Adult Basic Education

Mathematics

Mathematics 3104C
Probability, Circles and Ellipses

Study Guide

Prerequisites: Math 1104A, Math 1104B and Math 1104C
Math 2104A, Math 2104B and Math 2104C
Math 3104A

Credit Value: 1

Text: *Mathematics 12*. Alexander and Kelly; Addison Wesley, 1999.

Required Mathematics Courses **[Degree and Technical Profile]**
Math 1104A
Math 1104B
Math 1104C
Math 2104A
Math 2104B
Math 2104C
Math 3104A
Math 3104B
Math 3104C

Table of Contents

To the Student

I. Introduction to Mathematics 3104C

This course has two distinct topics. The first is a continuation of the study of probability that was begun in previous courses. The second topic is an introduction to circles and ellipses as conic shapes, and a development of the circle as a trigonometric function. The second topic, in particular, is an essential component of all post-secondary Math courses.

You should be able to calculate simple probabilities and create tree diagrams. You should be able to evaluate expressions that involve fractions and simplify fractional expressions that have common factors in the numerator and denominator. You should be able to use the translation rules that stretch or translate graphs and equations. You should also be familiar with right triangle trigonometry and be able to apply the sine and cosine definitions in terms of opposite, adjacent and hypotenuse.

When skills need to be reviewed, see your instructor for **Prerequisites** exercises.

II. Resources

You will require the following:

- *Addison Wesley Mathematics 12*, Western Canadian edition Textbook
- Scientific calculator
- graph paper
- Access to a TI-83 Plus Graphing Calculator (see your instructor) and/or *Graphmatica* or *Winplot* graphing software

Notes concerning the textbook:

Glossary: Knowledge of mathematical terms is essential to understand concepts and correctly interpret questions. Written explanations will be part of the work you submit for evaluation, and appropriate use of vocabulary will be required.

Your text for this course includes a Glossary where definitions for mathematical terms are found. Be sure you understand such definitions and can explain them in your own words. Where appropriate, you should include examples or sketches to support your definitions.

To the Student

Examples: You are instructed to study carefully the **Examples** in each section of the textbook and see your instructor if you have any questions. These **Examples** provide full solutions to problems that can be of great use when answering assigned **Exercises**.

Notes concerning technology:

It is important that you have a **scientific** calculator for your individual use. Ensure that your calculator has the word "scientific" on it, as there are calculators designed for business or statistics which would not have the functions needed for this course. Scientific calculators are sold everywhere and are fairly inexpensive. You should have access to the manual for any calculator that you use. It is a tool that can greatly assist the study of mathematics but, as with any tool, the more efficient its use, the better the progress.

You will require access to some sort of technology in order to meet some of the outcomes in this course. Since technology has become a significant tool in the study of Mathematics, your textbook encourages you to become proficient in its use by providing you with step-by-step exercises that will teach you about the useful functions of the TI-83 Plus graphing calculator. **See your instructor concerning this**. Please note that a graphing calculator is not essential for success in this course but it is useful.

While graphing calculators and graphing software (*Graphmatica* or *Winplot*) are useful tools, they cannot provide the same understanding that comes from working paper and pencil exercises.

III. Study Guide

This Study Guide is required at all times. It will guide you through the course and you should take care to complete each unit of study in the order given in this Guide. Often, at the beginning of each unit, you will be instructed to see your instructor for **Prerequisite** exercises. Please do not skip this step! It should only take a few minutes for you and your instructor to discover what, if any, prerequisite skills need review.

To be successful, you should read the **References and Notes** first and then, when indicated by the ⊡⊡ symbols, complete the **Work to Submit** problems. Many times you will be directed to see your instructor, and this is vital, especially in a Mathematics course. If you only have a hazy idea about what you just completed, nothing will be gained by continuing on to the next set of problems.

To the Student

Reading for this Unit:
In this box, you will find the name of the text, and the chapters, sections and pages used to cover the material for this unit. As a preliminary step, skim the referenced section, looking at the name of the section, and noting each category. Once you have completed this overview, you are ready to begin.

References and Notes	**Work to Submit**
This left hand column guides you through the material to read from the text. It will also refer to specific **Examples** found in each section. You are directed to study these **Examples** carefully and see your instructor if you have any questions. The **Examples** are important in that they not only explain and demonstrate a concept, but also provide techniques or strategies that can be used in the assigned questions. The symbols ▶▶ direct you to the column on the right which contains the work to complete and submit to your instructor. *You will be evaluated on this material.* Since the answers to **Discussing the Ideas** and **Communicating the Ideas** are not found in the back of the student text, you **must** have these sections corrected by your instructor **before** going on to the next question. This column will also contain general **Notes** which are intended to give extra information and are not usually specific to any one question.	There are four basic categories included in this column that correspond to the same categories in the sections of the text. They are **Investigate, Discussing the Ideas**, **Exercises**, and **Communicating the Ideas**. **Investigate:** This section looks at the thinking behind new concepts. The answers to its questions are found in the back of the textbook. **Discussing the Ideas**: This section requires you to write a response which clarifies and demonstrates your understanding of the concepts introduced. The answers to these questions are not in the student text and will be provided when you see your instructor. **Exercises**: This section helps reinforce your understanding of the concepts introduced. There are three levels of **Exercises**: **A:** direct application of concepts introduced; **B:** multi-step problem solving and some real-life situations; **C:** problems of a more challenging nature. The answers to the **Exercises** questions are found in the back of the textbook. **Communicating the Ideas:** This section helps confirm your understanding of the lesson of the section. If you can write a response, and explain it clearly to someone else, this means that you have understood the topic. The answers to these questions are not in the student text and will be provided when you see your instructor This column will also contain **Notes** which give information about specific questions.

