learning

One of the important skills of problem-solving in this unit will be the ability to determine the difference between combinations and permutations. Encourage students to recognize the conditions or requirements that signal whether or not **order** is important.

The representation for a combination is $_n C_r$. This is read as: *a combination of n objects, taking r objects at a time*. Students should be aware of the combination and/or factorial functions of their calculator.

Suggestions for Assessment

Study Guide questions 1.13 to 1.16 will meet the objectives of Outcome 1.3.

See Chapter 6, *Teacher's Resource Book*, pages 11 to 14, for questions that can be used for assessment or reinforcement.

The following questions are suggested for assessment:

1.   Melvin wants to buy 10 different books but can only afford four of them. In how many ways can he make his selection? **Note:** The four books selected must be distinct (no repetitions allowed) and since the order of the four chosen has no importance, this is a combination.

2.   Every member of a business would like to attend a special event this weekend, but only ten employees will be allowed to attend. In how many ways could the ten be selected if there are a total of 48 employees?

3.   From a group of 10 candidates, four will be elected for the executive positions of the student council: president; vice-president; secretary and treasurer. How many different possibilities are there for filling these four executive positions? **Note:** Positions will be filled according to number of votes. Order is not important.

4.   A committee of 3 is to be assigned from a group of 10 people. How many ways can this be done?

5.   A committee of three and a committee of four are to be assigned from a group of 10 people. How many ways can this be done if no person is assigned to both committees?

Resources

Mathematics 12, Section 6.4, Combinations, pages 371 - 378

Teacher's Resource Book, Mathematics 12, Chapter 6, Permutations and Combinations, pages 11 - 14

Independent Study Guide, Mathematics 12, pages 73 - 80

Outcomes

1.4   Distinguish between the use of the Fundamental Counting Principle, permutations and combinations.

Notes for Teaching and Learning

The particular conditions of a problem will determine which specific technique to use. The determining factor will be if *order* is important. If order is important, as in the question, "How many ways can three of eight people line up at a ticket counter?", then use permutations.

If order is not important, as in the question, "How many ways can a committee of three be selected from a group of twelve people?", then use combinations.

If there are several parts to the final answer, the results of the different parts can be multiplied together using the Fundamental Counting Principle.

Suggestions for Assessment

Study Guide questions 1.17 and 1.18 will meet the objectives of Outcome 1.4.

See **Review**, page 397, questions 1 to 6, for exercises that can be used for assessment or reinforcement.

Masters 6.3 to 6.7 contain questions (with answers) that can be used for assessment. Since not all of Chapter 6 was covered in this unit, the instructor should carefully choose relevant questions.

Resources

Mathematics 12, Review , pages 397

Independent Study Guide, Mathematics 12, pages 73 - 80

Teacher's Resource Book, Mathematics 12, Chapter 6, Masters 6.3 to 6.7

Outcomes

2.1   Use experimental and theoretical probability to solve problems.

   2.1.1   Distinguish between experimental probability and theoretical probability.

   2.1.2   Calculate the probability of an event A, given $P(A) = \dfrac{r}{n}$ where n is a number of equally likely outcomes and r is the number of favorable outcomes.

Notes for Teaching and Learning

Note: The instructor should assign review questions from **Prerequisites**, Chapter 7, in the *Teacher's Resource Book*.

The calculation of probability will be found by dividing the number of favorable outcomes of the event by the total number of outcomes.

In probability, the goal is to assign a number between 0 and 1 inclusive to events that interest us, but for which we don't know the outcome. (Remember: 0 would mean the event will never occur and 1 would mean the event occurred all the time). If we have a bag of marbles which contains 3 red marbles and 2 blue marbles, and we are wondering about the probability of selecting a red marble, we need to consider the number of ways of selecting a ***red*** marble compared to the total number of ways of selecting ***any*** marble. In selecting a red marble, we would have 3 ways of making the selection. To select any marble, we would have 5 choices. Therefore the probability of selecting a red marble would be 3 out of 5 (0.6).

Probability can be expressed in decimal or fraction form.

Note: It may help the students to list the possibilities by using a tree diagram which is a systematic method that can be used to determine the number of favorable outcomes as well as the total number of outcomes.

Suggestions for Assessment

Study Guide questions 2.1 to 2.4 will meet the objectives of Outcome 2.1

See Chapter 7, in the *Teacher's Resource Book* on pages 4 to 6, for questions that can be used for assessment or reinforcement.

The following are sample questions that can be asked to assess a student's understanding of the concepts.

1. Michelle Brown wants to have exactly two daughters. Assuming that the chance of having a boy or girl are equal, determine the probability of success in each of the following cases:
    i)   She has two children altogether.
    ii) She has three children altogether.

2. Find the probability of being dealt each of the following poker hands:
    i)   P(full house) (three of one denomination and two of another, ie., three 4s and two 5s)
    ii) P(royal house) (the five highest cards of a single suit)

Resources

Mathematics 12, Section 7.1 Experimental and Theoretical Probability, pages 402 - 407

Teacher's Resource Book, Mathematics 12, Chapter 7, pages 4 - 6

Independent Study Guide, Mathematics 12, pages 81 - 90

Outcomes

2.2   Distinguish between related events, A and B:
    i) the event that A does not occur,
    ii) the event that both A and B occur,
    iii) the event that A or B (or both) occur.

    2.2.1   Define the following:
        i) sample space
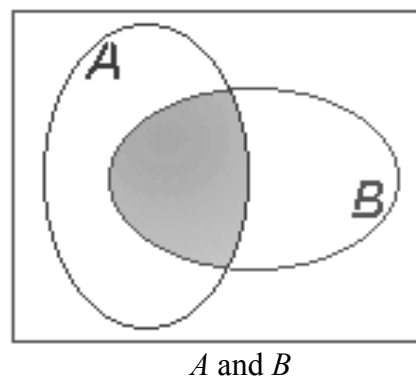   ii) complement
        iii) Venn diagram

    2.2.2   Demonstrate, using a Venn diagram or an example, an understanding of the event that both A and B occur, P(A and B).

