Mathematics 3201 June 2014 Public Exam Outcome Report

This examination follows the specifications, conventions and standards set out in the:

Mathematics Public Examination Standards

Chapters: 1 Set Theory

2 Counting Method

3 Probability

4 Rational Expressions and

Equations

5 Polynomial Functions

6 Exponential Functions

7 Logarithmic Functions

8 Sinusoidal Functions

9 Financial Mathematics: Borrowing Money

PART I: Selected Response—Total Value: 50%

Item	Curriculum Guide Pages	Outcome	Cognitive Level	Outcome Description	
1	26	LR2	L2M	Given two non-disjoint (intersecting) sets, identify the elements in the intersection of the sets.	
2	28	LR2	L2M	Given two non-disjoint sets, identify the elements in the complement of the union of the sets.	
3	22	LR2	L2A	Given one finite and one infinite set, each described using set notation, determine which statement about the two sets is true.	
4	24	LR2	L2A	Given a description of a situation involving collected data, identify the Venn diagram that correctly illustrates the data.	
5	62	P5	L2M	Identify the notation which indicates the number of ways in which r distinct elements can be selected from n distinct elements, where $r < n$ and where order is important.	

6	54	P4	L2M	Given multiple copies of three distinct objects, identify the numeric expression that indicates the number of ways in which one copy of each distinct object can be selected.
7	56	P4	L2M	Determine the number of passwords that can be created using the numbers 0 to 9 given the number of digits in the password, that repetition of numbers is permitted, and that there is a restriction on the numbers used for a digit of the password.
8	72	P6	L2A	Given the number of defensemen and the number of forwards on a hockey team, determine the notation which indicates the number of ways in which a starting lineup of 5 players (no goalie) can be selected.
9	68	P5	L2A	Given multiple copies of three distinct objects, identify the numerical expression which indicates the number of ways in which all of the objects can be arranged if order is not important.
10	60	P5	L2A	Solve an equation where one side of the equation involves an algebraic fraction containing factorials in the numerator and the denominator and the other side of the equation is a constant.
11	92	P1	L2M	Identify the numeric expression which indicates the probability of a given outcome when a fair coin is tossed more than once.
12	80	P1	L2M	Given the probability of an event as a percent, determine the odds in favour of the event.
13	80	P1	L2A	Given a tree diagram showing the outcomes when a fair coin is tossed three times, determine the odds in favour of obtaining a given outcome.

14	92	P3	L2A	Given multiple copies of three different coins, determine the probability of selecting (without replacement) two coins which have a given, combined total.
15	90	P2	L3	Given the number of elements in the universal set $(n(U))$, the percentage of elements in each of two non-disjoint sets A and B , and $n(A \cup B)'$, determine $n(A \cap B)$.
16	102	RF1	L2M	Identify equivalent rational expressions.
17	108	RF2	L2M	Determine the lowest common denominator for the sum of two rational expressions given in factored form.
18	104	RF1	L2A	Given a rational expression with binomials in the numerator and denominator, both in non-factored form, determine the simplified form of the rational expression.
19	106	RF2	L2A	Simplify the quotient of two rational expressions, each given in factored form.
20	108	RF2	L2A	Simplify the sum of two rational expressions, each with monomials in the numerators and denominators.
21	110	RF3	L2A	Solve an equation equating two rational expressions of the form $\frac{c}{ax+b}$.
22	126	RF7	L2M	Given the equation of a cubic function in standard form, determine the <i>y</i> -intercept and the end behaviour of the graph of the function.