IV. Recommended Evaluation

Written Notes	10%
Assignments	10%
Test(s)	30%
Final Exam *(entire course)*	50%
	100%

The overall pass mark for the course is 50%.

Unit 1 : Permutations and Combinations

To meet the objectives of this unit, students should complete the following:

Reading for this unit: *Mathematics 12*

Chapter 6:
- Section 6.1: pages 352 - 358
- Section 6.2: pages 360 - 365
- Section 6.4: pages 371 - 378
- Review: page 397

References and Notes	Work to Submit
In **Section 6.1**, study **Examples 1 - 3**. See your instructor if you have any questions. Answer the following questions. ▶/▶	1.1 See your instructor for **Prerequisites** exercises before you begin this unit. 1.2 Read **Investigate Counting without Counting** on page 354. Answer questions 1 - 4. 1.3 **Discussing the Ideas**, page 356 Answer question 1. 1.4 **Exercises**, pages 356 - 359 Answer questions 1 - 3, 5 - 7, 11, 13 - 14.
Before moving to the next set of questions, see your instructor to have **Discussing the Ideas** and **Communicating the Ideas** corrected.	1.5 **Communicating the Ideas**, page 359

Unit 1 : Permutations and Combinations

References and Notes	Work to Submit
In **Section 6.2**, answer the following question. ▶▶	1.6 Read **Investigate Permutations Involving Different Objects** on pages 360-361. Answer questions 1 - 6.
Carefully study **Examples 1** and **2**. See your instructor if you have any questions.	
Answer the following questions. ▶▶	1.7 Write a definition for *factorial notation*. Include an example of how a factorial number would be written and what its answer would be. Show how to find the answer using pencil and paper and ensure you know how to find the same answer using the factorial function on your calculator.
	1.8 Prove the following without a calculator: i) $4! = 24$ ii) $5! = 120$ iii) $\frac{8!}{5!} = 336$ iv) $\frac{10!}{4!(10-4)!} = 210$
	1.9 Write each of the following as a ratio of factorials: a) $7 \times 6 \times 5$ b) $10 \times 9 \times 8 \times 7 \times 6$

Unit 1 : Permutations and Combinations

References and Notes	Work to Submit
	1.10 Define *permutation* and give an example.
	1.11 **Exercises**, pages 364 - 365 Answer questions 1 - 6 and 9 - 14.
	1.12 **Communicating the Ideas**, page 365
	1.13 **Discussing the Ideas,** page 374 Answer questions 1 and 4.
Have **Communicating the Ideas** and **Discussing the Ideas** corrected by your instructor before going on to the next set of questions.	
In **Section 6.4**, study **Examples 1 - 2** and work through the given solutions. See your instructor if you have any questions.	
Answer the following questions. ▶▶	1.14 Define *combination* and give an example.
	1.15 **Exercises**, pages 374 - 378 Answer questions 1, 3 - 5, 7 - 10, 12, 14, 17 - 19 and 28.
	1.16 **Communicating the Ideas**, page 378.

Unit 1 : Permutations and Combinations

References and Notes	Work to Submit
See your instructor for answers to 1.17.	1.17 For each of the following, decide whether permutations or combinations are involved. Give reasons for your answers. (a) Find the number of committees of two that can be formed from a group of 12 people. (b) Find the number of possible lineups for a baseball team that can be formed from 12 people with regard to position (a baseball team consists of 9 players, as follows: pitcher; catcher; first, second, and third basemen; shortstop; right center, and left fielders.) c) Find the number of license plates that can be formed from 12 different letters. d) As a promotion, Pierre was the one millionth customer and was allowed to select 4 tapes and 4 compact discs. To find how many selections that Pierre can make, does one use permutations or combinations? Explain.
Answer the following questions. ⏩⏩	1.18 **Review**, page 397 Answer questions 1 - 6.

Unit 2: Probability

To meet the objectives of this unit, students should complete the following:

Reading for this unit: *Mathematics 12*
Chapter 7: Section 7.1: pages 402 - 407
Section 7.2: pages 410 - 416
Section 7.3: pages 420 - 424

References and Notes	Work to Submit
In **Section 7.1**, read **Investigate.** Study **Examples 1** and **2**. See your instructor if you have any questions. Answer the following questions. ▶/▶	2.1 Write a definition for *probability*. Include an example with your answer. 2.2 Explain the difference between *experimental* and *theoretical* probability. Give an example for each to support your answers.
Before moving to the next set of questions, have your instructor correct the questions in **Discussing the Ideas**.	2.3 **Discussing the Ideas**, page 405 Answer questions 1 and 2. *(See note below for question 2.)* **Note:** In question 2, there are not just two (2) but 201 possible outcomes. 2.4 **Exercises**, pages 405 - 407 Answer questions 1 - 5. *(See note below for question 5.)* Answer questions 9 and 11. **Note:** A tree diagram may help in question 5.

Unit 2: Probability

References and Notes	Work to Submit
In **Section 7.2**, study **Examples 1** and **2**. Read **Visualizing Related Events** on page 413. Answer the following questions. ▶/▶	2.5 **Investigate**, page 410 Answer questions 1-5. 2.6 Define the following terms in your own words: (i) sample space (ii) complement
See your instructor for correction of **Discussing the Ideas** before moving on to the next set of questions.	2.7 **Discussing the Ideas**, page 414 Answer questions 2, 4 and 5. *(See note below for question 5.)* **Note:** Try to explain question 5 using Venn diagrams. 2.8 **Exercises**, pages 414 - 416 Answer questions 2 - 14.

Unit 3: Circles

References and Notes

In **Section 7.3**, study **Examples 1** and **2**.

Answer the following questions.

See your instructor for correction of **Discussing the Ideas** and **Communicating the Ideas** before going on to the next set of questions.

