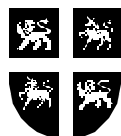


Calculators in Mathematics

Instruction and Assessment



GOVERNMENT OF
NEWFOUNDLAND
AND LABRADOR
Department of Education
Division of Program Development

A Position Statement
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Paul Gosse, Test Development Specialist - Mathematics, Division of
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Shawn Foss, Department Head - Mathematics, Holy Cross Junior High
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Introduction and Rationale

Introduction

Currently, there is much interest and debate concerning the role, nature and place of calculators in teaching and assessment in the school system. Calculators have a range of functions making them useful in many subject areas. These functions include:

- connectivity to other calculators, data-gathering devices, computers and the Internet;
- upgradeable software;
- applications such as simulations, place value and dynamic geometry activities;
- multiple graphic formats such as pie graphs, bar graphs and scatter plots.

Specific curriculum outcomes prescribed by the Department of Education direct teaching and assessment. Learning resources (e.g., student textbooks and teacher resource books) provide teachers with opportunities to use a wide range of supports, including manipulative materials, calculators and other technologies. Provincial mathematics curriculum and resource materials should guide the teacher in deciding where, when and how the calculator should be used. The calculator should be used when it supports the instructional pedagogy of the curriculum outcomes. The calculator may be useful in developing and consolidating a concept; however, it may not always be appropriate or essential in assessing that concept.

The National Council of Teachers of Mathematics (NCTM) provides a vision for the teaching and learning of mathematics in North America. NCTM's Technology Principle states: "Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning." (NCTM, 2000; page 24). Current research in technology and mathematics education has found "[e]lectronic technologies—calculators and computers—are essential tools for teaching, learning, and doing mathematics..." (NCTM, 2000; pages 24-25) and students can learn more mathematics more deeply with the appropriate use of technology.

"Technology should not be used as a replacement for basic understandings and intuitions; rather, it can and should be used to foster those understandings and intuitions. In mathematics-instruction programs, technology should be used widely and responsibly, with the goal of enriching students' learning of mathematics." (NCTM, 2000: page 25).

Calculators serve a similar function in mathematics classrooms as a word processor does in language arts. A word processor cannot 'create' an essay but it does help significantly in the creation of an essay. A calculator cannot 'understand' a mathematics problem but it can help significantly in the solution of the mathematics problem.

Rationale

This document is intended to support grade level and course specific curriculum guides. It is not intended to provide specific grade level guidance in calculator use. This document illustrates, by level, examples of calculator and non-calculator activities and provides general guidance relative to the role of the calculator in assessment. Teachers are referred to curriculum documents and authorized resource materials for more specific and wide-ranging examples of appropriate calculator usage for a particular grade level or course. This document addresses the appropriate use of calculators and is not intended as a tool in addressing the broader issues of technology use in the classroom.

Mental math, estimation, paper and pencil, calculators and computers comprise the range of tools to help students work through the computations and manipulations necessary for solving problems. In many problem situations, calculators free students from tedious calculations so that valuable classroom time can be spent on higher order thinking and reasoning.

Calculators in Classrooms

The ability to reason logically, work with different representations, solve problems and communicate understandings is the cornerstone of mathematics education in the 21st century. Students are citizens of a global village where facility with statistics, numbers, estimation, algebra and logical reasoning is essential. Comfort with technological tools and the ability to keep step with the rapid pace of technological change is an essential skill in this society. Appendix A provides a brief timeline on the evolution of calculating devices. Today's mathematics curriculum reflects this growing need for proficiency with technology.

The NCTM has made specific recommendations on calculators in school mathematics:

- All students should have access to calculators to explore mathematical ideas and experiences, to develop and reinforce skills, to support problem-solving activities and to perform calculations and manipulations.
- Mathematics teachers at all levels should promote the appropriate use of calculators to enhance instruction by modeling calculator applications, by using calculators in instructional settings, by integrating calculator use in assessment and evaluation, by remaining current with state-of-the-art calculator technology and by considering new applications of calculators to enhance the study and the learning of mathematics.
- Professional development activities should be provided that enhance teachers' understanding and application of state-of-the-art calculator technology.
- Teacher education institutions should develop and provide preservice and in-service programs that use a variety of calculator technology.
- Those responsible for the selection of curriculum materials should remain cognizant of how technology—in particular, calculators—affects the curriculum.
- Authors, publishers, and writers of assessment, evaluation and mathematics competition instruments should integrate calculator applications into their published work.
- Mathematics educators should inform students, parents, administrators, school boards, and others of research results that document the advantages of including the calculator as one of several tools for learning and teaching mathematics. (NCTM, 1998).

