Mathematics 8

Curriculum Guide 2015

Newfoundland Labrador

Education and Early Childhood Development

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Acknowledgements

The Department of Education would like to thank Western and Northern Canadian Protocol (WNCP) for Collaboration in Education, *The Common Curriculum Framework for K-9 Mathematics* - May 2006 and *The Common Curriculum Framework for Grades 10-12* - January 2008. Reproduced (and/or adapted) by permission. All rights reserved.

We would also like to thank the provincial Grade 8 Mathematics curriculum committee, the Alberta Department of Education, the New Brunswick Department of Education, and the following people for their contribution:

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INTRODUCTION

Background

The curriculum guide communicates high expectations for students.

The Mathematics curriculum guides for Newfoundland and Labrador have been derived from *The Common Curriculum Framework for K-9 Mathematics: Western and Northern Canadian Protocol*, 2006. These guides incorporate the conceptual framework for Grades Kindergarten to Grade 9 Mathematics and the general outcomes, specific outcomes and achievement indicators established in the common curriculum framework. They also include suggestions for teaching and learning, suggested assessment strategies, and an identification of the associated resource match between the curriculum and authorized, as well as recommended, resource materials.

Mathematics 8 was originally implemented in 2009.

Beliefs About Students and Mathematics

Mathematical understanding is fostered when students build on their own experiences and prior knowledge

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in developing mathematical literacy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. Through the use of manipulatives and a variety of pedagogical approaches, teachers can address the diverse learning styles, cultural backgrounds and developmental stages of students, and enhance within them the formation of sound, transferable mathematical understandings. Students at all levels benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions provide essential links among concrete, pictorial and symbolic representations of mathematical concepts.

The learning environment should value and respect the diversity of students' experiences and ways of thinking, so that students feel comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. They must come to understand that it is acceptable to solve problems in a variety of ways and that a variety of solutions may be acceptable.

Affective Domain

To experience success, students must learn to set achievable goals and assess themselves as they work toward these goals.

A positive attitude is an important aspect of the affective domain and has a profound impact on learning. Environments that create a sense of belonging, encourage risk taking and provide opportunities for success help develop and maintain positive attitudes and self-confidence within students. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must learn to set achievable goals and assess themselves as they work toward these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting, asssessing and revising personal goals.

Goals For Students

Mathematics education must prepare students to use mathematics confidently to solve problems.

The main goals of mathematics education are to prepare students to:

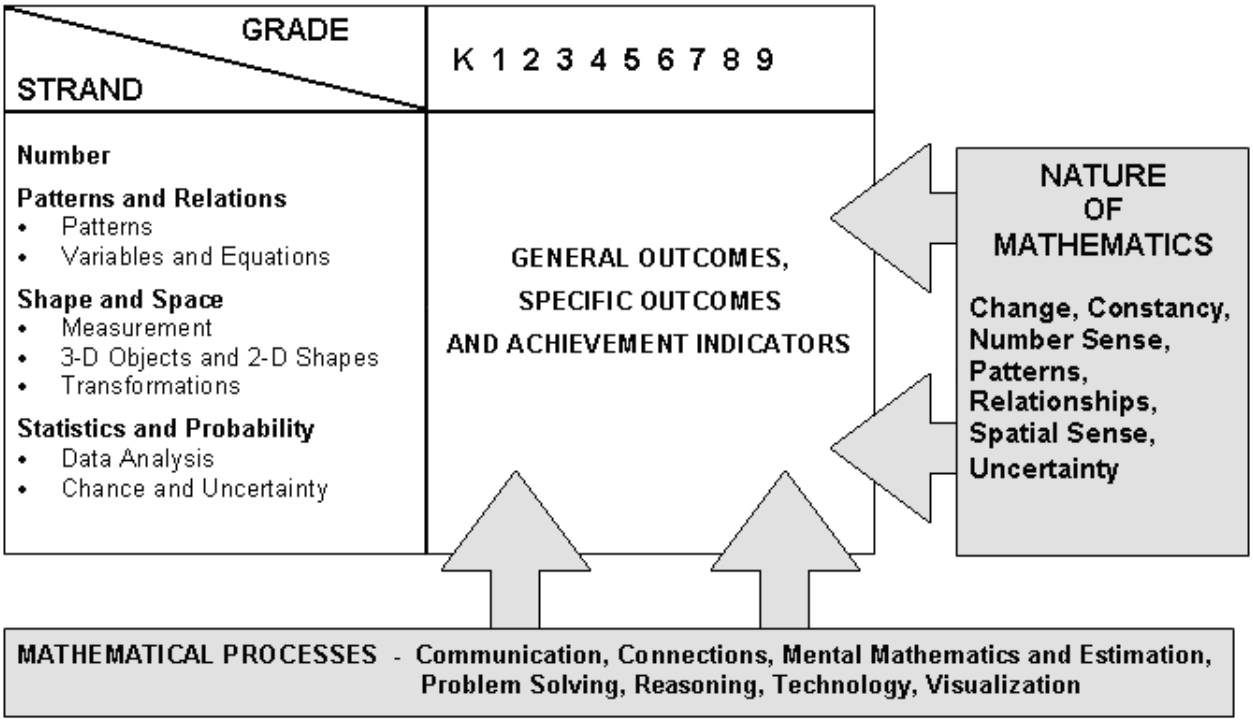
- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity.

CONCEPTUAL FRAMEWORK FOR K - 9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



Mathematical Processes

- *Communication [C]*
- *Connections [CN]*
- *Mental Mathematics and Estimation [ME]*
- *Problem Solving [PS]*
- *Reasoning [R]*
- *Technology [T]*
- *Visualization [V]*

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and embrace lifelong learning in mathematics.
Students are expected to:

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and for solving problems
- develop visualization skills to assist in processing information, making connections and solving problems.