Work to Submit

2.9 Explain what is meant by the term *mutually exclusive events*. Give an example to support your explanation.

2.10 State P(A or B) when A and B are mutually exclusive events.

2.11 State P(A or B) when A and B are not mutually exclusive events**.**

2.12 **Discussing the Ideas**, page 422
Answer questions 1 - 4.

2.13 **Exercises**, pages 423 - 424
Answer questions 1 - 8 and 11.
(See notes below for questions 4, 5 and 8.)

Note: In questions 4 and 5, you can use Venn diagrams to solve these problems or the formula
P(A or B) = P(A) + P(B) − P(A and B).

Note: In question 8, how many customers did <u>not</u> learn about the sale?

2.14 **Communicating the Ideas**, page 424

Unit 3: Circles

To meet the objectives of this unit, students should complete the following:

Reading for this unit: *Appendix A*
Pages 17 to 24

References and Notes

Note: The following formulas may be needed.

Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint: (x_m, y_m)

$$x_m = \frac{x_1 + x_2}{2} \; ; \; y_m = \frac{y_1 + y_2}{2}$$

Referring to pages 19 - 21 in Appendix A, carefully study **Examples 1 - 3**. Answer the following questions.

Work to Submit

3.1 See your instructor for **Prerequisite** exercises before you begin this unit.

3.2 Determine the equation of a circle with centre (0,0) and radius 5. Draw a sketch and use the distance formula.

Unit 3: Circles

References and Notes	Work to Submit
When ever possible, draw a sketch to make sure that you understand the stated conditions and are using the distance formula correctly. See your instructor for correction of **Discussing the Ideas** before going on to the next set of questions**.**	3.3 **Discussing the Ideas**, Appendix A, page 21 Answer questions 1 - 3. *(See note below for question 3.)* **Note**: For question 3, remember that a function is a rule that gives a single output number for every valid input number. The graph of a function is identified by the Vertical Line Test. If no two points on a graph can be joined by a vertical line then the graph represents a function. 3.4 **Exercises**, Appendix A, pages 22 and 23 Answer questions 1 - 9. *(See note below for questions 6 and 9.)* Answer questions 10a), 10b), 11 and 13. *(See note below for question 13.)* **Note:** In question 6c), if the centre of the circle is on the line $y = x$, then $h = k$. **Note:** In questions 9a) and 9b), first calculate the diameter or radius by using the distance formula. In part a), determine the centre by finding the midpoint of AB.
See your instructor for correction of **Communicating the Ideas** before going on to the next set of questions**.**	**Note:** In Question 13, $x^2 + y^2 = 16$ defines two functions: $y = \sqrt{16 - x^2}$ *and* $y = -\sqrt{16 - x^2}$. Using your graphing calculator, enter these two functions into the Y = menu. 3.5 **Communicating the Ideas**, page 24

Unit 4: Circles and Ellipses

To meet the objectives of this unit, students should complete the following:

Reading for this unit: *Mathematics 12*

Chapter 9:
- Section 9.1: page 520
- Section 9.2: pages 525, 528, 532-533
- Section 9.3: pages 534, 535, 537-538, 540-544
- Section 9.4: pages 545, 547-549
- Section 9.6: pages 561-563

References and Notes	Work to Submit
On the pages referenced you will study material **only** on circles and ellipses. Omit parabolas and hyperbolas.	4.1 See your instructor for **Prerequisite** exercises from **Section 9.2** before beginning this unit.
On page 520 only read the description of how the intersection of a plane and cone generates a circle.	
Read page 525, page 528 (bottom half), and pages 532 and 533. Answer the following questions. ▶▶	4.2 Explain the difference between the *standard form* and *general form* of an equation. Give examples to support your explanation. 4.3 State the standard form of the circle equation, indicating the centre and radius. 4.4 **Exercises,** pages 532 - 533 Answer questions 7a)i, and 9. 4.5 State the characteristics of the circle equation which has centre (0,0).

Unit 4: Circles and Ellipses

References and Notes

Read page 534, (left column on circles only).

If you have access to *Graphmatica*, or have the CONICS program on a TI-83, answer the following questions. ▶▶

Read page 537 (bottom of page).

Read **Visualizing** and carefully study **Example 1** on page 538.

On page 541, study **Standard Equations of Conic Sections Centred at (0,0)** (Read top of page on ellipse only.)

See your instructor for correction of **Discussing the Ideas** before going on to the next set of questions**.**

Work to Submit

4.6 **Investigate**, page 535
Answer questions 1- 4.

4.7 On a diagram, identify the following features of an ellipse: *centre*, *major axis*, and *minor axis*. Identify on both a horizontal ellipse and a vertical ellipse.

4.8 State the standard equation of an ellipse and describe what makes it different from the standard equation of a circle.

4.9 **Discussing the Ideas**, page 540
Answer questions 1, 2, and 5 (for ellipse only).

4.10 **Exercises,** pages 542 - 544

Answer questions:
1a), 1d) and 1f),
2, 3, 4, 5,
8a), (identify only the circle and ellipse),
14a), 14b) and 14e),
15a), 15b), 15e) and 15g).

