Adult Basic Education (ABE)

Level III Mathematics

Mathematics 3101B

Rational Expressions and Equations/Polynomial Functions/Exponential Functions

Curriculum Guide

Resource: *Principles of Mathematics 12. Nelson. 2012. ISBN-13: 978-0-17-654038-8*

Level III Degree and Technical/Business-Related College Profiles Mathematics Courses (Academic)
Mathematics 1101A: Measurement/Trigonometry/Factors and Products
Mathematics 1101B: Roots and Powers/Relations and Functions
Mathematics 1101C: Linear Functions/Systems of Linear Equations
Mathematics 2101A: Reasoning/Angles and Triangles/Trigonometry
Mathematics 2101B: Radicals/Statistics/Quadratic Functions
Mathematics 2101C: Quadratic Equations/Proportional Reasoning
Mathematics 3101A: Set Theory/Counting Methods/Probability
Mathematics 3101B: Rational Expressions and Equations/Polynomial Functions/Exponential Functions
Mathematics 3101C: Logarithmic Functions/Sinusoidal Functions/Borrowing Money

Newfoundland Labrador

Contents

General Information

Introduction

Mathematics 3101B when completed with **Mathematics 3101A** and **C** is equivalent to the Newfoundland and Labrador senior high school **Mathematics 3201 (Academic**) course.

Pre-requisite

Students must have passed **Mathematics 3101A**.

Resources

The student resource for this course is*:*

- *Principles of Mathematics 12. Nelson. 2012. ISBN-13: 978-0-17-654038-8.*

The instructor resources for this course are:

- *Principles of Mathematics 12 Teacher's Resource. Nelson. 2012. ISBN-13: 978-0-17-654044-9.*
- *Principles of Mathematics 12 Teacher's Resource CD-ROM.*

Instructors may also supplement with other resources at their discretion.

Study Guide

The Study Guide provides the student with Required Work for the course. It guides the student through the course by assigning relevant reading and exercises from the student resource. Sometimes the Study Guide provides important points for students to think about, to remember or to note. The Study Guide is designed to give students some degree of independence in their work. There is information in the Curriculum Guide applicable to teaching, learning and assessment that is not included in the Study Guide. Instructors should review this information and decide how to use it when teaching students.

Instructors can also exercise professional judgment and make minor alterations to the Required Work in the Study Guide. For example, an instructor may decide that it is unnecessary to assign students all the exercises to complete within each lesson.

Curriculum Guide

The Curriculum Guide includes the specific curriculum outcomes and achievement indicators for the course. The specific curriculum outcomes are listed numerically, and the achievement indicators are listed alphabetically. Suggestions for teaching, learning and assessment are also provided to support student achievement of the outcomes. Some of these suggestions will also be repeated in the curriculum guides for other mathematics courses as appropriate. The curriculum guide also states the pre-requisite for each Level III mathematics course.

Mathematics 3101B Outcomes/Achievement Indicators

Unit 1: Rational Expressions and Equations

1. Demonstrate equivalent forms of rational expressions (limited to numerators and denominators that are monomials and binomials).
    a) Explain why a given value is non-permissible for a given rational expression.
    b) Determine the non-permissible values for rational expressions.
    c) Compare the strategies for writing equivalent forms of rational expressions to the strategies for writing equivalent forms of rational numbers.
    d) Determine a rational expression that is equivalent to a given rational expression by multiplying the numerator and denominator by the same factor (limited to monomial or a binomial), and state the non-permissible values of the equivalent rational expression.
    e) Simplify a rational expression.
    f) Explain why the non-permissible values of a given rational expression and its simplified form are the same.
    g) Identify and correct errors in a given simplification of a rational expression, and explain the reasoning.
2. Perform operations on rational expressions (limited to numerators and denominators that are monomials and binomials.
    a) Compare the strategies for performing a given operation on rational expressions to the strategies for performing the same operation on rational numbers.
    b) Determine the non-permissible values when performing operations on rational expressions.
    c) Determine, in simplified form, the product or quotient of rational expressions.
    d) Determine, in simplified form, the sum or difference of rational expressions that have the same denominator.
    e) Determine, in simplified form, the sum or difference of two rational expressions that have different denominators.