23	126	RF7	L2M	Determine the possible number of turning points on the graph of a function with a given degree.	
24	122	RF7	L2A	Given a table of values and the corresponding scatter plot, determine the correct statement concerning the value(s) of a and c in the equation of the quadratic regression, $y = ax^2 + bx + c$.	
25	126	RF7	L2A	Given the graph of a cubic polynomial, determine the corresponding polynomial equation in standard form.	
26	130	RF7	L2A	Given the polynomial equation of the curve of best fit in standard form and the scatter plot containing the curve of best fit, determine the value of <i>y</i> for a given <i>x</i> value.	
27	126-7	RF7	L2A	Given the equation of a quadratic function in vertex form, determine the coordinates of the vertex and the <i>y</i> -intercept of the corresponding graph of the function.	
28	150	RF5	L2M	Given an exponential equation of the form $P(t) = a(b)^{\frac{t}{c}}$ which describes the growth of a population over time, determine the initial population and the rate of growth of the population.	
29	138	RF6	L2M	Given an exponential function in the form $f(x) = a(b)^x$, determine the corresponding graph.	
30	146	RF5	L2A	Solve an exponential equation involving powers with variable exponents in which the bases can be made equal.	
31	138	RF6	L2A	Given two exponential functions in the form $f(x) = b^x$, determine which statement about the relative sizes of the two functions is true.	

32	150	RF5	L2A	Given a table of values which models exponential growth, determine the initial amount and the rate of growth.	
33	154	RF6	L2A	Given the initial dollar value of an investment and the simple interest rate, determine the value of the investment after a given time period.	
34	170	RF4	L2M	Rewrite a given exponential equation in logarithmic form.	
35	164	RF6	L2M	Given a logarithmic function in the form $y = \log_b x$, identify the graph which illustrates the function and its reflection in the line $y = x$.	
36	176	RF5	L2A	Given the formula $\beta = 10(\log I + 12)$ and the value of the sound intensity in W/m ² , determine the sound level in decibels.	
37	174	RF5	L2A	Evaluate a logarithmic expression consisting of the sum and difference of logarithms with equal bases.	
38	178	RF5	L2A	Find the exact solution to an exponential equation, equating a power with a variable exponent and a constant, in which the bases cannot be made equal.	
39	174	RF4	L3	Identify the step in which the error occurs in the "simplification" of a logarithmic expression containing the sum and difference of logarithms with equal bases.	
40	196	RF8	L2M	Given the graph of a sinusoidal function, determine the amplitude of the graph.	
41	200	RF8	L2M	Given the equation of a sinusoidal function in the form $y = a\cos x + d$, identify the corresponding graph of the function.	

42	190	RF8	L2A	Convert an angle measure from radians to degrees.	
43	198	RF8	L2A	Given a sinusoidal graph of the height of a Ferris wheel over time, determine the height of the center of the Ferris wheel.	
44	200	RF8	L2A	Given the sinusoidal function in the form $h(t) = a \sin b(t - c) + d$, which models the motion of a Ferris wheel over time, determine the height of the Ferris wheel after a given amount of time.	
45	196,200	RF8	L2A	Determine the range of a sinusoidal function.	
46	214	F1	L2M	Determine the loan balance after a given payment period given an initial loan amount, the interest compounding period, the amortization period, and the amortization table.	
47	216	F1	L2M	Determine the number of years required to repay a loan given a table indicating the present value of a loan, a future value of 0, the annual interest rate, the number of times per year that interest is compounded, the payment, the number of payment periods per year, and the total number of payments.	
48	224	F2	L2M	Determine which object is most likely to depreciate over time.	
49	212	F1	L2A	Given the future value of an investment, the annual interest rate, the compounding period, and the investment time, determine the present value of the investment.	
50	212	F1	L2A	Given an exponential equation that models a bank loan, in the form $A = P(1+i)^n$, the compounding period, and the repayment time, determine the annual interest rate.	