Notes for Teaching and Learning

In this section, diagrams will be very useful to students by assisting them with their understanding of what the words "and" and "or" mean when considering favorable outcomes. The instructor should use Venn diagrams to support the discussion of these concepts. Venn diagrams may also prove to be useful to students as a problem-solving strategy when answering probability questions.

Note: Venn diagrams are not used to explain the following concepts but, as with tree diagrams in an earlier unit, they are a way of visualizing relationships that may aid understanding.

Students should be able to identify what A and B represent and to understand that a favorable outcome is one in which both A and B occur. For example, if A is a black card and B is a face card, to find P(A and B), the favorable outcome would be a black face card which would be indicated by the intersection of A and B in the Venn diagram that follows.

A and *B*

Suggestions for Assessment

Study Guide questions 2.5 to 2.8 will meet the objectives of Outcome 2.2**.**

See Chapter 7, Probability, *Teacher's Resource Book*, pages 8 to 10, for questions that can be used for assessment or reinforcement.

The following are sample questions that can be used for further assessment or practice.

1. If one number is selected randomly from the set (*1, 2, 3, 4, 5, 6, 7, 8, 9, 10)*, find the probability that:
    i) the number will be odd or a multiple of 3, P(odd or multiple of 3).
    ii) the number will be odd and a multiple of 3, P(odd and multiple of 3).

2. Suppose a single card is to be drawn from a standard deck of 52 cards. Find the probability that:
    i) that the card will be other than a king, P(not a king).
    ii) the card will be a spade and a red card, P(spade and red card).
    iii) the card will be a spade or a red card, P(spade or red card).

Resources

Mathematics 12, Section 7.2, Related Events, pages 410 - 416

Teacher's Resource Book, Mathematics 12, Chapter 7, Probability, pages 8 - 10

Independent Study Guide, Mathematics 12, pages 81 - 90

Outcomes

2.2.3   Demonstrate, using a Venn diagram or an example, an understanding of the event that A or B (or both) occur, P(A or B).

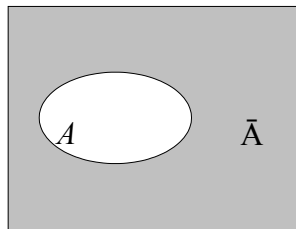## Notes for Teaching and Learning

The instructor should help students identify what A and B represent and understand that a favorable outcome would be one in which either A or B or both occur. For example, if A is a black card and B is a face card, to find the P(A or B), the favorable outcome would be a black card, a face card or a black face card.

This would describe areas A and B in the Venn diagram below as well as the intersecting area between A and B.

A or *B*

2.2.4   Demonstrate, using a Venn diagram or an example, an understanding of the event that A does not occur, and that
$$P(A) + P(\bar{A}) = 1.$$
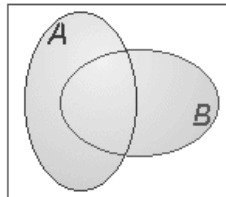
With a Venn diagram, this becomes an easy rule to grasp.

Complement of *A*
$$P(A) + P(\bar{A}) = 1$$

Suggestions for Assessment

Study Guide questions 2.5 to 2.8 will meet the objectives of Outcome 2.2**.**

See Chapter 7, Probability, *Teacher's Resource Book*, pages 8 to 10, for questions that can be used for assessment or reinforcement.

The following are sample questions that can be used for further assessment or practice.

1.   If one number is selected randomly from the set (1, 2, 3, 4, 5, 6, 7, 8, 9, 10), find the probability that:
         i)   the number will be odd or a multiple of 3, P (odd or multiple of 3).
         ii)   the number will be odd and a multiple of 3, P(odd and multiple of 3).

2.   Suppose a single card is to be drawn from a standard deck of 52 cards. Find the probability that:
         i) that the card will be other than a king, P(not a king).
         ii) the card will be a spade and a red card, P(spade and red card).
         iii) the card will be a spade or a red card, P(spade or red card).

Resources

Mathematics 12, Section 7.2, Related Events, pages 410 - 416

Teacher's Resource Book, Mathematics 12, Chapter 7, Probability, pages 8 - 10

Independent Study Guide, Mathematics 12, pages 81 - 90

Outcomes

Notes for Teaching and Learning

2.3   Determine the probability of Event *A* or *B* involving exclusive events, and non-mutually exclusive events.

In the previous section, probability was determined by counting the outcomes in the sample space. However, in this section formulas will be applied. Initially, the events used will be mutually exclusive, meaning they can't occur at the same time, ie., a card that is a spade and red.

   2.3.1   Describe events that are mutually exclusive.

If *A* and *B* are mutually exclusive events,
$$P(A \text{ or } B) = P(A) + P(B)$$
and
$$P(A \text{ and } B) = 0 \text{ .}$$

Students will now calculate probabilities of the "or" group by *adding* the probabilities of the different outcomes that are favorable. The instructor should remind the students of the rules for adding and simplifying fractions and ensure that they are aware of the fraction key (if available) on their calculators. Students should be encouraged to do the calculations with pencil and paper first.

   2.3.2   Calculate P(A or B) as P(A) + P(B) where A and B are mutually exclusive events.

   2.3.3   Calculate P(A or B) as P(A) + P(B) – P(A and B) where A and B are not mutually exclusive events.

If A is a spade and B is a black card, the P(A or B) would mean a spade or a black card would be a favorable outcome. In a standard 52 card deck this would mean 13 spades and 13 clubs, as black cards, would be favorable outcomes. The 13 spades, as spades, would also qualify as favorable outcomes. However, the cards that are both black **and** spades, P(A and B), will be subtracted from the total number of favorable outcomes because they have already been counted in either the black group or the spade group.