Unit 4: Circles and Ellipses

References and Notes	Work to Submit
Read page 545 (top of page only). On page 547, read **Standard Equations of Conic Sections Centred at (*h, k*)** (top of page on ellipse only). Answer the following questions. ▶▶	4.11 See your instructor for **Prerequisite** exercises from **Section 9.4** before continuing this unit. 4.12 **Exercises**, pages 548 Answer questions 1b), 2, 3a) and 3b), 4a), 4b) and 4e), 8a), 8d) and 8f).
Read page 561 (top section only). Study **Example 1**. This conic is an hyperbola. However, study the example for the purpose of reviewing how to change a general equation to a standard equation by completing the square. If necessary, see your instructor for more review material on completing the square. Answer the following questions. ▶▶ See your instructor for correction of **Discussing the Ideas** and **Communicating the Ideas** before leaving this unit.	4.13 Suppose an equation of an ellipse is given in standard form. Explain how to determine a) the coordinate of the centre, b) the length of the major and minor axis and c) if the major axis is vertical or horizontal. 4.14 See your instructor for **Prerequisite** exercises for **Section 9.6** before completing this unit. 4.15 **Discussing the Ideas**, page 562 Answer questions 1a) and 1b). 4.16 **Exercises**, page 563 Answer questions 1a) and 1d), 2a) and 2d), write 2a) and 2d) in *standard* form, 3a) and 3b). 4.17 **Communicating the Ideas**, page 563

Unit 5: The Circle as a Trigonometric Function

To meet the objectives of this unit, students should complete the following:

Reading for this unit: *Mathematics 12*
Chapter 3: Section 3.3: pages 170-175
Section 3.4: pages 176-181
Section 3.5: pages 184-189

References and Notes

In **Section 3.3**, carefully read page 170 and study **Examples 1-3**.
See your instructor if you have any questions.

Answer the following questions.
▶▶

In **Section 3.4**, carefully read pages 176 and 177. Study **Example** and **Visualizing** on page 178.
See your instructor if you have any questions.

Work to Submit

5.1 See your instructor for **Prerequisite** exercises from **Section 3.3** before completing the following questions.

5.2 Define the following terms and draw a sketch where appropriate.

(i) standard position of an angle
(ii) coterminal angles
(iii) initial arm
(iv) terminal arm
(v) clockwise rotation
(vi) counterclockwise rotation

5.3 **Exercises**, pages 173-175
Answer questions 1-9.

5.4 **Communicating the Ideas**, page 175

5.5 See your instructor for **Prerequisite** exercises from **Section 3.4** before completing the following questions.

Unit 5: The Circle as a Trigonometric Function

References and Notes

Answer the following questions.
▶▶
See your instructor for correction of **Discussing the Ideas** before leaving this unit.

Work to Submit

5.6 **Discussing the Ideas**, page 179
Answer questions 1-5.

5.7 **Exercises**, pages 179-181
Answer questions 1-3.
(See note below on questions 2 and 3.)

Answer question 4.
(See note below on question 4.)

Answer questions 5-10 and 16.
(See note below on question 16.)

Note: When completing questions 2 and 3, recall that an angle in degrees must have a degree symbol, but an angle in radians does not have a unit.

Note: In question 4, it will be helpful if you choose increments of 30^0 for θ, (0^0, 30^0, 60^0, 90^0 ...) and record the values of $\cos\theta$ and $\sin\theta$ in a table.

Note: In question 16, use Pythagorean Theorem.

5.8 **Communicating the Ideas**, page 181

Unit 5: The Circle as a Trigonometric Function

References and Notes

In **Section 3.5**, carefully read page 184.
Study **Examples 1** and **2** and **Visualizing** on page 187.
See your instructor if you have any questions.
Answer the following questions.
▶▶
See your instructor for correction of **Discussing the Ideas** before leaving this unit.

Work to Submit

5.9 See your instructor for **Prerequisites** exercises on **Section 3.5** before completing the following exercises.

5.10 **Discussing the Ideas**, page 187
Answer questions 1 - 3.

5.11 **Exercises**, pages 188 and 189
Answer questions 1 - 3.
(See note below on question 3.)

Answer questions 4 - 6.

Note: When finding exact value in question 3, you should draw the angle to determine the sign of the trigonometric function, then find the reference angle.

Appendix A

The Equation of a Circle

Many farms in western North America use an automated centre-pivot irrigation system. A long pipe sprays water as it rotates about the centre. Distinctive circular traces are left by the wheels and, since the end of the pipe is always the same distance from the centre, the area watered forms a circle.

On a coordinate grid, we can determine the equation of a circle, just as we can determine equations of lines and parabolas.

Example 1

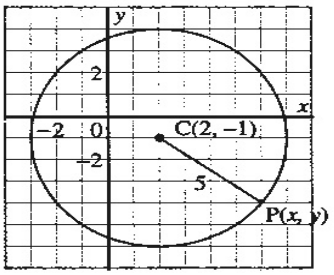
Consider the circle with centre C(2, −1) and radius 5 units.

a) Graph the circle.

b) Determine the equation of the circle.

Solution

a) Draw a circle with centre C(2, −1) and radius 5.



b) Let P(x, y) be any point on the circle.

Since CP is a radius, CP = 5

Use the distance formula.

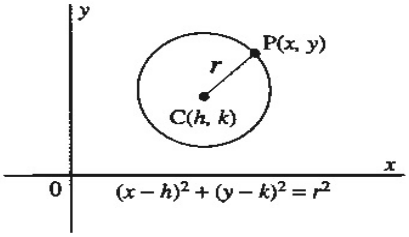
$\sqrt{(x-2)^2 + (y+1)^2} = 5$

Square each side to eliminate the radical.

$(x-2)^2 + (y+1)^2 = 25$

This is the equation of the circle.

We can use the method of *Example 1* to determine the equation of any circle with centre C(h, k) and radius r.
Let P(x, y) be any point on the circle.
Then,



$$CP = r$$
$$\sqrt{(x-h)^2 + (y-k)^2} = r \quad \text{Square each side.}$$
$$(x-h)^2 + (y-k)^2 = r^2$$

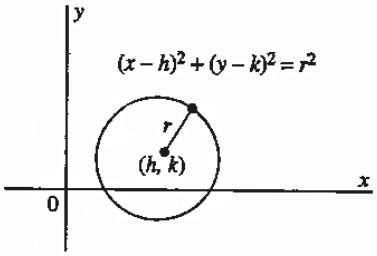
Equation of a Circle

The equation of a circle with centre (h, k) and radius r is:

$(x - h)^2 + (y - k)^2 = r^2$

When the centre is O(0, 0), the equation is:

$x^2 + y^2 = r^2$



Example 2

Graph the circle defined by $(x - 5)^2 + (y + 1)^2 = 20$.