PART II: Constructed Response—Total Value: 50%

Item	Curriculum Guide Page	Outcome	Cognitive Level	Value	Outcome Description
51	36	LR2	L3	2	Given $n(U)$, $n(S)$, $n(E)$, $n(M)$, $n((S \cap E) \setminus M)$, $n((E \cap M) \setminus S)$, and $n(S \cap E \cap M)$, determine $n(S \setminus E \setminus M)$.
52a	75	P5	L2A	3	Given an equation in the form $_{n+c}C_r=k$, where $c, r,$ and k are constants, algebraically solve the equation for n .
52b(i)	56	P4	L2A	1	Determine the total number of ways in which a specific number of letters and digits can be selected from the 26 letters of the alphabet and the digits 0 through 9, if repetition is allowed.
52b(ii)	56	P4	L2A	1	Determine how the situation would change in 52b(i) if repetition of letters is not allowed and there is a restriction on the digit used in a particular position.
53a(i)	84	P6	L2A	1	Given the number of girls and the number of boys on a student council, determine the probability that sub-committee of size <i>n</i> has <i>n</i> girls.
53a(ii)	84	P6	L2A	1	Given the number of girls and the number of boys on a student council, determine the probability that sub-committee of size n has x girls and $n-x$ boys.
53a(iii)	84	P6	L3	2	Given the number of girls and the number of boys on a student council, determine the probability that sub-committee of size <i>n</i> students has at least one girl.

53b(i)	52,80	P1, P4	L2A	2	Using the digits 0 to 9 and the 26 letters of the alphabet, a password is created that must begin and end with a letter and have digits for the remaining characters. Determine the probability that the password starts and ends with a vowel if letters are not case sensitive and no repeating characters are allowed.
53b(ii)	52,80	P1, P4	L3	2	Indicate whether the probability in 53b(i) changes if the letters are case sensitive. Justify the answer.
54a(i)	112	RF3	L2A	3	Solve a given rational equation, based on a real-life situation, in which one side of the equation consists of the difference of 2 rational expressions with different denominators and the other side of the equation is a constant.
54a(ii)	112	RF3	L3	1	Adjust the rational equation provided in 54a(i) if the situation on which the equation is based is changed.
54b(i)	108	RF3	L3	1	Specifically identify the error in the subtraction of two rational expressions.
54b(ii)	108	RF3	L2A	1	Provide the correct simplification for the rational expression in 54b(i).
55a	120-8	RF7	L2A	2	Sketch two different polynomial graphs which have the same end behaviour, number of turning points, and <i>y</i> -intercept.
55b(i)	128	RF7	L2A	2	Given a polynomial graph, determine the end behaviour, the number of turning points, the <i>y</i> -intercept of the graph and the degree of the corresponding polynomial function.

55b(ii)	128	RF7	L3	2	Sketch the resulting graph if the <i>y</i> -intercept of the graph in 55b(i) is changed while maintaining the <i>x</i> -intercept(s) and the end behaviour. Describe how the change in the <i>y</i> -intercept affects the graph.
56a	146	RF5	L2A	3	Solve an exponential equation with variable exponents in which the bases can be made equal.
56b	150	RF5	L3	2	Given two exponential equations, each modeling how an object cools, in the form $A(t) = A_o(b)^{\frac{t}{c}}$ and the numerical values of A_o , b , and c , determine which object is cooling more quickly and explain the reasoning.
57a(i)	176	RF5	L2A	2	Given the formula for pH $(p(x) = -\log x)$ and a table containing the pH scale and examples of solutions with each pH, determine the pH of an unknown solution given the hydrogen ion concentration, identify the unknown solution, and compare the pH of the unknown solution to that of second solution.
57a(ii)	176	RF5	L2A	2	From the given the pH table, algebraically determine the hydrogen ion concentration of a solution.
57b(i)	150,182	RF5/6	L2A	1	Given an exponential function for half-life in the form $A(t) = A_{\rm o} \left(\frac{1}{2}\right)^{\frac{t}{h}}$, and the numerical values of $A_{\rm o}$ and h , algebraically determine the amount remaining after a given period of time.

57b(ii)	150,182	RF5/6	L2A	3	Using the function provided, algebraically determine the time taken for the sample to decay to a specific amount.
58a	194	RF8	L2A	4	Given the graph of a sinusoidal function, determine the period, the amplitude, the equation of the midline of the graph, and the maximum and minimum values of the function.
58b	196	RF8	L3	2	On the grid provided, sketch the graph of a sinusoidal function given its domain, range, period and <i>y</i> -intercept.
59	210, 156	F1, RF6	L3	4	Given two loan repayment options with different annual interest rates and repayment terms but equal compounding frequencies, create an exponential function that models each option and use the functions to determine which option is most favourable for the borrower.