This curriculum guide incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students must be able to communicate mathematical ideas in a variety of ways and contexts.

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication helps students make connections among concrete, pictorial, symbolic, oral, written and mental representations of mathematical ideas.

Connections [CN]

Through connections, students begin to view mathematics as useful and relevant.

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. “Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding … Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching” (Caine and Caine, 1991, p.5).

Mental Mathematics and Estimation [ME]

Mental mathematics and estimation are fundamental components of number sense.

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math" (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "... become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001, p. 442).

Mental mathematics "... provides the cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers" (Hope, 1988, p. v).

Estimation is used for determining approximate values or quantities or for determining the reasonableness of calculated values. It often uses benchmarks or referents. Students need to know when to estimate, how to estimate and what strategy to use.

Estimation assists individuals in making mathematical judgements and in developing useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels.

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you know?" or "How could you ...?", the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

A problem-solving activity requires students to determine a way to get from what is known to what is unknown. If students have already been given steps to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly seek and engage in a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics.

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for students to develop their ability to reason. Students can explore and record results, analyze observations, make and test generalizations from patterns, and reach new conclusions by building upon what is already known or assumed to be true.

Reasoning skills allow students to use a logical process to analyze a problem, reach a conclusion and justify or defend that conclusion.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

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Technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- create geometric patterns
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and which estimation strategies to use (Shaw and Cliatt, 1989).

Nature of Mathematics

- *Change*
- *Constancy*
- *Number Sense*
- *Relationships*
- *Patterns*
- *Spatial Sense*
- *Uncertainty*

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this curiculum guide. The components are change, constancy, number sense, patterns, relationships, spatial sense and uncertainty.

Change

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Change is an integral part of mathematics and the learning of mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- the number of a specific colour of beads in each row of a beaded design
- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

Constancy

Constancy is described by the terms stability, conservation, equilibrium, steady state and symmetry.

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS-Benchmarks, 1993, p.270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The ratio of the circumference of a teepee to its diameter is the same regardless of the length of the teepee poles.
- The sum of the interior angles of any triangle is 180°.
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

An intuition about number is the most important foundation of a numerate child.

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p.146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Mastery of number facts is expected to be attained by students as they develop their number sense. This mastery allows for facility with more complex computations but should not be attained at the expense of an understanding of number.

Number sense develops when students connect numbers to their own real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. The evolving number sense typically comes as a by product of learning rather than through direct instruction. It can be developed by providing rich mathematical tasks that allow students to make connections to their own expereinces and their previous learning.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns.

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands of mathematics.

Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with, and understanding of, their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps students develop algebraic thinking, which is foundational for working with more abstract mathematics.

Relationships

Mathematics is used to describe and explain relationships.

Mathematics is one way to describe interconnectedness in a holistic worldview. Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves collecting and analyzing data and describing relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense offers a way to interpret and reflect on the physical environment.

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes and to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of shapes and objects. Spatial sense allows students to make predictions about the results of changing these dimensions; e.g., doubling the length of the side of a square increases the area by a factor of four. Ultimately, spatial sense enables students to communicate about shapes and objects and to create their own representations.

Uncertainty

Uncertainty is an inherent part of making predictions.

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Essential Graduation Learnings

Essential graduation learnings are statements describing the knowledge, skills and attitudes expected of all students who graduate from high school. Essential graduation learnings are cross-curricular in nature and comprise different areas of learning: *aesthetic expression, citizenship, communication, personal development, problem solving, technological competence* and *spiritual and moral development*.

Aesthetic Expression

Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.

Citizenship

Graduates will be able to assess social, cultural, economic and environmental interdependence in a local and global context.

Communication

Graduates will be able to use the listening, viewing, speaking, reading and writing modes of language(s) and mathematical and scientific concepts and symbols to think, learn and communicate effectively.

Personal Development

Graduates will be able to continue to learn and to pursue an active, healthy lifestyle.

Problem Solving

Graduates will be able to use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts.

Technological Competence

Graduates will be able to use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

Spiritual and Moral Development

Essential graduation learnings are statements describing the knowledge, skills and attitudes expected of all students who graduate from high school. Essential graduation learnings are cross-curricular in nature and comprise different areas of learning: *aesthetic expression, citizenship, communication, personal development, problem solving, technological competence* and *spiritual and moral development*.

Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.

Strands

- *Number*
- *Patterns and Relations*
- *Shape and Space*
- *Statistics and Probability*

The learning outcomes in the mathematics program are organized into four strands across the grades K–9. Some strands are further divided into substrands. There is one general outcome per substrand across the grades K–9.

The strands and substrands, including the general outcome for each, follow.

Number

Number

- Develop number sense.

Patterns and Relations

Patterns

- Use patterns to describe the world and to solve problems.

Variables and Equations

- Represent algebraic expressions in multiple ways.

Shape and Space

Measurement

- Use direct and indirect measurement to solve problems.

3-D Objects and 2-D Shapes

- Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

Transformations

- Describe and analyze position and motion of objects and shapes.

Statistics and Probability

Data Analysis

- Collect, display and analyze data to solve problems.

Chance and Uncertainty

- Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

Outcomes and Achievement Indicators

The curriculum is stated in terms of general outcomes, specific outcomes and achievement indicators.

General Outcomes

General outcomes are overarching statements about what students are expected to learn in each course.

Specific Outcomes

Specific outcomes are statements that identify the specific skills, understanding and knowledge that students are required to attain by the end of a given grade.

In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are not requirements that must be addressed to fully meet the learning outcome.