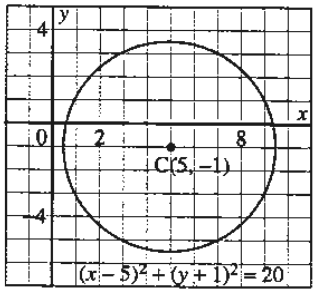
Solution

The equation $(x - h)^2 + (y - k)^2 = r^2$ represents a circle with centre (h, k) and radius r.

Hence, $(x - 5)^2 + (y + 1)^2 = 20$ represents a circle with centre $(5, -1)$ and radius $\sqrt{20}$.

Plot the point C(5, −1) on a grid.

With centre C, draw a circle with radius $\sqrt{20} \doteq 4.47$.



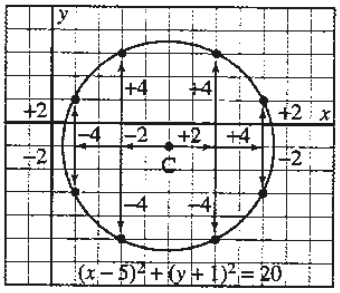
Visualizing

Consider the graph in *Example 2*. Observe that $r^2 = 20 = 2^2 + 4^2$.

Start at the centre. Points on the circle can be located by moving:

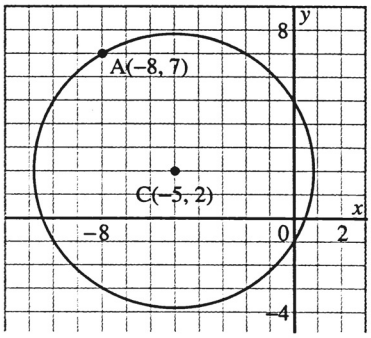
2 left, 4 up	2 right, 4 up
4 left, 2 up	4 right, 2 up
2 left, 4 down	2 right, 4 down
4 left, 2 down	4 right, 2 down

This method can only be used when the square of the radius is the sum of two squares.



Example 3

Determine the equation of this circle.



Solution

Think...

To determine the equation of the circle, we need to know the radius and the coordinates of the centre. The coordinates of the centre are given, and we can calculate the radius using the distance formula.

The radius is the length of CA.

$$\begin{aligned} CA &= \sqrt{(-8 + 5)^2 + (7 - 2)^2} \\ &= \sqrt{9 + 25} \\ &= \sqrt{34} \end{aligned}$$

The radius of the circle is $\sqrt{34}$. The centre is $(-5, 2)$.

Use the equation $(x - h)^2 + (y - k)^2 = r^2$.

The equation of the circle is $(x + 5)^2 + (y - 2)^2 = 34$.

DISCUSSING THE IDEAS

1. In *Example 1*, the equation of the circle was written as $(x - 2)^2 + (y + 1)^2 = 25$. Why did we not expand the left side of the equation?
2. For the equation of the circle in *Example 3*:
   a) Why did we write +5 in $(x + 5)^2$, but −2 in $(y - 2)^2$?
   b) What happened to the radical sign in $\sqrt{34}$?
3. a) Does the graph of a circle represent a function? Explain.
   b) Does the equation of a circle represent a function? Explain.

Exercises

A **1.** Determine if each point is on the circle defined by $x^2 + y^2 = 85$.

a) A(9, –2) **b)** B(–5, 8) **c)** C(–7, –6) **d)** D(4, 8)

2. State the radius and the coordinates of the centre of the circle defined by each equation.

a) $(x - 3)^2 + (y + 4)^2 = 81$ **b)** $(x + 2)^2 + (y - 1)^2 = 5$

c) $(x + 4)^2 + y^2 = 15$ **d)** $x^2 + (y - 6)^2 = 48$

e) $x^2 + y^2 = 64$ **f)** $x^2 + y^2 = 12$

3. a) Write the equation of the circle with each given centre and radius.

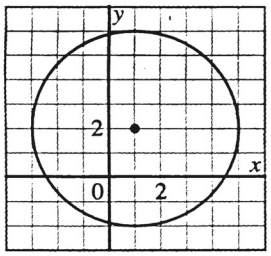
i) O(0, 0), 3 **ii)** O(0, 0), 7 **iii)** A(5, 3), 4 **iv)** B(–2, 6), 5

v) C(4, 0), 6 **vi)** D(0, –3), 9 **vii)** E(2, 0), $\sqrt{5}$ **viii)** F(3, –5), $\sqrt{10}$

b) Choose one circle from part a. Write to explain how you determined its equation.

4. Write the equation of each circle.

a)



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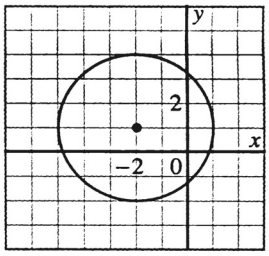

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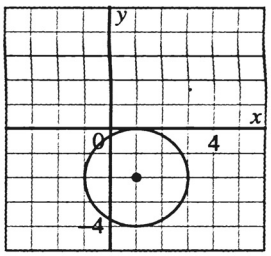


5. What does each equation represent? Explain.

a) $x^2 + y^2 = 1$ **b)** $(x - 3)^2 + (y - 2)^2 = 2$

B **6.** State which of these circles have:

a) radius less than 5

b) centre on the x-axis

c) centre on the line $y = x$

i) $(x - 3)^2 + (y - 3)^2 = 1$ **ii)** $(x + 2)^2 + (y - 4)^2 = 9$

iii) $x^2 + (y + 7)^2 = 13$ **iv)** $(x - 5)^2 + y^2 = 20$

v) $(x + 1)^2 + (y + 1)^2 = 25$ **vi)** $x^2 + y^2 = 32$

7. On the same grid, graph the equations in each list. Write to explain how the patterns in the equations are related to the patterns in the graphs.

a) $(x-2)^2+(y-5)^2=10$
$(x+2)^2+(y-5)^2=10$
$(x-2)^2+(y+5)^2=10$
$(x+2)^2+(y+5)^2=10$

b) $(x-2)^2+(y+5)^2=5$
$(x-2)^2+(y+5)^2=10$
$(x-2)^2+(y+5)^2=20$
$(x-2)^2+(y+5)^2=40$

8. a) Graph the circle $(x-3)^2+(y+5)^2=100$ and the points R(10, 2), S(9, −13), and T(−6, 0).

b) Which, if any, of the points R, S, and T are on the circle?