3. Solve problems that involve rational equations (limited to numerators and denominators that are monomials and binomials).
    a) Determine the non-permissible values for the variable in a rational equation.
    b) Determine the solution to a rational equation algebraically and explain the strategy used to solve the equation.
    c) Explain why a value obtained in solving a rational equation may not be a solution of the equation.
    d) Solve a contextual problem that involves a rational equation.

Unit 2: Polynomials Functions

1. Represent the data using polynomial functions (of degree ≤3), to solve problems.
   a) Describe, orally and in written form, the characteristics of a polynomial function by analyzing its graph.
   b) Describe, orally and in written form, the characteristics of a polynomial function by analyzing its equation.
   c) Match equations in a given set to their corresponding graphs.
   d) Graph data, and determine the polynomial function that best approximates the data.
   e) Interpret the graph of a polynomial function that models a situation, and explain the reasoning.
   f) Solve, using technology, a contextual problem that involves data that is best represented by graphs of polynomial functions, and explain the reasoning.

Unit 3: Exponential Functions

1. Represent data using exponential and logarithmic functions to solve problems.
    a) Describe, orally and in written form, the characteristics of an exponential function by analyzing its graph.
    b) Describe, orally and in written form, the characteristics of an exponential by analyzing its equation.
    c) Match exponential equations in a given set to their corresponding graphs.
2. Solve problems that involve exponential functions.
    a) Determine the solution of an exponential equation in which the bases are powers of one another; e.g., $2^{x-1}=4^{x-2}$.
    b) Determine the solution of an exponential equation in which the bases are not powers of one another; e.g., $2^{x-1}=3^{x+1}$.
    c) Solve problems that involve the application of exponential equations.
    d) Graph data, and determine the exponential function that best approximates the data.
    e) Interpret the graph of an exponential function that models a situation, and explain the reasoning.
    f) Solve, using technology, a contextual problem that involves data that is best represented by graphs of exponential functions and explain the reasoning.

Recommended Evaluation

Written Notes (Including all the Required Work)	10%
Assignments	20%
Tests	20%
Final Exam (entire course)	50%
Total	**100%**

Instructors have the discretion to make minor changes to this evaluation scheme.

Unit 1: Rational Expression and Equations—Suggestions for Teaching and Learning

- In this unit, students will simplify rational expressions and determine non-permissible values. They will also add, subtract, multiply and divide rational expressions, limited to numerators and denominators that are monomials and binomials.
- Discuss with students that a rational expression is any expression that can be written as the quotient of two polynomials in the form, $P(x)/Q(x)$ where $Q(x)\neq 0$.
- Discuss with students that non-permissible values are the values of a variable that make the denominator of a rational expression equal zero.
- Discuss with students that a common error occurs when students generalize that the non-permissible value is zero rather than looking at the value(s) of x that produce a denominator of zero.
- Encourage students to substitute the non-permissible value(s) for x back into the denominator to verify the denominator results in 0.
- Discuss with students that simplifying a rational expression to lowest terms means the process of simplifying fractions. In both cases common factors in the numerator and denominator form a ratio of 1 and can be simplified.
- Ensure that students understand the benefit of simplifying rational expressions, whether it is for evaluating or performing operations.

Unit 1: Rational Expression and Equations—Suggestions for Teaching and Learning

- Ensure students are also able to identify incorrect solutions, including why errors might have occurred and how they can be corrected.
- Discuss with students that multiplying and dividing rational expressions is very similar to the process used to multiply and divide rational numbers.
- Discuss with students multiplication of rational expressions follows the same procedure as multiplying rational numbers, but with the added necessity of determining the non-permissible values for variables.
- Ensure that students first write the fractions with a common denominator before beginning to add or subtract.
- Remind students to be careful when subtracting rational expressions containing negative signs. The use of brackets can help eliminate errors involving the negative sign.
- Ensure that students understand how to eliminate the denominator by first multiplying both sides of the equation by the lowest common denominator.
- Ensure students recognize that solutions which are non-permissible values are extraneous roots. Remind students to verify their solutions to avoid extraneous roots.

Unit 1: Rational Expressions and Equations —Suggestions for Assessment

- Instructors can use the BLM's on the CD-ROM to further reinforce the unit concepts.
- The BLM's on the CD-ROM can be useful for developing unit tests and the final exam.
- Instructors have discretion to combine the last unit test with the final exam if beneficial to the student.
- Students must pass the final exam with a minimum grade of 50% to receive credit for this course.
- Instructors should encourage students to reflect on the math concepts in this unit to relate to everyday life.
- Instructors should engage students in discussions to verbalize student thinking on the math concepts.
- Instructors should require students to always show complete calculations with correct units when relevant.
- Instructors can use their own professional judgment to design assessment tools (additional exercises, word problems, assignments, reflections, math journals, etc.) to meet individual student needs.