Achievement Indicators

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome.

Specific curriculum outcomes represent the means by which students work towards accomplishing the general outcomes and ultimately, the essential graduation learnings.

Summary

The conceptual framework for K - Grade 9 Mathematics (p. 3) describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed. The components are not meant to stand alone. Activities that take place in the mathematics classroom should result from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between topics.

ASSESSMENT AND EVALUATION

Purposes of Assessment

What learning is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others about what is really valued.

Assessment techniques are used to gather information for evaluation. Information gathered through assessment helps teachers determine students' strengths and needs in their achievement of mathematics and guides future instructional approaches.

Teachers are encouraged to be flexible in assessing the learning success of all students and to seek diverse ways in which students might demonstrate what they know and are able to do.

Evaluation involves the weighing of the assessment information against a standard in order to make an evaluation or judgment about student achievement.

Assessment has three interrelated purposes:

- assessment *for* learning to guide and inform instruction;
- assessment *as* learning to involve students in self-assessment and setting goals for their own learning; and
- assessment *of* learning to make judgements about student performance in relation to curriculum outcomes.

Assessment *for* Learning

Assessment *for* learning involves frequent, interactive assessments designed to make student understanding visible. This enables teachers to identify learning needs and adjust teaching accordingly. It is an ongoing process of teaching and learning.

Assessment *for* learning:

- requires the collection of data from a range of assessments as investigative tools to find out as much as possible about what students know
- provides descriptive, specific and instructive feedback to students and parents regarding the next stage of learning
- actively engages students in their own learning as they assess themselves and understand how to improve performance.

Assessment *as* Learning

Assessment *as* learning actively involves students' reflection on their learning and monitoring of their own progress. It focuses on the role of the student as the critical connector between assessment and learning, thereby developing and supporting metacognition in students.

Assessment *as* learning:

- supports students in critically analysing their learning related to learning outcomes
- prompts students to consider how they can continue to improve their learning
- enables students to use information gathered to make adaptations to their learning processes and to develop new understandings.

Assessment *of* Learning

Assessment *of* learning involves strategies to confirm what students know, demonstrate whether or not they have met curriculum outcomes, or to certify proficiency and make decisions about students' future learning needs. Assessment *of* learning occurs at the end of a learning experience that contributes directly to reported results.

Traditionally, teachers relied on this type of assessment to make judgments about student performance by measuring learning after the fact and then reporting it to others. Used in conjunction with the other assessment processes previously outlined, however, assessment *of* learning is strengthened.

Assessment *of* learning:

- provides opportunities to report evidence to date of student achievement in relation to learning outcomes, to parents/guardians and other stakeholders
- confirms what students know and can do
- occurs at the end of a learning experience using a variety of tools.

Because the consequences of assessment *of* learning are often far-reaching, teachers have the responsibility of reporting student learning accurately and fairly, based on evidence obtained from a variety of contexts and applications.

Assessment Strategies

Assessment techniques should match the style of learning and instruction employed. Several options are suggested in this curriculum guide from which teachers may choose, depending on the curriculum outcomes, the class and school/district policies.

Observation (formal or informal)

This technique provides a way of gathering information fairly quickly while a lesson is in progress. When used formally, the student(s) would be aware of the observation and the criteria being assessed. Informally, it could be a frequent, but brief, check on a given criterion. Observation may offer information about the participation level of a student for a given task, use of a concrete model or application of a given process. The results may be recorded in the form of checklists, rating scales or brief written notes. It is important to plan in order that specific criteria are identified, suitable recording forms are ready, and all students are observed within a reasonable period of time.

Performance

This curriculum encourages learning through active participation. Many of the curriculum outcomes promote skills and their applications. In order for students to appreciate the importance of skill development, it is important that assessment provide feedback on the various skills. These may be the correct manner in which to use a manipulative, the ability to interpret and follow instructions, or to research, organize and present information. Assessing performance is most often achieved through observing the process.

Paper and Pencil

These techniques can be formative or summative. Whether as part of learning, or a final statement, students should know the expectations for the exercise and how it will be assessed. Written assignments and tests can be used to assess knowledge, understanding and application of concepts. They are less successful at assessing processes and attitudes. The purpose of the assessment should determine what form of paper and pencil exercise is used.

Journal

Journals provide an opportunity for students to express thoughts and ideas in a reflective way. By recording feelings, perceptions of success, and responses to new concepts, a student may be helped to identify his or her most effective learning style. Knowing how to learn in an effective way is powerful information. Journal entries also give indicators of developing attitudes to mathematical concepts, processes and skills, and how these may be applied in the context of society. Self-assessment, through a journal, permits a student to consider strengths and weaknesses, attitudes, interests and new ideas. Developing patterns may help in career decisions and choices of further study.

Interview

This curriculum promotes understanding and applying mathematics concepts. Interviewing a student allows the teacher to confirm that learning has taken place beyond simple factual recall. Discussion allows a student to display an ability to use information and clarify understanding. Interviews may be a brief discussion between teacher and student or they may be more extensive. Such conferences allow students to be proactive in displaying understanding. It is helpful for students to know which criteria will be used to assess formal interviews. This assessment technique provides an opportunity to students whose verbal presentation skills are stronger than their written skills.

Presentation

The curriculum includes outcomes that require students to analyze and interpret information, to be able to work in teams, and to communicate information. These activities are best displayed and assessed through presentations. These can be given orally, in written/pictorial form, by project summary, or by using electronic systems such as video or computer software. Whatever the level of complexity, or format used, it is important to consider the curriculum outcomes as a guide to assessing the presentation. The outcomes indicate the process, concepts and context for which a presentation is made.