Suggestions for Assessment

Study Guide questions 2.9 to 2.14 will meet the objectives of Outcome 2.3.

See Chapter 7, in the *Teacher's Resource Book* on pages 13 to 15, and Appendix B on pages 43 to 50 for questions that can be used for assessment or reinforcement.

The following are sample questions that can be used for further assessment or practice:

1. Of the twenty television programs to be aired this evening, Marc plans to watch one, which he will pick at random by throwing a dart at the TV schedule. If 8 of the programs are educational, 9 are interesting and 5 are both educational and interesting, find the probability that the show he watches will have at least one of the attributes.

2. Determine if the following pairs of events are mutually exclusive:
    a) The weather is fine. I walk to work.
    b) I cut a deck of cards and have a Queen; you cut a five.
    c) I cut a deck and have a red card; you cut a card with an odd number.
    d) I select a voter who registered Liberal; you select a voter who is registered Tory.
    e) I found a value for x to be greater than -2; you found x to have a value greater than 3.
    f) I selected two cards from a deck, the first was a face card, the second was red.

Resources

Mathematics 12, Section 7.3, The Event A or B, pages 420 - 424

Teacher's Resource Book, Mathematics 12, Chapter 7, Probability, pages 13 - 15

Independent Study Guide, Mathematics 12, pages 81 - 90

Unit 3 - Circles and Coordinate Geometry

Outcomes

3.1 Translate between equations, graphs and written descriptions of circles.

    3.1.1 Use the distance formula to write the equation of a circle when given the centre and radius.

    3.1.2 Find the centre and radius of a circle when given the equation.

    3.1.3 Find the centre and radius of a circle when given its graph.

    3.1.4 Graph a circle when given the standard equation.

    3.1.5 Determine the equation of a circle when given a set of conditions.

Notes for Teaching and Learning

Note: Assign review questions from **Prerequisites**, pages 4, 14 and 23, Chapter 9, Coordinate Geometry, *Teacher's Resource Book.*

Students should become familiar with the characteristics of the equation of a circle with centre (0,0) and centre (*h,k*).

The graph is a familiar shape and the information needed to describe the circle from either the graph or the equation will be the radius and the coordinates of the centre.

Students may need some review on a) eliminating the radical in a radical equation such as

$$\sqrt{(x-3)^2 + y^2} = 6,$$

b) using the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2},$$

and c) using the midpoint formula:

$$\text{midpoint: } (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).$$

When doing the questions in the Study Guide, students should use diagrams whenever possible to ensure that they understand the stated conditions and are using the distance formula properly.

Unit 3 - Circles and Coordinate Geometry

Suggestions for Assessment

Study Guide questions 3.1 to 3.3 will meet the objectives of Outcome 3.1

Resources

Appendix A, pages 32 - 37

www.cdli.ca
Math 3204, Unit 04,
Section 04

Unit 4 - Circles and Ellipses

Outcomes

4.1   Demonstrate a knowledge of the distinction between a circle equation and an ellipse equation.

   4.1.1   Classify the circle and ellipse by their generation from the intersection of a cone and a plane.

   4.1.2   State the properties of a circle and an ellipse.

   4.1.3   State the difference between the standard form and general form of an equation.

   4.1.4   Recognize and graph equations of circles and ellipses with centres (0,0).

   4.1.5   Recognize and graph equations of circles and ellipses with centres (h,k).

   4.1.6   Convert an equation of a circle or ellipse from general to standard form, and vice versa.

Notes for Teaching and Learning

Circles and ellipses are similar in that both involve squared variables that are positive. The feature that distinguishes one from the other is the coefficient in front of the squared variables; in the circle, the coefficients will be the same and for the ellipse, they will be different. One of the first skills to develop in this section is to be able to identify from the equations what conic shape will be seen in the graph.

The ability to interpret information from the equation is a useful one and can give the student knowledge about the graph before any graphing is done.

Suggestions for Assessment

Study Guide questions 4.1 to 4.13 will meet the objectives of Outcome 4.1.

Resources

Mathematics 12, Section 9.2 to Section 9.4, pages 525 - 549

Teacher's Resource Book, Mathematics 12, Chapter 9, pages 6-22

www.cdli.ca
Math 3204, Unit 04, Section 04

Outcomes

4.2   Convert equations from the general form to the standard form.

   4.2.1   Convert ellipse and circle equations from a general form to a standard form.

   4.2.2   Describe the characteristics of the ellipse graph from the standard ellipse equation: centre, intercepts, major and minor axes and lengths of those axes.

   4.2.3   Solve problems involving the equations and characteristics of circles and ellipses.

Notes for Teaching and Learning

The technique of completing the square will be used in this conversion and may require some review.

Unit 4 - Circles and Ellipses

Suggestions for Assessment

Study Guide questions 4.14 to 4.17 will meet the objectives or Outcome 4.2.

The instructor could choose questions on the circle and ellipse from Masters 9.5 to 9.8.

The following are sample questions that can be used to assess the student's understanding of this unit.

1. The dome of an arena is elliptical in shape. If the height of the dome is 28m, and it has a span of 75m, find a possible equation for this ellipse.

2. An ellipse is given by the equation
$25x^2 + 4y^2 + 100x - 16y + 16 = 0$
What are the coordinates of the centre?

3. The equation of a circle is given by
$x^2 + y^2 - 4x + my - 18 = 0$
if A(7,3) is a point on the circle, find the value of m.