Unit 2: Polynomial Functions—Suggestions for Teaching and Learning

- In this unit, students work with polynomial functions of degree of three or less. They will compare polynomial functions with respect to degree of the leading coefficient, and the constant term (y-intercept).
- Discuss the definition of end behavior of the graph of a function as the behavior of the y-values and x becomes large in the positive or negative direction.
- Students should describe the characteristics of the graph using the degree of the function, the sign of the leading coefficient, and the value of the constant term.
- Students should observe that the degree of a polynomial function determines the shape of a function; e.g., for a cubic function, the end behavior is opposite on the left and right sides of the graph.
- Students should be able to match a polynomial function with its complementing graph. Ensure that students can justify their matches in relation to characteristics such as end behavior, inflection points, y-intercept and number of x-intercepts.
- Provide students the opportunity to become comfortable with using technology to find the regression equations.
- It is important for students to observe that the polynomial regression results in an equation of a line (or curve) that balances the points on both sides of the line (or curve).
- Students can interpolate or extrapolate values by tracing along the line/curve of best fit or by substituting values into the equation of the regression function.
- Remind students that the regression equation is a model that best suits the data as a whole, rather than at any one point on the scatter plot.

Unit 2: Polynomial Functions—Suggestions for Assessment

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Unit 3: Exponential Functions—Suggestions for Teaching and Learning

- Students should investigate the characteristics of an exponential function using graphing technology or a table of values.
- Discuss with students the concept of asymptote and the connection to the range of an exponential function.
- Students should be aware that a polynomial of degree 1 is linear, degree 2 is quadratic and degree 3 is cubic.
- Discuss with students that in order to solve some exponential equations algebraically, students may need to rewrite both sides of the equation as a power of the same base.
- Note that when the exponent is binomial, students sometimes forget to apply the distributive property. Ensure students place brackets around the exponent and then use the distributive property.
- Discuss with students systematic trial and graphing technology as strategies to solve exponentials with variable exponents. Those strategies are important later so that students will be better able to determine the reasonableness of solutions during work with logarithms.
- Ensure that students are able to use graphing technology to determine the solution for an exponential function.
- Remind students that an exponential expression arises when a quantity changes by some factor for each unit of time; e.g., a population doubles every year, a bank account increases by .1% each month, etc.
- Discuss with students the half-life exponential function, and discuss the unknown variables.
- Students should solve problems where the exponential equation is given. Depending on the context of the problem, students should substitute values into the function.
- Discuss with students that they performed linear, quadratic and cubic regressions when working with polynomial functions, and they should continue to use technology to create a scatter plot and determine the equation of the exponential regression function that models the data.
- Discuss with students financial applications involving exponential equations. Introduce students to the concepts of simple interest and compound interest.

Unit 3: Exponential Functions—Suggestions for Teaching and Learning

- Discuss with students the exponential function that models compound interest.
- Discuss with students that simple interest is calculated only in terms of the original amount invested, not on the accumulated interest.
- Discuss with students that the accumulated value or amount A is the sum of the principal and the accumulated interest (Prt).
- Guide students as they set up the table for compound interest investment and work through the calculations. Encourage students to identify any patterns they notice.
- Students should be able to identify that compound interest is determined by applying the interest rate to the sum of the principal and any accumulated interest.
- Discuss with students problems where a value depreciates (e.g., the depreciation in the value of a vehicle).
- Discuss with students that exponential functions also apply to loan payments. Whether someone can afford a loan payment depends on whether he/she can afford the periodic payment.

Unit 3: Exponential Functions—Suggestions for Assessment

- Instructors can use the BLM's on the CD-ROM to further reinforce the unit concepts.
- The BLM's on the CD-ROM can be useful for developing unit tests and the final exam.
- Instructors have discretion to combine the last unit test with the final exam if beneficial to the student.
- Students must pass the final exam with a minimum grade of 50% to receive credit for this course.
- Instructors should encourage students to reflect on the math concepts in this unit to relate to everyday life.
- Instructors should engage students in discussions to verbalize student thinking on the math concepts.
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