Portfolio

Portfolios offer another option for assessing student progress in meeting curriculum outcomes over a more extended period of time. This form of assessment allows the student to be central to the process. There are decisions about the portfolio, and its contents, which can be made by the student. What is placed in the portfolio, the criteria for selection, how the portfolio is used, how and where it is stored, and how it is evaluated are some of the questions to consider when planning to collect and display student work in this way. The portfolio should provide a long-term record of growth in learning and skills. This record of growth is important for individual reflection and self-assessment, but it is also important to share with others. For all students, it is exciting to review a portfolio and see the record of development over time.

INSTRUCTIONAL FOCUS

Planning for Instruction

Consider the following when planning for instruction:

- Integration of the mathematical processes within each strand is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency and must be integrated throughout the program.
- There is to be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using manipulatives and be developed concretely, pictorially and symbolically.
- Students bring a diversity of learning styles and cultural backgrounds to the classroom. They will be at varying developmental stages.

Teaching Sequence

The curriculum guide for Mathematics 8 is organized by units. This is only a suggested teaching order for the course. There are a number of combinations of sequences that would be appropriate.

Each two page spread lists the topic, general outcome, and specific outcome.

Instructional Time per Unit

The suggested number of weeks of instruction per unit is listed in the guide at the beginning of each unit. The number of suggested weeks includes time for completing assessment activities, reviewing and evaluating. The timelines at the beginning of each unit are provided to assist in planning. The use of these timelines is not mandatory. However, it is mandatory that all outcomes are taught during the school year, so a long term plan is advised. Teaching of the outcomes is ongoing, and may be revisited as necessary.

Resources

The resource selected by Newfoundland and Labrador for students and teachers is *Math Makes Sense 8* (Pearson). Column four of the curriculum guide references *Math Makes Sense 8* for this reason. Teachers may use any other resource, or combination of resources to meet the required specific outcomes.

GENERAL AND SPECIFIC OUTCOMES

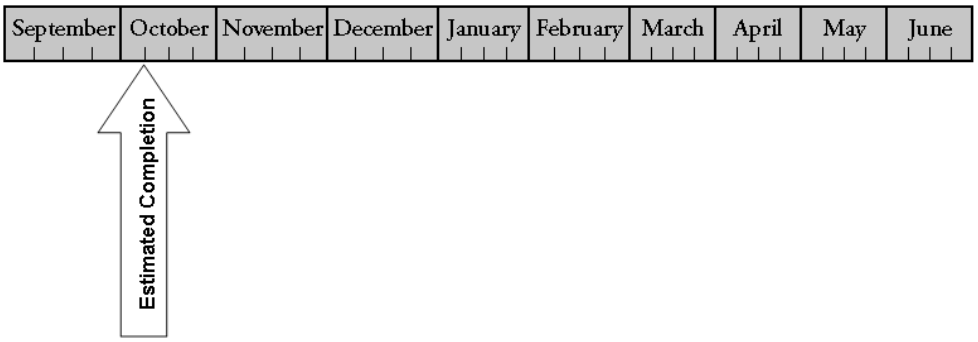
GENERAL AND SPECIFIC OUTCOMES WITH ACHIEVEMENT INDICATORS (pages 19-212)

This section presents general and specific outcomes with corresponding achievement indicators and is organized by unit. The list of indicators contained in this section is not intended to be exhaustive but rather to provide teachers with examples of evidence of understanding that may be used to determine whether or not students have achieved a given specific outcome. Teachers may use any number of these indicators or choose to use other indicators as evidence that the desired learning has been achieved. Achievement indicators should also help teachers form a clear picture of the intent and scope of each specific outcome.

Mathematics 8 is organized into eight units: *Square Roots and the Pythagorean Theorem, Integers, Operations with Fractions, Measuring Prisms and Cylinders, Percent, Ratio, and Rate, Linear Equations and Graphing, Data Analysis and Probability,* and *Geometry.*

Square Roots and the Pythagorean Theorem

Suggested Time: 4 Weeks



Unit Overview

Focus and Context

In this unit, students will explore perfect squares and square roots. They will relate the side lengths of squares to square roots and the areas to perfect square numbers. They will determine if numbers are perfect squares through the use concrete materials, such as grid paper or pattern blocks, as well as using prime factorization and examining the factors of the given numbers. Students will estimate the square root of a non-perfect square to the nearest tenth.

Students will develop the Pythagorean Theorem through exploration and use their previously learned knowledge of squares and square roots to determine the lengths of sides of right triangles.

The Pythagorean Theorem is used in many fields including architecture and construction, navigation, and surveying. Exploring such situations will strengthen student understanding of the concepts in this unit.

Outcomes Framework

GCO
Develop Number Sense

SCO 8N1

Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers).

SCO 8N2

Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).

GCO
Use direct and indirect measurement to solve problems.

SCO 8SS1

Develop and apply the Pythagorean theorem to solve problems.

SCO Continuum

Grade 7	Grade 8	Grade 9
Number		
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
7N1 Determine and explain why a number is divisible by 2, 3, 4, 5, 6, 8, 9, or 10, and why a number cannot be divided by 0. [C, R]	8N1 Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V] 8N2 Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [C, CN, ME, R, T]	9N5 Determine the square root of positive rational numbers that are perfect squares. [C, CN, PS, R, T] 9N6 Determine an approximate square root of positive rational numbers that are non-perfect squares. [C, CN, PS, R, T]
Shape and Space (Measurement)		
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
	8SS1 Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V]	9SS1 Solve problems and justify the solution strategy using the following circle properties: • the perpendicular from the centre of a circle to a chord bisects the chord • the measure of the central angle is equal to twice the measure of the inscribed angles subtended by the same arc • the inscribed angles subtended by the same arc are congruent • a tangent to a circle is perpendicular to the radius at the point of tangency. [C, CN, PS, R, T, V]

Mathematical Processes

[C] Communication	**[PS] Problem Solving**
[CN] Connections	**[R] Reasoning**
[ME] Mental Mathematics and Estimation	**[T] Technology**
	[V] Visualization

Number

Specific Outcomes

Students will be expected to

8N1 Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers).