4. Express each equation in standard form then sketch the graph.
a) $x^2 + y^2 - 8x - 9 = 0$
b) $x^2 + y^2 - 10x - 6y - 2 = 0$
c) $2x^2 + 2y^2 - 4x + 6y - 2 = 0$
d) $2x^2 + 2y^2 + 5y = 0$
e) $x^2 + y^2 - 8x + 6y - 11 = 0$

Resources

Mathematics 12, Section 9.6, Converting Equations from General to Standard Form, pages 561 - 563

Teacher's Resource Book, Mathematics 12, Chapter 9 Masters 9.5 - 9.8

Unit 5 - The Circle as a Trigonometric Function

Outcomes

5.1   Demonstrate an understanding of the relationship between angle rotation and the coordinates of a rotating point.

   5.1.1   Define the following terms:
      i) standard position of an angle
      ii) coterminal angles
      iii) initial arm
      iv) terminal arm
      v) clockwise rotation
      vi) counterclockwise rotation

   5.1.2   Draw an angle in standard position and determine the measures of coterminal angles.

   5.1.3   Apply the definitions of sine and cosine in right triangle trigonometry to the definitions of these functions in terms of the unit circle.

   5.1.4   Express the coordinates of any point, *P*, on a unit circle as (*cos θ, sin θ*).

Notes for Teaching and Learning

Note: Assign review questions from **Prerequisites**, Chapter 3, *Teacher's Resource Book,* pages 9 and 13. The instructor should emphasize that when an angle is in standard position, the initial arm is always on the positive *x*-axis and does not move.

The instructor should point out to students that the answers in **Example 1** could vary. Usually the smallest positive and negative coterminal angles are given as answers.

Instructors should remind students how quadrants are numbered and that angles such as $\frac{3\pi}{2}$ or 270^0 are quadrantal angles since the terminal arm is on an axis.

Unit 5 - The Circle as a Trigonometric Function

Suggestions for Assessment

Study Guide questions 5.1 to 5.11 will meet the objectives of
Outcome 5.1.

The *Teacher's Resource Book, Mathematics 12* has extra
problems in **Supplementary Examples** and **Assessing the
Outcome** which could be used for extra practice, homework or
assessment.

There is also a **Written Test** and **Multiple Choice Test** in the
Teacher's Resource Book which could be used for further
assessment.

Resources

Mathematics 12, Section
3.3 to Section 3.5 pages 170
- 189

*Teacher's Resource Book,
Mathematics 12*, Chapter 3,
pages 9 - 22

*Independent Study Guide,
Mathematics 12*

www.cdli.ca
Math 3204, Unit 04,
Section 05

Unit 5 - The Circle as a Trigonometric Function

Outcomes

5.1.5 Apply unit circle definitions to find the trigonometric values for the quadrantal angles.

5.1.6 Extend the trigonometric definitions of sine and cosine to a circle with radius, *r* and express any point, *P,* on the circle as trigonometric functions of the central angle, θ, (*r cos θ, r sin θ*).

5.1.7 Determine the exact value of the sine and cosine functions of the special angles, $\frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{6}$ and their multiples.

Notes for Teaching and Learning

Note: Assign review questions from **Prerequisites**, *Teacher's Resource Book, Mathematics 12,* Chapter 3, page 18.

Suggestions for Assessment

Resources

Mathematics 12, Section 3.3 to Section 3.5 pages 170 - 189

Teacher's Resource Book, Mathematics 12, Chapter 3, pages 9-22

Appendix A

The Equation of a Circle

Many farms in western North America use an automated centre-pivot irrigation system. A long pipe sprays water as it rotates about the centre. Distinctive circular traces are left by the wheels and, since the end of the pipe is always the same distance from the centre, the area watered forms a circle.

On a coordinate grid, we can determine the equation of a circle, just as we can determine equations of lines and parabolas.

Example 1

Consider the circle with centre C(2, −1) and radius 5 units.

a) Graph the circle.

b) Determine the equation of the circle.

Solution

a) Draw a circle with centre C(2, −1) and radius 5.

b) Let P(x, y) be any point on the circle.

Since CP is a radius, CP = 5

Use the distance formula.

$\sqrt{(x-2)^2 + (y+1)^2} = 5$

Square each side to eliminate the radical.

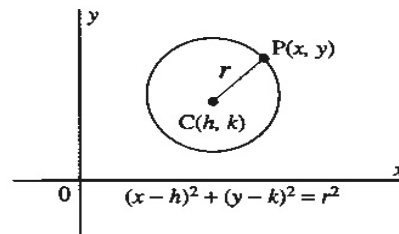$(x-2)^2 + (y+1)^2 = 25$

This is the equation of the circle.
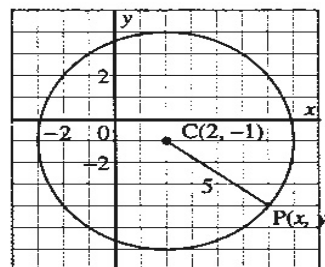
We can use the method of *Example 1* to determine the equation of any circle with centre C(h, k) and radius r.

Let P(x, y) be any point on the circle. Then,

$$CP = r$$
$$\sqrt{(x-h)^2 + (y-k)^2} = r \quad \text{— Square each side.}$$
$$(x-h)^2 + (y-k)^2 = r^2$$
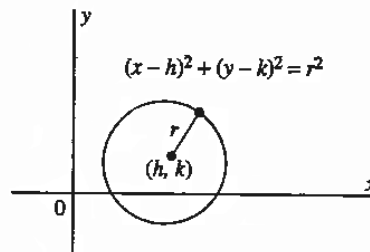

$(x-h)^2 + (y-k)^2 = r^2$

Equation of a Circle

The equation of a circle with centre (h, k) and radius r is:

$(x - h)^2 + (y - k)^2 = r^2$

When the centre is $O(0, 0)$, the equation is:

$x^2 + y^2 = r^2$

Example 2

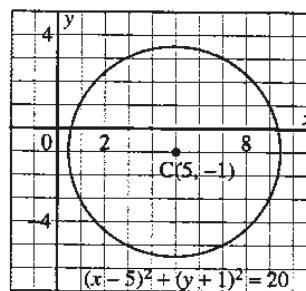Graph the circle defined by $(x - 5)^2 + (y + 1)^2 = 20$.