Since curriculum outcomes and teaching methodologies promote technology use, it is important to establish guidelines that help set clear expectations for the role and nature of calculator use in instruction and assessment.

Role of Parents

Societal change dictates that students' mathematical needs today are in many ways different from those of their parents. These differences are manifested not only with respect to mathematical content, but also with respect to instructional approach. As a consequence, it is important that educators take every opportunity to discuss changes in mathematical pedagogy with parents and why these changes are significant. Parents who understand the reasons for changes in instruction and assessment will be better able to support their students in mathematical endeavours by fostering positive attitudes towards mathematics, stressing the importance of mathematics in their students' lives, assisting students with mathematical activities at home and, ultimately, helping to ensure that their children become confident, independent learners of mathematics.

Parents often feel that if their children use calculators they will not learn basic facts or standard computational algorithms. Consequently, it is important that educators discuss with parents the appropriate and inappropriate use of calculators and emphasize that basic arithmetic skills are just as important as they have traditionally been. When calculator use is appropriate, the traditional issues concerning computational proficiency are less prevalent.

Calculators in Instruction

Calculators and Manipulatives for Teaching and Learning

Teachers have a range of strategies and resources available to them to help students develop, consolidate and extend their mathematical understanding. The use of manipulative materials has been shown to help students deepen their understanding of key mathematical concepts. In the primary grades, this includes using counters and bean sticks to develop the concept of number. In the elementary level, fractions and decimals are explored using pattern blocks and base 10 materials. At the intermediate and senior high school levels, materials such as interlocking cubes, geometric solids and algebra tiles are effective tools in the development and consolidation of student learnings in algebra and geometry. Calculators are used to serve many of the same purposes as other concrete materials in the teaching and learning of mathematics.

Both manipulatives and calculators assist in:

- enhancing student understanding
- solving problems
- exploring concepts and connections through multiple representations
- developing student understanding

Manipulatives and calculators may bring student learning to a point where students no longer need to use them. It is important to balance the use of both manipulatives and calculators with mental mathematics, estimation skills, and paper and pencil proficiency. Students should not become dependent on manipulatives or calculators. Both should be used to develop understanding, to extend learning, and to assist in problem solving situations. Problem solving skills and the strategic use of appropriate tools are fast becoming the benchmark of mathematical understanding.

Researchers have been examining the impact of calculator use in classrooms for the past two decades. Research has shown the **appropriate** use of calculators has no effect on development of computational proficiency. However, inappropriate use as well as the lack of appropriate use can have negative impact. Emphasis must continue to be on mental math and estimation and appropriate calculator use. Ruthuen (1990) compared the performance of 47

secondary students in England who used graphing calculators in a year of pre-calculus study with 40 students who did not have access. The study revealed that students with access to the graphing calculator were better able to make links between graphic and algebraic representations of functions.

Primary

The primary grades provide the foundation for future mathematics learning. In primary, the mathematics program is one in which “thinking is encouraged, uniqueness is valued and exploration is supported.” (NCTM, 2001; pg. 74) At this level children need programs that support their ability to think and reason mathematically. These programs must include much more than “short-term learning of rote procedures.” (NCTM, 2001; pg. 76)

The calculator and other manipulatives have a role to play in the primary mathematics classroom. This role is primarily instructional. “Guided work with calculators can enable students to explore number and pattern, focus on problem-solving processes and investigate realistic applications. With teacher guidance, children should recognize when using a calculator is appropriate and when it is more efficient to compute mentally.” (NCTM, 2001; pg. 77) Sometimes at the primary level children may see the calculator as more efficient for basic computation simply because they have not yet acquired basic facts at a recall level. It is important that students are regularly expected to compute mentally utilizing various strategies and concrete materials. Basic skill development can be interfered with if a calculator is used as soon as a problem with skill development is encountered. It would be appropriate to ask students to own a basic four function calculator as long as this requirement goes hand-in-hand with proper communication to parents on the role the calculator plays in students’ mathematical development and the dangers associated with inappropriate or indiscriminant use.