[C, CN, R, V]

Achievement Indicators:

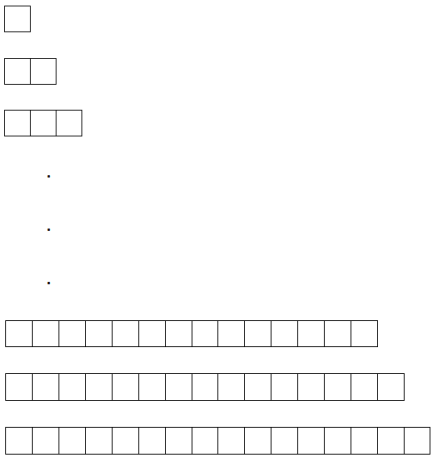
8N1.1 *Represent a given perfect square as a square region using materials, such as grid paper or square shapes.*

8N1.2 *Determine whether or not a given number is a perfect square using materials and strategies, such as square shapes, grid paper or prime factorization, and explain the reasoning.*

Suggestions for Teaching and Learning

In Grade 6, students developed and applied the formula for the area of a rectangle. In Grade 7, they extended this knowledge to include the areas of triangles, parallelograms, and circles. They are familiar with using square units to represent area. In this unit, students will use their knowledge of squares to develop an understanding of perfect square numbers and square roots.

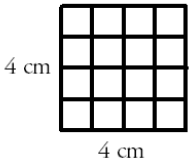
To introduce perfect square numbers, teachers could give students a set of strips such as the ones shown below (alternatively, students could use pattern blocks):



Students should use the smaller blocks in each strip to try to create a square with each area. Ask them questions such as:

- Which strips were you able to rearrange into a square?
- What is the side length of each square you made?
- How is the side length of the square related to its area?

Students should conclude that a perfect square is the product of two identical factors. The diagram below, for example, shows that 16 is a perfect square.



The area of this square is 4 cm × 4 cm = 16 cm^2. Teachers should emphasize the relationship between the side length of the square and the area.

General Outcome: Develop Number Sense.

Suggested Assessment Strategies

Performance

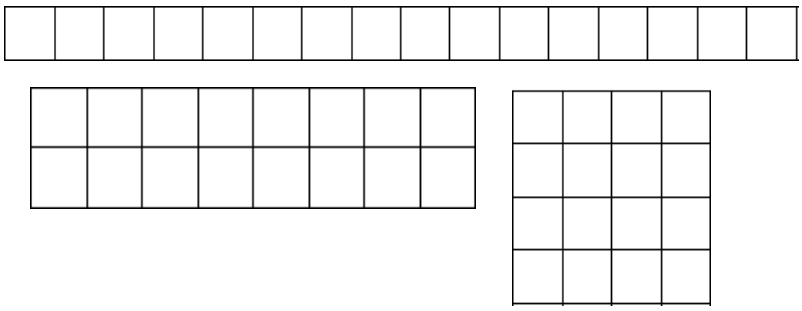
- For the activity Commit and Toss, ask students to write a whole number on a piece of paper, and indicate whether or not their number is a perfect square. They should provide a justification. Students crumple their papers into a ball and toss the paper balls around the room until their teacher tells them to stop and pick up or hold on to one paper. Students share the answer and explanation that is described on the paper they are holding.

  (8N1.1)

- Ask students to represent given perfect squares (16 and 25, for example) using 2-D square tiles. They should identify the factors of each perfect square.

  (8N1.1)

- Students could use grid paper to create as many rectangles as possible with an area of 16 cm^2. For example:



Ask them to answer the following:

(i) Is 16 a perfect square?

(ii) How did your created rectangles help you determine this?

Students could also complete this activity for non-square numbers. This will strengthen their understanding of perfect square numbers.

(8N1.1, 8N1.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 1.1: Square Numbers and Area Models

ProGuide: pp. 4-8

Master 1.15

CD-ROM: Master 1.24

Student Book (SB): pp. 6-10

Practice and Homework Book (PB): pp. 4-6

Number

Specific Outcomes

Students will be expected to

8N1 Continued...

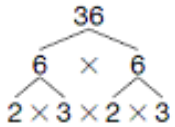
Achievement Indicators:

8N1.1 *(Continued) Represent a given perfect square as a square region using materials, such as grid paper or square shapes.*

8N1.2 *(Continued) Determine whether or not a given number is a perfect square using materials and strategies, such as square shapes, grid paper or prime factorization, and explain the reasoning.*

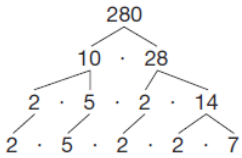
Suggestions for Teaching and Learning

In Grade 6, students completed factor trees to determine the prime factors of a whole number. Prime factorization can help them determine whether or not a given number, such as 36, is a perfect square. Students should construct a factor tree for the given number:



These factors can be arranged into two equal groups: 36 = (2×3)×(2×3). If they think about the two equal groups as two identical factors, students should conclude that 36 is a perfect square.

Students should also explore examples that are non-perfect squares. If they construct the factor tree for 280, for example, students should recognize that the prime factors cannot be arranged into two equal groups.



280 = 2×2×2×5×7

They should conclude that 280 is not a perfect square. Through various explorations, some students may recognize that a perfect square requires that each prime factor occur an even number of times.