Solution

The equation $(x - h)^2 + (y - k)^2 = r^2$ represents a circle with centre (h, k) and radius r.

Hence, $(x - 5)^2 + (y + 1)^2 = 20$ represents a circle with centre $(5, -1)$ and radius $\sqrt{20}$.

Plot the point $C(5, -1)$ on a grid.

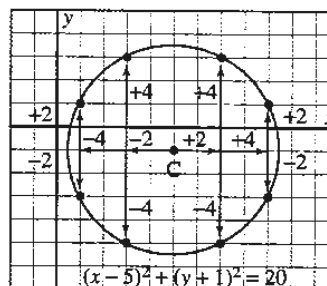With centre C, draw a circle with radius $\sqrt{20} \doteq 4.47$.

Visualizing

Consider the graph in *Example 2*. Observe that $r^2 = 20 = 2^2 + 4^2$.
Start at the centre. Points on the circle can be located by moving:

|	
2 left, 4 up	2 right, 4 up
4 left, 2 up	4 right, 2 up
2 left, 4 down	2 right, 4 down
4 left, 2 down	4 right, 2 down

This method can only be used when the square of the radius is the sum of two squares.

Example 3

Determine the equation of this circle.

Solution

Think...

To determine the equation of the circle, we need to know the radius and the coordinates of the centre. The coordinates of the centre are given, and we can calculate the radius using the distance formula.
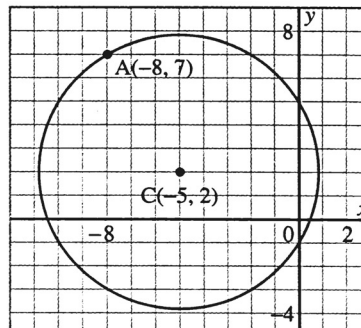
The radius is the length of CA.

$$CA = \sqrt{(-8 + 5)^2 + (7 - 2)^2}$$
$$= \sqrt{9 + 25}$$
$$= \sqrt{34}$$

The radius of the circle is $\sqrt{34}$. The centre is $(-5, 2)$.

Use the equation $(x - h)^2 + (y - k)^2 = r^2$.

The equation of the circle is $(x + 5)^2 + (y - 2)^2 = 34$.

DISCUSSING THE IDEAS

1. In *Example 1*, the equation of the circle was written as $(x - 2)^2 + (y + 1)^2 = 25$. Why did we not expand the left side of the equation?

2. For the equation of the circle in *Example 3*:
   a) Why did we write $+5$ in $(x + 5)^2$, but -2 in $(y - 2)^2$?
   b) What happened to the radical sign in $\sqrt{34}$?

3. a) Does the graph of a circle represent a function? Explain.
   b) Does the equation of a circle represent a function? Explain.

Exercises

(A) 1. Determine if each point is on the circle defined by $x^2 + y^2 = 85$.

    a) A(9, −2)   **b)** B(−5, 8)   **c)** C(−7, −6)   **d)** D(4, 8)

2. State the radius and the coordinates of the centre of the circle defined by each equation.

    a) $(x − 3)^2 + (y + 4)^2 = 81$ **b)** $(x + 2)^2 + (y − 1)^2 = 5$

    c) $(x + 4)^2 + y^2 = 15$ **d)** $x^2 + (y − 6)^2 = 48$

    e) $x^2 + y^2 = 64$ **f)** $x^2 + y^2 = 12$

3. a) Write the equation of the circle with each given centre and radius.

    i) O(0, 0), 3   **ii)** O(0, 0), 7   **iii)** A(5, 3), 4   **iv)** B(−2, 6), 5

    v) C(4, 0), 6   **vi)** D(0, −3), 9   **vii)** E(2, 0), $\sqrt{5}$ **viii)** F(3, −5), $\sqrt{10}$

    b) Choose one circle from part a. Write to explain how you determined its equation.

4. Write the equation of each circle.
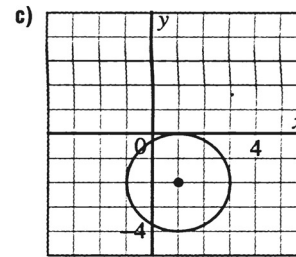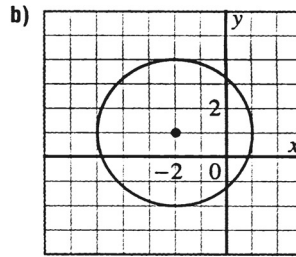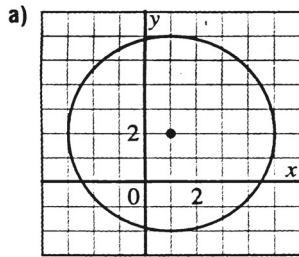
5. What does each equation represent? Explain.

    a) $x^2 + y^2 = 1$ **b)** $(x − 3)^2 + (y − 2)^2 = 2$

(B) 6. State which of these circles have:

    a) radius less than 5

    b) centre on the *x*-axis

    c) centre on the line $y = x$

        i) $(x − 3)^2 + (y − 3)^2 = 1$ **ii)** $(x + 2)^2 + (y − 4)^2 = 9$

        iii) $x^2 + (y + 7)^2 = 13$ **iv)** $(x − 5)^2 + y^2 = 20$

        v) $(x + 1)^2 + (y + 1)^2 = 25$ **vi)** $x^2 + y^2 = 32$

7. On the same grid, graph the equations in each list. Write to explain how the patterns in the equations are related to the patterns in the graphs.

a) $(x - 2)^2 + (y - 5)^2 = 10$
   $(x + 2)^2 + (y - 5)^2 = 10$
   $(x - 2)^2 + (y + 5)^2 = 10$
   $(x + 2)^2 + (y + 5)^2 = 10$

b) $(x - 2)^2 + (y + 5)^2 = 5$
   $(x - 2)^2 + (y + 5)^2 = 10$
   $(x - 2)^2 + (y + 5)^2 = 20$
   $(x - 2)^2 + (y + 5)^2 = 40$

8. a) Graph the circle $(x - 3)^2 + (y + 5)^2 = 100$ and the points R(10, 2), S(9, −13), and T(−6, 0).

   b) Which, if any, of the points R, S, and T are on the circle?