Assessment

In the primary grades student assessment is multi-representational. Techniques such as conferencing, portfolios, interviews, class work, homework and paper and pencil testing all have their place in the assessment of mathematics outcomes. Students are permitted, and even encouraged, to use manipulatives to demonstrate their understanding. While the calculator has a place in this multi-representational assessment, approximately 90-95% of the primary mathematics outcomes do not require the use of a calculator and should be assessed without its use. Teacher understanding of the outcomes is essential in determining when and how the calculator may be used in assessment.

Sample Activities

The following activities are meant to provide examples of when it would be appropriate to use a calculator and when other methods of computation should be employed. Curriculum documents and authorized resources will offer further examples of integration of calculator activities in the curriculum.

Non-calculator Activities

1. Students could be asked to explain why it might be easier to answer $123 - 99$ than $123 - 87$.
2. Students could be asked to explain an easy way to find the sum of \$1.99, \$2.98 and \$4.99.

Calculator Activities

Some of the activities in the primary grades are enhanced through calculator use.

3. This activity includes using a hundreds board and a calculator.

The task is to compare counting sequences on your calculator with the patterns they generate on a hundreds board. Press the keys on the calculator to observe your counting sequence displayed on the hundreds board. For example, press $4 + 4$, $=$, $=$, $=$, $= \dots$. Mark each result in the hundreds board as it appears on the calculator. Try the same process with other numbers. What patterns do you see? Can you predict what numbers will be marked? [An activity like this might be suitable for calculator use in early primary, whereas in late primary this same activity should be a mental exercise.]

4. Explore different counting patterns using the calculator.
 - a) Skip counting by what numbers will include 100 as part of the pattern. (If counting by threes, start with three. If counting by eights, start with eight.)
 - b) What patterns do you see when you count by twos and begin with two?
 - c) What do you notice when you count by twos and begin with one? Why?
 - d) Will there be a pattern if you skip-count by fives and begin with three? Why?
 - e) What happens if you skip-count by tens and start with thirty seven? What do you notice?
 - f) How does adding ten to any number relate to skip-counting by tens beginning with that number? (NCTM, 2002)
5. This activity combines mental math and calculator use.

I have four \$10 bills. I want to buy a CD that costs \$17.99 and a birthday gift for my little sister that cost \$4.50.

 - a) Estimate - How many \$10 bills I will have left?
 - b) Use your calculator to determine **exactly** how much was spent and how much change was received.

Note: A grade 3 student may be expected to try part 5(b) using pencil and paper.

Elementary

At the elementary level the calculator is used primarily as an instructional aid to explore mathematical ideas and to verify results. The calculator is used to help children in the development of new mathematical processes based on the processes previously learned. Students are encouraged to use all four methods of mathematical computation: mental computation, computational estimation, pencil and paper, and calculators. The emphasis should be on the first three methods. Students are encouraged to select a strategy to estimate and check the reasonableness of their answers before using a calculator.

As in the primary grades, the calculator has a role to play in mathematics instruction and assessment along with other manipulatives. The calculator can enhance the achievement of outcomes but teachers need to consider carefully the place and role of the calculator. When computational algorithms (e.g., adding decimals) are being developed, the use of the calculator can be counterproductive. The calculator should be used to facilitate problem solving and to encourage student interest in mathematics and mathematics related subjects. It would be appropriate to ask students to own a basic four function calculator as long as this requirement goes hand-in-hand with proper communication to parents on the role the calculator plays in mathematical development as well as the dangers associated with inappropriate or indiscriminant use.