Students could also list the factors of a given number to determine whether it is a perfect square. The factors for 36, for example, are shown below:

$1 \times 36 = 36$

$2 \times 18 = 36$

$3 \times 12 = 36$

$4 \times 9 = 36$

$6 \times 6 = 36$

Students should recognize that a square can be constructed having a side length of 6. They should also conclude that a square number has an odd number of factors.

General Outcome: Develop Number Sense.

Suggested Assessment Strategies

Journal

- Ask students to respond to the following:
  - (i) Is there a perfect square between 900 and 961? Explain. Would you use prime factorization to determine whether 900 is a perfect square? Why or why not?

    (8N1.2)
  - (ii) Explain, using an example, why a square number has an odd number of factors.

    (8N1.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 1.1: Square Numbers and Area Models

ProGuide: pp. 4-8

Master 1.15

CD-ROM: Master 1.24

SB: pp. 6-10

PB: pp. 4-6

Number

Specific Outcomes

Students will be expected to

8N1 Continued...

Achievement Indicators:

8N1.3 *Determine a square root of a given perfect square and record it symbolically.*

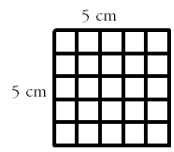
8N1.4 *Determine the factors of a given perfect square, and explain why one of the factors is a square root and the others are not.*

8N1.5 *Determine the square of a given number.*

Suggestions for Teaching and Learning

A square root is a number which, when multiplied by itself, results in a given number. The square root of 25, for example, is 5 since $5 \times 5 = 25$. While each perfect square has a positive and a negative square root, the focus for Mathematics 8 is on the principal (positive) square root. Introduce students to the square root symbol, $\sqrt{\ }$, which is used to represent positive square roots.

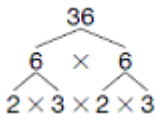
Students should now make the connection between the side length of their perfect square representations and the square root. When asked to determine $\sqrt{25}$, for example, they could use pattern blocks or grid paper to construct a square having an area of 25 square units. The side length of their square is the square root:



$$\sqrt{25} = 5$$

To verify that the result is correct, students should multiply the number by itself. While students do not explore powers until Grade 9, a discussion around an exponent of 2 is essential. Emphasize that multiplying a number by itself is called squaring the number. Squaring a number can be written using an exponent of 2 (e.g., $5 \times 5 = 5^2$).

Prime factorization should also be used to determine the square root of a given perfect square. After arranging the prime factors into equal groups, the square root is the product of the factors contained in one of the groups. Consider 36, for example.



$$36 = (2 \cdot 3) \times (2 \cdot 3)$$

$$\sqrt{36} = 2 \times 3 = 6$$

The square root of 36 is the product of 2 and 3. Students should be encouraged to square 6 to verify their answer.

Earlier, students listed the factors of a given number to determine whether it was a perfect square. They should now use this strategy to determine its square root. Consider 36, for example:

$1 \times 36 = 36$
$2 \times 18 = 36$
$3 \times 12 = 36$
$4 \times 9 = 36$
$6 \times 6 = 36$

1, 2, 3, 4, 5, 6, 9, 12, 18, 36

General Outcome: Develop Number Sense.

Suggested Assessment Strategies

Journal

- The factors of 81 are 1, 3, 9, 27, and 81. Ask students to use pictures, numbers and words to explain how they know if 81 is a perfect square and, if so, which factor is the square root of 81. (8N1.2, 8N1.3, 8N1.4)
- 361 has only 3 factors: 1, 19, and 361. Ask students to explain how how they know that 361 is a perfect square. (8N1.2)

Performance

- Ask students to play the game *Find the Matching Squares.* (8N1.3, 8N1.5)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 1.1: Square Numbers and Area Models

Lesson 1.2: Squares and Square Roots

ProGuide: pp. 4-8, 9-14

CD-ROM: Unit 1 Masters

SB: pp. 6-10, 11-16

PB: pp. 7-8

Suggested Resource

http://www.k12pl.nl.ca/curr/k-6/math/grade-8/links/unit1.html

- Weblink for the *Find the Matching Squares* game

Number

Specific Outcomes

Students will be expected to

8N1 Continued...

Achievement Indicators:

8N1.3 *(Continued) Determine a square root of a given perfect square and record it symbolically.*

8N1.4 *(Continued) Determine the factors of a given perfect square, and explain why one of the factors is a square root and the others are not.*

8N1.5 *(Continued) Determine the square of a given number.*

Suggestions for Teaching and Learning

Students should recognize that the square has a side length of 6. The square root of 36 is 6, or $\sqrt{36} = 6$. Alternatively, students can list the factors in ascending order. The middle factor represents the square root.

Using various strategies, students should be able to make statements such as the following:

$\sqrt{49} = 7$ or $7^2 = 49$

$\sqrt{100} = 10$ or $10^2 = 100$

$\sqrt{144} = 12$ or $12^2 = 144$

None of the previous strategies for identifying perfect squares and determining square roots require the use of a calculator. Calculator usage can be discussed, but the focus should be on non-technological techniques.

A discussion of inverse operations is appropriate. An inverse operation reverses the result of another operation. Relating the concept of an inverse to non-mathematical situations may help students gain a better understanding of inverse operations. Ask students, for example, to identify the inverse of walking up a set of stairs. Many students will recognize that the inverse would be walking down a set of stairs. Ask students questions such as:

- What is the inverse operation for addition?
- What is the inverse operation for multiplication?
- What is the inverse operation for squaring a number?

Students should realize that squaring a number and taking a square root are inverse operations. Using the concept of inverse operations, students should be able to calculate mentally the value of expressions such as $\sqrt{3^2}$ or $\sqrt{14 \times 14}$.