9. Determine the equation of each circle.

   a) The line segment with endpoints A(−2, 0) and B(6, −6) is a diameter of the circle.

   b) The circle passes through C(1, −6) and has centre D(4, 2).

   c) The circle just touches the x-axis and has centre E(5, 4).

10. Determine the equation of each circle.

    a) Its centre is C(3, −2), and R(−1, 1) is a point on the circle.

    b) The endpoints of a diameter are M(5, 1) and N(−3, 3).

    c) The circle passes through A(2, 2) and B(5, 3), and its centre is on the line defined by $y = x + 1$.

11. The centre of a circle lies on the x-axis, 3 units from the origin. The circle passes through A(6, −4). Determine the possible equations of the circle.

12. A circle has centre O(0, 0) and radius 6 units.

    a) Write the equation of the circle.

    b) The point B(4, k) is on the circle. Determine the value of k.

13. To graph any equation on a graphing calculator, the equation must be expressed in this form: "$y =$"

    a) Solve the equation $x^2 + y^2 = 16$ for y. Write two equations.

    b) Graph the equations from part a on the same grid. Describe the graph.

    c) By tracing and zooming, determine to two decimal places:

       i) the value(s) of m when $(2, m)$ is on the circle

       ii) the value(s) of k when (k, k) is on the circle.

14. A treasure map has this information.
    The treasure is buried 3 m from point A and 8 m from point B.
    Point A is located at O(0, 0). Point B has coordinates (5, 0).
    What are the possible locations of the treasure?

15. a) A circle has centre A(3, 2) and radius 5 units.

    i) The point C(m, 3) is on the circle. Determine the value of m.

    ii) The point D(2, n) is on the circle. Determine the value of n.

  b) Choose one point from part a. Write to explain how you determined the value of the variable.

16. A circle has x-intercepts 0 and 4, and y-intercepts 0 and 6. Determine the equation of the circle.

17. Describe the graph of each equation.

  a) $x^2 + y^2 = 0$ **b)** $x^2 + y^2 = -9$

  c) $y = \sqrt{25 - x^2}$ **d)** $x = \sqrt{25 - y^2}$

C **18.** Determine if the circles defined by each pair of equations intersect.

  a) $(x - 3)^2 + (y + 1)^2 = 5$; $(x - 1)^2 + (y + 2)^2 = 32$

  b) $(x - 5)^2 + (y + 1)^2 = 8$; $(x + 3)^2 + (y + 7)^2 = 49$

19. Determine the equation of the circle that passes through the points J(−3, 2), K(4, 1), and L(6, 5).

20. Four circles, each with radius 5, touch the x- and y-axes and a smaller circle with centre O(0, 0). Determine the equation of the smaller circle.
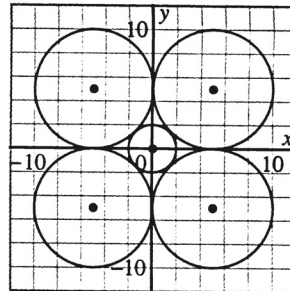
21. The equation $x^2 + y^2 = 25$ represents a circle with centre O(0, 0) and radius 5. Use a calculator to determine what each equation represents. Sketch the graph of each equation on plain paper.

  a) $x^2 - y^2 = 9$ **b)** $x^2 + 2y^2 = 36$

  c) $x^4 + y^4 = 625$ **d)** $\sqrt{x} + \sqrt{y} = 4$

COMMUNICATING THE IDEAS

Write to explain how the equation of a circle and the Pythagorean Theorem are related. Assume the centre of the circle is any point, not necessarily the origin.

Appendix B

Factorial Notation

Evaluate the following:

1. 9!	2. 11!	3. 13!	4. 15!	5. 17!
6. $\dfrac{9!}{8!}$	7. $\dfrac{5!}{2!}$	8. $\dfrac{8!}{6!}$	9. $\dfrac{9!}{6!}$	10. $\dfrac{10!}{9!}$
11. $\dfrac{12!}{8!}$	12. $\dfrac{8!}{8!}$	13. $\dfrac{60!}{1!}$	14. $\dfrac{10!}{2!}$	15. $\dfrac{13!}{10!}$

Simplify the following:

16. 5! − 4!	17. 2(3! + 1!)
18. (5 − 4)!	19. [2(3 + 1)]!
20. $\dfrac{6!-4!}{5!-3!}$	21. $\dfrac{(6-4)!}{(5-3)!}$

Tree Diagrams and Fundamental Counting Principle

Fill in the table below:

#	Experiment	Tree Diagram	Number of Possible Outcomes (show workings)
1	Four coins are tossed one after another.		
2	At a private school, students wear school uniforms. Each student must wear either white dress shirt, a blue dress shirt, or a white golf shirt. Students must also wear dress pants in grey, black, navy or white.		

#	Experiment	Tree Diagram	Number of Possible Outcomes (show workings)
3	When purchasing a new car there is a choice of two upholstery materials (cloth and leather) and four colours (black, ivory, grey and blue).		
4	An all star baseball team has a roster of seven pitchers and three catchers.		