9. Determine the equation of each circle.

a) The line segment with endpoints A(−2, 0) and B(6, −6) is a diameter of the circle.

b) The circle passes through C(1, −6) and has centre D(4, 2).

c) The circle just touches the x-axis and has centre E(5, 4).

10. Determine the equation of each circle.

a) Its centre is C(3, −2), and R(−1, 1) is a point on the circle.

b) The endpoints of a diameter are M(5, 1) and N(−3, 3).

c) The circle passes through A(2, 2) and B(5, 3), and its centre is on the line defined by $y=x+1$.

11. The centre of a circle lies on the x-axis, 3 units from the origin. The circle passes through A(6, −4). Determine the possible equations of the circle.

12. A circle has centre O(0, 0) and radius 6 units.

a) Write the equation of the circle.

b) The point B(4, k) is on the circle. Determine the value of k.

13. To graph any equation on a graphing calculator, the equation must be expressed in this form: "$y=$"

a) Solve the equation $x^2+y^2=16$ for y. Write two equations.

b) Graph the equations from part a on the same grid. Describe the graph.

c) By tracing and zooming, determine to two decimal places:

i) the value(s) of m when $(2, m)$ is on the circle

ii) the value(s) of k when (k, k) is on the circle.

14. A treasure map has this information.
The treasure is buried 3 m from point A and 8 m from point B.
Point A is located at O(0, 0). Point B has coordinates (5, 0).
What are the possible locations of the treasure?

15. a) A circle has centre A(3, 2) and radius 5 units.

i) The point C(m, 3) is on the circle. Determine the value of m.

ii) The point D(2, n) is on the circle. Determine the value of n.

b) Choose one point from part a. Write to explain how you determined the value of the variable.

16. A circle has x-intercepts 0 and 4, and y-intercepts 0 and 6. Determine the equation of the circle.

17. Describe the graph of each equation.

a) $x^2 + y^2 = 0$

b) $x^2 + y^2 = -9$

c) $y = \sqrt{25 - x^2}$

d) $x = \sqrt{25 - y^2}$

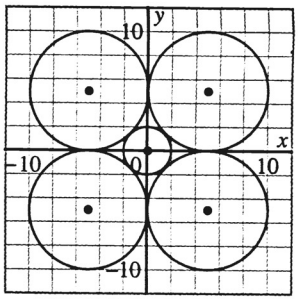
C **18.** Determine if the circles defined by each pair of equations intersect.

a) $(x - 3)^2 + (y + 1)^2 = 5$; $(x - 1)^2 + (y + 2)^2 = 32$

b) $(x - 5)^2 + (y + 1)^2 = 8$; $(x + 3)^2 + (y + 7)^2 = 49$

19. Determine the equation of the circle that passes through the points J(–3, 2), K(4, 1), and L(6, 5).

20. Four circles, each with radius 5, touch the x- and y-axes and a smaller circle with centre O(0, 0). Determine the equation of the smaller circle.



21. The equation $x^2 + y^2 = 25$ represents a circle with centre O(0, 0) and radius 5. Use a calculator to determine what each equation represents. Sketch the graph of each equation on plain paper.

a) $x^2 - y^2 = 9$

b) $x^2 + 2y^2 = 36$

c) $x^4 + y^4 = 625$

d) $\sqrt{x} + \sqrt{y} = 4$

COMMUNICATING THE IDEAS

Write to explain how the equation of a circle and the Pythagorean Theorem are related. Assume the centre of the circle is any point, not necessarily the origin.

Appendix B

Factorial Notation

Evaluate the following:

1. $9!$	2. $11!$	3. $13!$	4. $15!$	5. $17!$
6. $\frac{9!}{8!}$	7. $\frac{5!}{2!}$	8. $\frac{8!}{6!}$	9. $\frac{9!}{6!}$	10. $\frac{10!}{9!}$
11. $\frac{12!}{8!}$	12. $\frac{8!}{8!}$	13. $\frac{60!}{1!}$	14. $\frac{10!}{2!}$	15. $\frac{13!}{10!}$

Simplify the following:

16. $5! - 4!$	17. $2(3! + 1!)$
18. $(5 - 4)!$	19. $[2(3 + 1)]!$
20. $\frac{6! - 4!}{5! - 3!}$	21. $\frac{(6-4)!}{(5-3)!}$

Tree Diagrams and Fundamental Counting Principle

Fill in the table below:

#	Experiment	Tree Diagram	Number of Possible Outcomes (show workings)
1	Four coins are tossed one after another.		
2	At a private school, students wear school uniforms. Each student must wear either white dress shirt, a blue dress shirt, or a white golf shirt. Students must also wear dress pants in grey, black, navy or white.		

#	Experiment	Tree Diagram	Number of Possible Outcomes (show workings)
3	When purchasing a new car there is a choice of two upholstery materials (cloth and leather) and four colours (black, ivory, grey and blue).		
4	An all star baseball team has a roster of seven pitchers and three catchers.		