Students should solve a variety of problems that require them to square a number or determine its square root. Given the area of a square, for example, they should determine the side length. Similarly, if given the side length of a square, they should determine its area.

Students should identify each of the perfect squares from 1 through 144 and their associated square roots. This will help them determine approximate square roots for numbers that are not perfect squares and to determine the square roots of larger numbers. Knowing that the square root of 25 is 5, for example, can be used to determine the square root of 2500 (50). This recognition is also important in future mathematics courses, such as Mathematics 1201, where students will simplify radicals by extracting the largest perfect square.

General Outcome: Develop Number Sense.

Suggested Assessment Strategies

Interview

- Ask students to determine each of the following:
  - (i) $\sqrt{100}$
  - (ii) $\sqrt{100^2}$
  - (iii) the square of 10
  - (iv) the square of $\sqrt{100}$

  (8N1.3, 8N1.5)
- Ask students to respond to the following:

  Jim measures each side of his mother's vegetable garden to be 3.2 m. Explain how Jim could determine the area of the vegetable garden.

  (8N1.5)

Paper and Pencil

- Ask students to determine the the factors of each perfect square, and use them to determine the square roots.

  (i) $\sqrt{9}$ (ii) $\sqrt{25}$ (iii) $\sqrt{81}$ (iv) $\sqrt{169}$
  (v) $\sqrt{36}$ (vi) $\sqrt{16}$ (vii) $\sqrt{64}$

  (8N1.4)
- Ask students to solve each of the following problems:
  - (i) Ruth wants a large picture window put in the living room of her new house. The window is to be square with an area of 49 square feet. How long should each side of the window be?

    (8N1.4)
  - (ii) The side length of a square is 11 cm. What is the area of the square?

    (8N1.5)
  - (iii) A miniature portrait is square and has an area of 196 square centimeters. How long is each side of the portrait?

    (8N1.4)

Journal

- Ask students to explain why prime factorization cannot be used to find the square root for numbers that are not perfect squares.

  (8N1.3)
- Ask students to explain why, when the factors of a square number are listed in ascending order, the middle factor represents the square root.

  (8N1.4)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 1.2: Squares and Square Roots

Lesson 1.3: Measuring Line Segments

ProGuide: pp. 9-14, 15-19

Master 1.16

CD-ROM: Master 1

SB: pp. 11-13, 15-16, 17-21

PB: pp. 7-8, 9-10

Number

Specific Outcomes

Students will be expected to

8N2 Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers).

[C, CN, ME, R, T]

Suggestions for Teaching and Learning

Once students have become comfortable with determining the square root of perfect squares, they should determine the approximate square root of numbers that are not perfect squares, to the nearest tenth. Students should use the square roots of perfect squares as benchmarks to determine the square roots of numbers that are not perfect squares. It is important that they understand the difference between an exact value and an approximation.

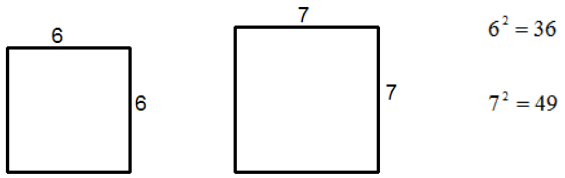
Achievement Indicators:

8N2.1 *Estimate, or approximate, a square root of a given number that is not a perfect square using the roots of perfect squares as benchmarks.*

Remind students of common perfect squares between 1 and 144 and their associated square roots. Fluency with these perfect squares will help students identify appropriate benchmarks. To estimate $\sqrt{55}$ to one decimal place, students should recognize that 55 lies between the perfect squares 49 and 64. The square root of 55, therefore, must be between 7 and 8. Since 55 is closer to 49, it follows that the square root of 55 is closer to 7. Students might suggest 7.3 as an approximation. They should check this value by squaring it: 7.3 × 7.3 = 53.29. Since this value is less than 55, they should refine their estimate by increasing their original value by one tenth: 7.4 × 7.4 = 54.76 . This approximation is closer to 55. Some students may suggest trying 7.5: 7.5 × 7.5 = 56.25. Based on the values obtained, they should conclude that the best approximation for $\sqrt{55}$ is 7.4. This can be written as $\sqrt{55} \approx 7.4$.

8N2.2 *Identify a whole number whose square root lies between two given numbers.*

Students should also be asked to identify a whole number, for example, whose square root lies between 6 and 7. Remind students that the square root is the side length of the square representation.



Since 36 and 49 are the corresponding perfect squares, any whole number between 36 and 49 will have a square root between 6 and 7. Students should realize that there are many answers to this type of question.

General Outcome: Develop Number Sense.

Suggested Assessment Strategies

Interview

- Ask students to respond to the following:
  - (i) If a whole number has an approximate square root of 5.66, is the whole number closer to 25 or 36? How do you know? (8N2.1)
  - (ii) In your own words, explain how you would estimate the square root of 75. (8N2.1)

Journal

- Ask students to respond to the following:

  How could you explain to a classmate that $\sqrt{10}$ is between 3 and 4? (8N2.1)

Performance

- Display a pair of whole numbers. Ask students to identify a whole number whose square root lies between the two given numbers. Students should write their answers on an index card, or piece of paper, and hold them up as a group. Discuss why all of the answers are not the same. (8N2.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 1.4: Estimating Square Roots

Technology: Investigating Square Roots with a Calculator

ProGuide: pp. 24-27

CD-ROM: Master 1.27

SB: pp. 29

PB: pp. 11-12

Number

Specific Outcomes

Students will be expected to

8N2 Continued...