Investigation #1

The table below lists the players on a hockey team. One player will be selected at random to attend a hockey school. He coach calculated the probability of selecting a girl or a defence. Because 10 people on the team are wither girls or defence, then

$$P(girl\ or\ defence) = \frac{10}{16}$$

| Position	Names		
Goal	Amy (female)	Burt (male)	
Defence	Cathy (f)	Fred (m)	Harold (m)
	Darren (m)	Greg (m)	Eve (f)
Left Wing	Ike (m)	Jacqui (f)	
Centre	Katie (f)	Mike (m)	Leo (m)
Right Wing	Nadine (f)	Pete (m)	Otto (m)

One of the players uses reasoning to get a different answer.

There are 6 girls, so $P(girl) = \dfrac{6}{16} = \dfrac{3}{8}$ AND   there are 6 defence, so $P(defence) = \dfrac{6}{16} = \dfrac{3}{8}$

Therefore, it seems reasonable that the $P(girl\ or\ defense) = \dfrac{3}{8} + \dfrac{3}{8} = \dfrac{6}{8}$.

The coach can see that even though the reasoning seems sound, it must be flawed because it gives the wrong answer.

One way to find where the player's reasoning is flawed is to look at a simpler problem. Think about two playing positions: defence and other, and boys and girls. The information from the table is shown in the Venn diagram below.
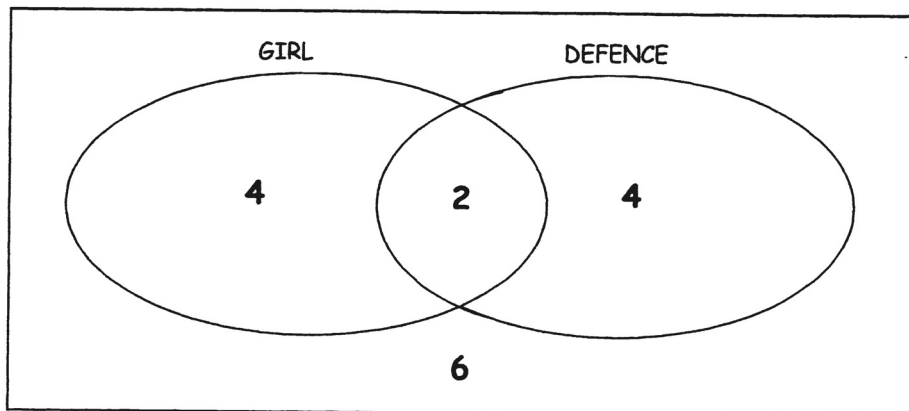
Purpose
Find an experiment for P(A or B).

Questions

1. Examine the Venn diagram. Fill out the table below.

Total number of ...	Answer
Girls	
Defence players	
Girls playing defence	

2. Suppose you were choosing one person from the lineup. Fill out the table below.

Event	Probability
Choosing a girl	P(girl) =
Choosing a defence player	P(defence) =
Choosing someone who is a girl AND a defence player	P(girl and defence) =
Choosing someone who either a girl OR a defence player	P(girl or defence) =

3. Is P(girl or defence) = P(girl) + P(defence)? Explain

_____

_____

_____

4. Find P(girl) + P(defence) - P(girl and defence).

_____

5. Compare the results from #4 to the probability in #3. What do you notice?

_____

6. Based on the table, why does it make sense to subtract P(girl and defence) from P(girl) + P(defence)?

_____

_____

_____

Let P(girl) be called P(A) and P(defence) be called P(B).
The **Addition Principle** states that,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Probability and Odds

The terms **probability** and **odds** are often used interchangeably. The fact is, though, that they mean different things.

Suppose you place three white balls and five red balls in a bag and randomly select one.

- The probability that you will select a white ball is $\dfrac{3}{8}$.

- The odds that you will select a white ball are 3 to 5, or 3:5 (same as $\dfrac{3}{5}$)

In general,

$$Pr\,obability = \frac{number\ of\ favourable\ outcomes}{number\ of\ possible\ outcomes}$$

$$Odds = number\ of\ favourable\ outcomes : number\ of\ unfavourable\ outcomes$$

1. Fill in the table below.

#	Experiment	Event	Probability	Odds
1	Selecting a ball from a bag that has three white balls and five red balls in it.	Selecting a red ball.		
2	Tossing a single coin.	Tossing a head.		
		Tossing a tail.		
3	Pulling out a coin from a pocket that has five quarters and six dimes.	Pulling out a quarter.		
		Pulling out a dime.		

2. Describe how the probability of an event occurring is similar to the odds of an event occurring.

_____

_____

_____

_____

| #	Experiment	Event	Probability	Odds
1	Spinning a spinner that will stop at any one of six equal sized sections.	A) Stopping on an even number.		
		B) Stopping on a multiple of three.		
		C) Stopping on an even number OR a multiple of three.		
		D) Stopping on a number less than three.		
2	Drawing a card from an ordinary deck.	A) Drawing a heart.		
		B) Drawing an ace.		
		C) Drawing the jack of spades.		
		D) Drawing a red card.		
		E) Drawing an ace or a king.		
3	Drawing a ball from a box containing 4 black balls, 7 white balls, and 3 red balls.	A) Drawing a black ball.		
		B) Drawing a red ball.		
		C) Drawing a white ball.		
		D) Drawing a red or white ball.		
		E) Drawing a black or white ball.		

Investigation #2

Suppose a coin is flipped and a die is rolled. You want to find the probability of the result being a number less than 3 and "heads."

Purpose

Investigate how multiplication can be used to find probabilities.

Procedure

Construct a tree diagram to show all possible outcomes.