Investigation #1

The table below lists the players on a hockey team. One player will be selected at random to attend a hockey school. He coach calculated the probability of selecting a girl or a defence. Because 10 people on the team are wither girls or defence, then

$$P(girl\ or\ defence) = \frac{10}{16}$$

Position	Names		
Goal	Amy (female)	Burt (male)	
Defence	Cathy (f)	Fred (m)	Harold (m)
	Darren (m)	Greg (m)	Eve (f)
Left Wing	Ike (m)	Jacqui (f)	
Centre	Katie (f)	Mike (m)	Leo (m)
Right Wing	Nadine (f)	Pete (m)	Otto (m)

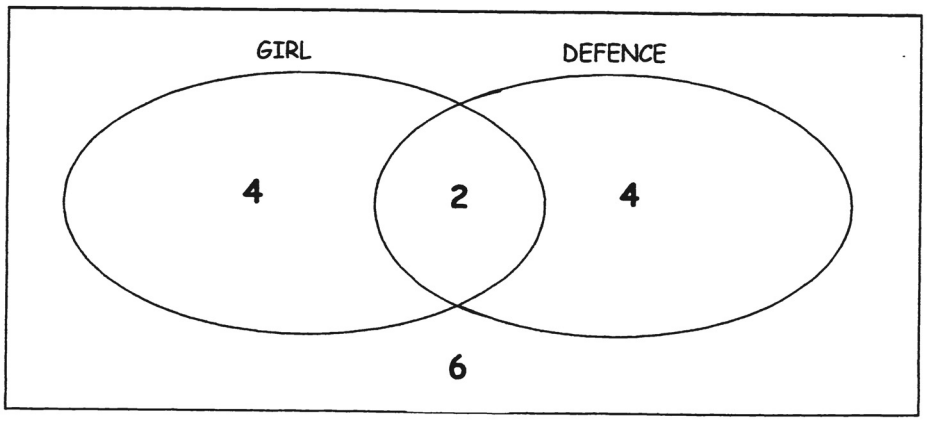
One of the players uses reasoning to get a different answer.

There are 6 girls, so P(girl) = $\frac{6}{16} = \frac{3}{8}$ AND there are 6 defence, so P(defence) = $\frac{6}{16} = \frac{3}{8}$

Therefore, it seems reasonable that the P(girl or defense) = $\frac{3}{8} + \frac{3}{8} = \frac{6}{8}$.

The coach can see that even though the reasoning seems sound, it must be flawed because it gives the **wrong answer**.

One way to find where the player's reasoning is flawed is to look at a simpler problem. Think about two playing positions: defence and other, and boys and girls. The information from the table is shown in the Venn diagram below.



Purpose

Find an experiment for P(A or B).

Questions

1. Examine the Venn diagram. Fill out the table below.

Total number of ...	Answer
Girls	
Defence players	
Girls playing defence	

2. Suppose you were choosing one person from the lineup. Fill out the table below.

Event	Probability
Choosing a girl	P(girl) =
Choosing a defence player	P(defence) =
Choosing someone who is a girl AND a defence player	P(girl and defence) =
Choosing someone who either a girl OR a defence player	P(girl or defence) =

3. Is P(girl or defence) = P(girl) + P(defence)? Explain

4. Find P(girl) + P(defence) - P(girl and defence).

5. Compare the results from #4 to the probability in #3. What do you notice?

6. Based on the table, why does it make sense to subtract P(girl and defence) from P(girl) + P(defence)?

Let P(girl) be called P(A) and P(defence) be called P(B).
The **Addition Principle** states that,

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Probability and Odds

The terms **probability** and **odds** are often used interchangeably. The fact is, though, that they mean different things.

Suppose you place three white balls and five red balls in a bag and randomly select one.

- The probability that you will select a white ball is $\frac{3}{8}$.
- The odds that you will select a white ball are 3 to 5, or 3:5 (same as $\frac{3}{5}$)

In general,

$$\text{Probability} = \frac{\textit{number of favourable outcomes}}{\textit{number of possible outcomes}}$$

$$\textit{Odds} = \textit{number of favourable outcomes} : \textit{number of unfavourable outcomes}$$

1. Fill in the table below.

#	Experiment	Event	Probability	Odds
1	Selecting a ball from a bag that has three white balls and five red balls in it.	Selecting a red ball.		
2	Tossing a single coin.	Tossing a head.		
		Tossing a tail.		
3	Pulling out a coin from a pocket that has five quarters and six dimes.	Pulling out a quarter.		
		Pulling out a dime.		

2. Describe how the probability of an event occurring is similar to the odds of an event occurring.

#	Experiment	Event	Probability	Odds
1	Spinning a spinner that will stop at any one of six equal sized sections.	A) Stopping on an even number.		
		B) Stopping on a multiple of three.		
		C) Stopping on an even number OR a multiple of three.		
		D) Stopping on a number less than three.		
2	Drawing a card from an ordinary deck.	A) Drawing a heart.		
		B) Drawing an ace.		
		C) Drawing the jack of spades.		
		D) Drawing a red card.		
		E) Drawing an ace or a king.		
3	Drawing a ball from a box containing 4 black balls, 7 white balls, and 3 red balls.	A) Drawing a black ball.		
		B) Drawing a red ball.		
		C) Drawing a white ball.		
		D) Drawing a red or white ball.		
		E) Drawing a black or white ball.		