Assessment

Student assessment in the elementary grades is multi-representational. Techniques such as conferencing, portfolios, interviews, class work, homework and paper and pencil testing have their place in the assessment of mathematics outcomes. Students may be allowed, and even encouraged, to use manipulatives to demonstrate their understanding. The calculator has a place in this multi-representational assessment. The outcomes being assessed determine when and how the calculator may be used in assessment. The vast majority of the outcomes should be assessed without the use of calculators, since the curriculum emphasizes mental math, estimation and paper and pencil algorithms in the numeracy strand. Assessment in the elementary grades often takes the form of a unit or culminating task. For many culminating tasks the focus is on decision making and the integration of a variety of outcomes addressed in the unit. For such tasks the use of the calculator can be often justified whereas a unit quiz is less likely to have a role for the calculator.

Sample Activities

The following activities are meant to provide examples of when it would be appropriate to use a calculator and when other methods of computation should be employed.

Non-calculator Activities

1. Four people are sharing a pizza which costs \$14.40. Ask students what each person's share of the cost would be. Have the students show that his/her answer is correct.
2. If the missing number in each of the open sentences below is a whole number, can you tell whether the open sentence is sometimes true, always true, or never true? Explain your thinking.
 - 3 ☐ is even.
 - 3 ☐ is multiple of 3.
 - 3 ☐ is greater than 500.
 - 3 ☐ is 0.

Calculator Activities

3. Pat is looking at skateboards in the window of the Mountain Sports Store. The Blue Lightning skateboard costs \$8.47 more than the Silver Streak Surfer. Magician skateboard costs \$21.42 more than the Silver Streak Surfer. The three skateboards together cost \$160.00. How much does each skateboard cost?

Find out

- What is the question you have to answer?
- What do you know about the price of the Blue Lightning skateboard?
- What do you know about the price of the Silver Surfer skateboard? The Magician skateboard?
- What is the total price for the three skateboards?

Choose a Strategy

- Make a guess about the price of the Silver Surfer skateboard. Check to see how close your guesses are. Use this information to make better guesses.

Estimate

- Estimate: Which skateboard is least expensive? Most expensive?

Calculate and Solve

- Begin with your guess. Let's say your guess is \$50. Begin with these keystrokes:
 $50 + 8.47 =$ Blue Lightning
 $50 + 21.42 =$ Magician
- Continue guessing and checking.

This activity allows students to explore a real world example. The calculator frees students from tedious or cumbersome calculations and allows them to focus on problem solving strategies.

4. Students could explore a short cut for multiplying 2-digit numbers by 11. Notice the following pattern for multiplying by 11.

42	53	62
$\times 11$	$\times 11$	$\times 11$
462	583	682
↓	↓	↓
(4 + 2)	(5 + 3)	(6 + 2)

Calculators facilitate identifying patterns for larger 2-digit numbers (e.g., 78 \times 11), 3-digit numbers (e.g., 243 \times 11), multiples of 11 (e.g., 49 \times 22), etc. *

Note: The focus of #4 is on the pattern as opposed to the computation.

* When multiplying $78 \times 11 = 858$ students can discuss why the pattern changed. They may realize that since $7 + 8 = 15$ the regrouping creates a 1 [really 100] which is added to the 1st digit of the answer. They can understand why this happens by analyzing the standard multiplication algorithm.

Intermediate

At the intermediate level, it is still important to use all four methods of mathematical computation: mental computations, computational estimation, pencil and paper and calculators. While calculators receive greater emphasis as students progress through the grades, it is important that teachers still emphasize the first three methods of mathematical computation. Teachers must monitor when, why and how students are using calculators. Students should be introduced to the use of a graphing calculator as familiarity with it will be helpful in aspects of the intermediate curriculum as well as throughout high school. All students should own a minimum of a scientific calculator in the intermediate grades.

Assessment

Students should be permitted to use calculators during assessment only when it is appropriate in relation to the outcome(s) being measured. Assessments should include both a calculator and non-calculator component where appropriate. It is important that teachers gauge the appropriateness of using the calculator by focusing on the intent of the outcome, which is addressed in the elaboration column of the curriculum guide. In grade 7, the majority of the outcomes should be assessed without calculator use, whereas by grade 9 calculator use becomes relevant for a greater portion of the outcomes. For example, the use of calculators for operations with fractions and integers should be avoided, whereas calculation involving surface area and volume would be an appropriate place for calculator use.

Sample Activities

The following activities provide a guide for appropriate use of calculators in the intermediate level.