COIN   DIE
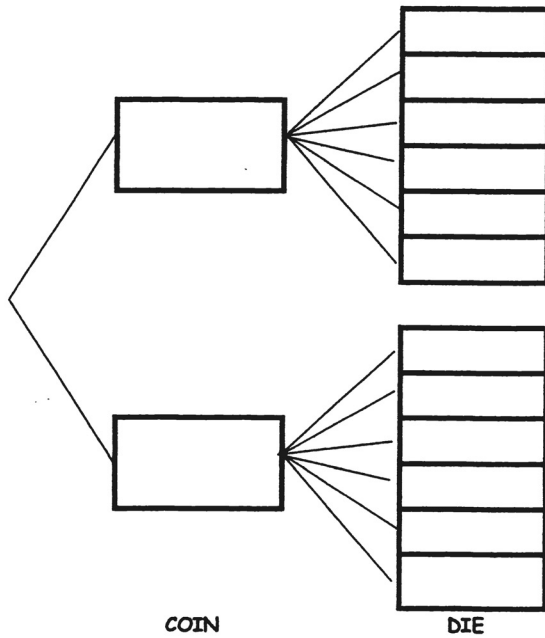
Questions

1. Count the number of favourable outcomes and the total number of outcomes. What is the probability of getting heads and a number less than 3?

    _____

    _____

2. What is the probability of getting heads when you flip a fair coin?

    _____

3. What is the probability of getting a number less than3 when you roll a die?

    _____

4. Multiply the results from #2 and #3. What do you notice?

    _____

Investigation #3

Carl is the assistant coach of a novice hockey team. He has taken six team members to one end of the ice. One player has been designated as the goalie and one defence has also been chosen. Carl is going to choose the three forwards from the remaining four players: Eddie, Jane Claudette, and Bruce.

Purpose

Find the number of groups that can be chosen with and without regard to order.

Procedure

CASE	Arrange three players from Eddie (E), Jane (J), Claudette(C), and Bruce (B) (Not all spaces below have to be filled!)			
(1) Carl will assign the players to play right wing, left wing, and centre. **(ordered groups)** Ex: E-J-C is *different* from E-C-J	E-J-C			
(2) Carl will just select the three forwards and will decide later which position each should play. **(non-ordered groups)** Ex: E-J-C is the *same* as E-C-J	E-J-C			

Questions

1. Which case results in more possibilities? Explain.

   _____

   _____

   _____

   Both Case 1 and Case 2 involve choosing groups of three from a set of four.

2. How do the cases differ?

   _____

   _____

   _____

3. Which is easier to determine? Why?

   _____

   _____

   _____

Investigation #4

A high school hockey team has 15 players. The players are introduces, one by one, as they skate into the ice. In how many different ways can the players skate onto the ice?

You can calculate the number of ways as follows:

15 x 14 x 13 x 12 x 11 x 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1

The calculation takes a long time to write and to do but can be simplifies through the use of **factorial** notation. You can write the above expression as 15!, which is read "15 factorial."
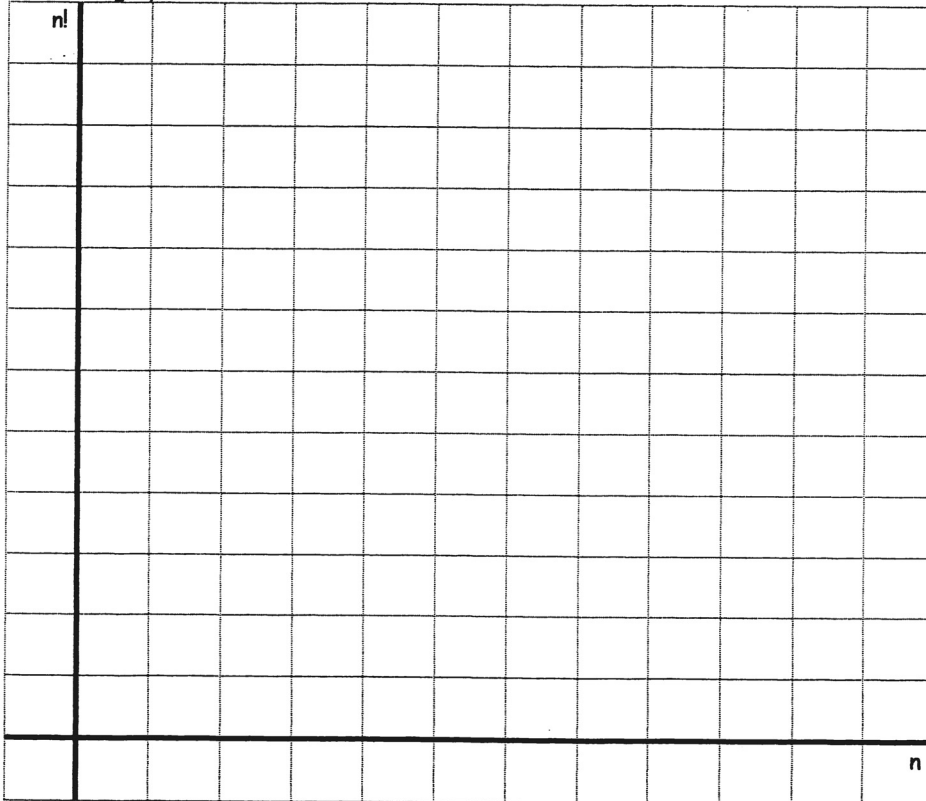
Purpose

Explore the meaning of factorial.

Procedure

• Complete the table below.

n	1	2	3	4	5
Calculation of n	1	2 x 1	3 x 2 x 1		
n!	1	2	6		

• Plot a graph on the axis below based on the table above.

Questions

1. Examine the graph and the table in the Procedure. What happens to the value of n! as n becomes larger?

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2. Determine the values of the following,

   A)   6!

            _____

   B)   7!

            _____

   C)   8!

            _____

   D)   9!

            _____

3. Explain whether your answers in #2 agree with your answer to #1.

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