Investigation #2

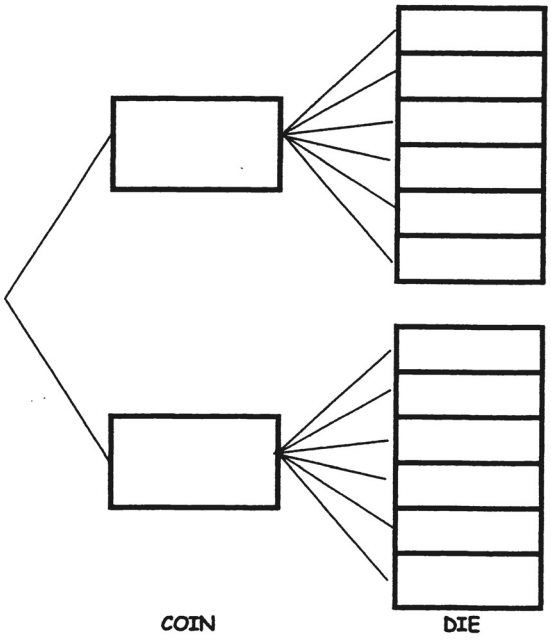
Suppose a coin is flipped and a die is rolled. You want to find the probability of the result being a number less than 3 and "heads."

Purpose

Investigate how multiplication can be used to find probabilities.

Procedure

Construct a tree diagram to show all possible outcomes.



Questions

1. Count the number of favourable outcomes and the total number of outcomes. What is the probability of getting heads and a number less than 3?

   _____

   _____

2. What is the probability of getting heads when you flip a fair coin?

   _____

3. What is the probability of getting a number less than3 when you roll a die?

   _____

4. Multiply the results from #2 and #3. What do you notice?

   _____

Investigation #3

Carl is the assistant coach of a novice hockey team. He has taken six team members to one end of the ice. One player has been designated as the goalie and one defence has also been chosen. Carl is going to choose the three forwards from the remaining four players: Eddie, Jane Claudette, and Bruce.

Purpose

Find the number of groups that can be chosen with and without regard to order.

Procedure

CASE	Arrange three players from Eddie (E), Jane (J), Claudette(C), and Bruce (B) (Not all spaces below have to be filled!)			
(1) Carl will assign the players to play right wing, left wing, and centre. **(ordered groups)** Ex: E-J-C is *different* from E-C-J	E-J-C			
(2) Carl will just select the three forwards and will decide later which position each should play. **(non-ordered groups)** Ex: E-J-C is the *same* as E-C-J	E-J-C			

Questions

1. Which case results in more possibilities? Explain.

Both Case 1 and Case 2 involve choosing groups of three from a set of four.

2. How do the cases differ?

3. Which is easier to determine? Why?

Investigation #4

A high school hockey team has 15 players. The players are introduces, one by one, as they skate into the ice. In how many different ways can the players skate onto the ice?

You can calculate the number of ways as follows:

$$15 \times 14 \times 13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

The calculation takes a long time to write and to do but can be simplifies through the use of **factorial** notation. You can write the above expression as 15!, which is read "15 factorial."

Purpose

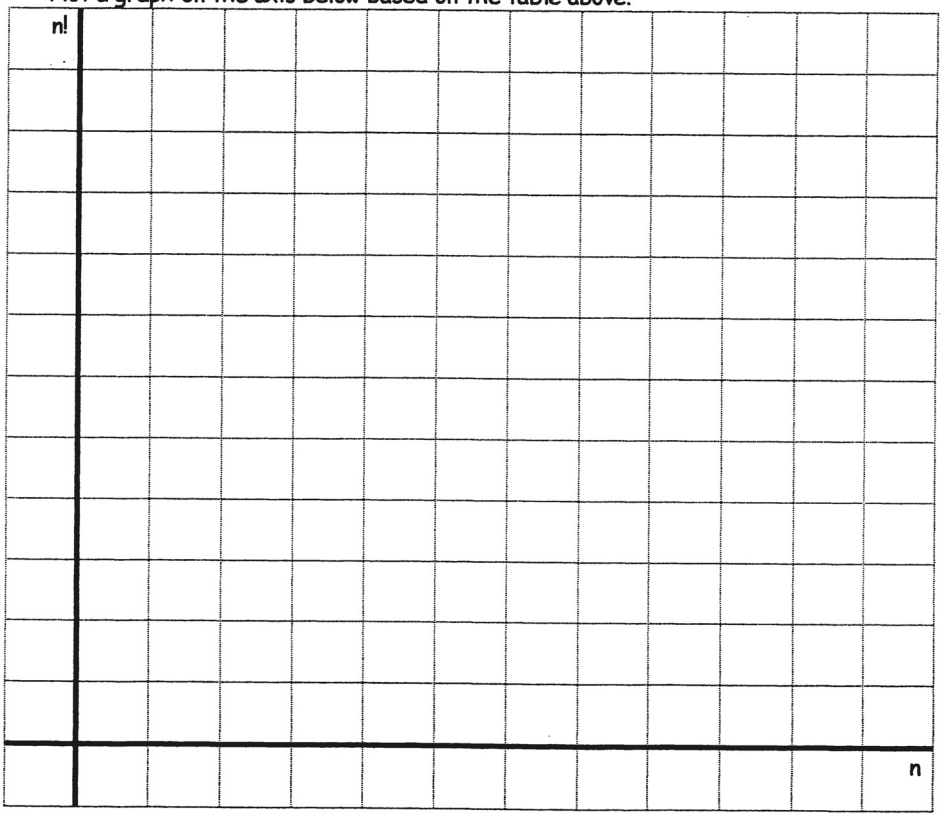
Explore the meaning of factorial.

Procedure

- Complete the table below.

n	1	2	3	4	5
Calculation of n	1	2 x 1	3 x 2 x 1		
n!	1	2	6		

- Plot a graph on the axis below based on the table above.



Questions

1. Examine the graph and the table in the Procedure. What happens to the value of n! as n becomes larger?

_____

_____

_____

_____

2. Determine the values of the following,

   A) $6!$

   _____

   B) $7!$

   _____

   C) $8!$

   _____

   D) $9!$

   _____

3. Explain whether your answers in #2 agree with your answer to #1.

_____

_____

_____