Non-calculator Activity

1. Patterns
 - a) What is the sum of the first two odd numbers?
 - b) What is the sum of the first three odd numbers?
 - c) What is the sum of the first four odd numbers?
 - d) Make a chart and look for a pattern.
 - e) Write a rule pertaining to the sum of the first n odd numbers.
2. A recipe requires in part, $1\frac{1}{3}$ cups of sugar, $2\frac{1}{2}$ cups of flour and $\frac{3}{4}$ cup of butter.
 - a) Double this recipe.
 - b) The recipe makes 3 dozen cookies. Adjust the quantities in the above recipe to make only 1 dozen cookies.

Calculator Activities

3. $\frac{1}{7} = 0.142857142857142857\dots$ Your calculator may only show $0.\overline{1428571}$.

We can produce other sevenths decimal expansions by comparing it to $\frac{1}{7}$ as reported on the calculator. Observe the pattern in the numbers shown.

$$\frac{1}{7} = 0.1428571$$

$$\frac{2}{7} = 0.2857142$$

.

.

.

$$\frac{6}{7} = ???$$

- a) Find the decimal expansions for $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$ using the pattern above.
- b) Use your calculator to investigate the decimal expansions of the following fractions:

i) $\frac{1}{13}$, $\frac{2}{13}$

ii) $\frac{1}{17}$, $\frac{2}{17}$

iii) $\frac{1}{19}$, $\frac{2}{19}$

- c) Use the information in part b) to find the decimal expansion of:

i) $\frac{5}{13}$, $\frac{6}{13}$

ii) $\frac{4}{17}$, $\frac{5}{17}$

iii) $\frac{4}{19}$, $\frac{5}{19}$, $\frac{8}{19}$

4. A sphere which is exactly 12 cm in diameter fits exactly inside a cube (that is, it touches all sides of the cube).
- a) Find the volume and surface area of the cube.
- b) Find the volume and surface area of the sphere.
- c) Find the volume of the space inside the cube not taken up by the sphere.
- d) The same sphere exactly fits inside a cylinder. Find the volume and surface area of the cylinder.

Senior High

The use of calculators in high school extends well beyond computation. The graphing capabilities of calculators allow statistics, probability and algebra to be explored in ways that would never have been considered without the graphing calculator. While graphing calculators are powerful tools that can be used for investigating concepts, solving problems, and verifying solutions, it is important to note that graphing calculators are not meant to replace skills with mental math, estimation and pencil and paper. These skills are a very important focus of the high school mathematics curriculum. The calculator is particularly efficient as a tool in trigonometry. All high school students should own a minimum of a scientific calculator. Graphing calculators are often necessary for class work. Graphing calculators have been provided to schools based on 20% of the high school population. This is ample to meet the needs if they are used only when their special capabilities are needed. For work with topics such as right triangle trigonometry and 3-D geometry a scientific calculator is quite sufficient.

Assessment

The use of calculators should be dependent on the outcome(s) being assessed. Assessment instruments should include, where appropriate, both a calculator and a non-calculator component. In high school, students should have consolidated their understanding of mental math and estimation strategies to the point where the calculator is not used when a process involves simple arithmetic. Teachers need to be mindful of the tools accessible to students when constructing assessment items. Some items are not affected when a calculator is permitted; others are made trivial when a calculator, especially a graphing calculator, is used.

Calculator Use in Public Examinations

The use of calculators and similar devices in public examinations is set out in the Department of Education Public Examination Handbook.

This is found at <http://www.gov.nl.ca/edu/pub/hsc/hsc.htm>

Sample Activities

The following activities provide a guide for appropriate use of calculators in high school.

Non-calculator Activities

1. Find the exact numerical value for:

$$\frac{\cos 45^\circ}{\cos 30^\circ} + \frac{\sin 45^\circ}{\sin 30^\circ}$$

2. Use your mental math skills to solve the following equation for x.

$$3^x - 2 = 25$$

3. Estimate the value of $\log_2 17$.
4. A parabola has vertex (1, 2) and passes through (3, 4). Find the equation of the parabola in transformational form.

Calculator Activities

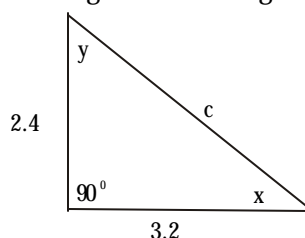
5. Statistics

John's class had the following set of marks for the Grade 10 math mid-term. Use this data to answer the following questions.

55	62	71	77	90	33	88	77	95	84	80
66	59	78	89	99	23	45	67	70	93	57
18	52	54	61	60	83	85	75	70	66	61
100	44	66	62	69	79	89	97	48	51	50

- a) What will happen to the mean if Cheryl, Joe and Mike wrote the exam the next day and the mean of their marks was 74%?
- b) What is the standard deviation for the mid-term marks?
- c) Would a mark of 86% fall within two standard deviations of the mean? Explain why or why not?
- d) Use technology to produce a histogram with appropriate bin widths.

4. Solve the following for the missing components:



- 5.* How many real roots does $f(x) = x^3 + 6x^2 + 12x - 72$ have on $-2 \leq x \leq 5$. How do you know? Estimate the root(s) to 3 places? Describe the roots of the function.
 - 6.* A sponge ball is thrown into the air. After $t = 1$ second the ball has a height of 4.8 m. After $t = 3$ seconds the ball has a height of 3.2 m. After $t = 4$ seconds the ball has a height of 1.8 m. Using a system of equations and matrix multiplication, find the quadratic function that describes the path of the ball. At what time will the ball land? How far from the starting point will the ball land?
- * These are samples of questions that teachers may ask students to do both using a calculator and algebraically depending on the nature of the solution.

Appendices

Calculating Machines - A Brief History

The following timeline provides a summary of the significant milestones in the evolution of calculating devices.

15000 BC	Tally Sticks - Notches were made on a piece of wood or bone as a means of recording.
250 BC	Abacus - The first mechanical calculating device.
1594AD	Napier's Bones - This invention permitted large, and small numbers to be multiplied and divided or to have powers or roots taken by using simple arithmetic on their related powers. John Napier spent 20 years of his life completing his tables of logarithms.
1622	Slide Rule - The slide rule was invented in 1622. This device was based on logarithms and saved a tremendous amount of computational time.
1641	First Calculating Machine - Blaise Pascal constructed the first calculating machine, foreshadowing modern day computers.
1820	Machine for Calculating Mathematical Tables - Charles Babbage began construction of a machine called the 'difference engine' for calculating mathematical tables. This is best known as the blue print for the modern computer and it corrected many errors in the log tables which had been developed by hand.
1948	Curta - A small handheld device, designed by Curt Herzstark, was developed to perform the four basic functions. This was the first truly portable calculating machine.
1967	Texas Instruments Calculator - This brought the ability to accurately and efficiently perform simple calculations to a very compact form.

- 1972** **First Scientific Pocket Calculator** - This was invented by Hewlett-Packard.
- 1985** **Graphing Calculator** - The first graphing calculator was invented by Casio though many companies brought other models to market rapidly thereafter.
- 1986** **Graphing Calculator** - This was the first graphing calculator with symbolic algebra capability.
- 1998** **Flash Technology** - Calculators were developed in which the operating system can be upgraded and application software installed.

Definitions

Calculator: simple four function calculating device.

Scientific Calculator: non-qwerty calculating device with one or two line display capable of returning, for example, basic trigonometric ratios, logarithms, powers and roots.

Scientific Calculator (Programmable): scientific calculator with ability to write and store programs, for example, the quadratic formula.

Graphing Calculator: scientific calculator capability with additional features of producing tables of values and plots for given functions on chosen domains; may include finance package, I/O port, and other unit specific functions.

Manipulatives: concrete devices able to be physically manipulated as representations of a mathematical concept or process. Selected examples include: stones, base-ten blocks, place-value mats, geoboards, algebra tiles.

QWERTY Keyboard: a device with key input arranged as on a typical typewriter/ computer keyboard. This acronym comes from the first five letters from top left to right on such keyboards. These devices often allow storage and retrieval of text as well as many other utilities.

Multiple Representations: refers to the various forms of capturing a mathematical concept or relationship. These forms include diagrams, tables, charts, graphical displays, words and symbolic/algebraic expressions.

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