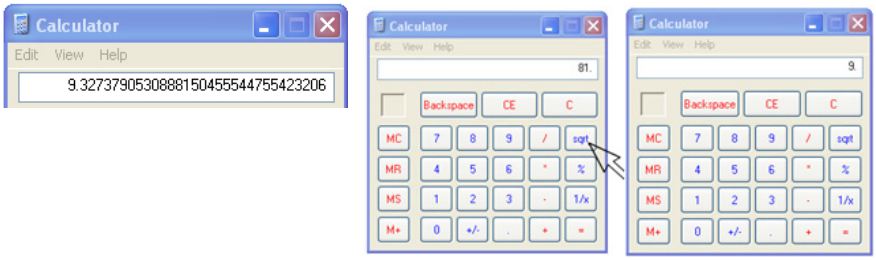
Achievement Indicators:

8N2.3 *Approximate a square root of a given number that is not a perfect square using technology, e.g. calculator, computer.*

8N2.4 *Explain why the square root of a number shown on a calculator may be an approximation.*

Suggestions for Teaching and Learning

Calculators provide an efficient means of approximating square roots. Encourage students to check previous approximations using their calculators. It also provides a good opportunity to emphasize the difference between exact and approximate values. When students use their calculators to determine an approximation for a number that is not a perfect square the result is a decimal that never terminates nor repeats. Ask students to compare, for example, the results for $\sqrt{87}$ and $\sqrt{81}$.



Since there is such variety with calculators, it is important that teachers provide guidance as students determine their approximations using technology.

Square roots can be approximated with calculators to any requested number of decimal places using rounding strategies.

General Outcome: Develop Number Sense.

Suggested Assessment Strategies

Performance

- Ask students to complete the following:
  (i) Estimate $\sqrt{10}$, to the nearest tenth.
  (ii) Use your calculator to square your estimate.
  (iii) Is your estimate less than 10 or greater than 10? How can you revise your estimate to determine the best approximation?
  (iv) Use your calcuator to determine $\sqrt{10}$. Round your answer to the nearest thousandth.

  (8N2.1, 8N2.3)

- Ask students to use a calculator to determine the following square roots and identify which of the numbers are perfect squares:
  (i) $\sqrt{1600}$
  (ii) $\sqrt{1681}$
  (iii) $\sqrt{1212}$
  (iv) $\sqrt{1000}$
  (v) $\sqrt{2468}$

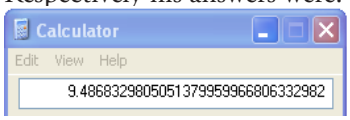
  (8N2.3)

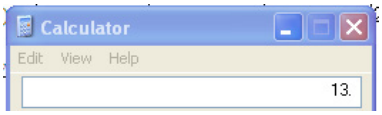
Journal

- Emily wanted to find the area of a rectangle having a length of 9 cm. The width of the rectangle was the same as the length of the sides of an adjacent square. The area of the square was 38 cm^2. To find the side lengths of the square, she used her calculator: $\sqrt{38} = 6.2$. She concluded that the area of the rectangle was 9 cm × 6.2 cm = 55.8 cm^2. Andrew solved the same problem by evaluating $\sqrt{38} \times 9 = 55.5$ cm^2. Why did they get different results?

  (8N2.3, 8N2.4)

- Kevin used his calculator to find the square roots of 90 and 169. Respectively his answers were:





  Are these exact answers? Explain your reasoning.

  (8N2.3, 8N2.4)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 1.4: Estimating Square Roots

Technology: Investigating Square Roots with a Calculator

ProGuide: pp. 24-27

CD-ROM: Master 1.27

SB: pp. 29

PB: pp. 11-12

Shape and Space

Specific Outcomes

Students will be expected to

8SS1 Develop and apply the Pythagorean theorem to solve problems.

[CN, PS, R, T, V]

Achievement Indicator:

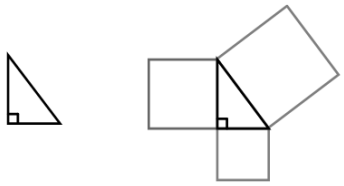
8SS1.1 *Model and explain the Pythagorean theorem concretely, pictorially or using technology.*

Suggestions for Teaching and Learning

Pythagoras was born in the late 6th century BC on the island of Samos. He was a Greek philosopher and religious leader who was responsible for important developments in mathematics, astronomy and the theory of music.

It is believed that the Egyptians and other ancient cultures used a 3-4-5 rule in construction. In Egypt, Pythagoras studied with the engineers, known as "rope-stretchers", who built the pyramids. They had a rope with 12 evenly spaced knots. If the rope was pegged to the ground in the dimensions 3-4-5, a right triangle would result. This enabled them to lay the foundations of their buildings accurately. Pythagoras generalized this relationship and is credited with its first geometrical demonstration.

Students should be given the opportunity to develop the Pythagorean theorem. Teachers could provide students with a right triangle and ask them to create three squares formed by the sides of the triangle:

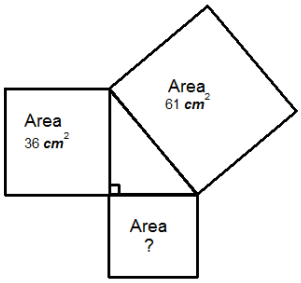


Ask students questions such as:

- What is the area of each square?
- What is the relationship between the areas of the two smaller squares and the square on the longer size?

Students should recognize that the sum of the areas of the two smaller squares is equal to the area of the larger square. This is the Pythagorean theorem.

Teachers could also provide students with a variety of right triangles with two of the three areas given and ask them to determine the missing area. They may find it more challenging to determine the area of one of the smaller squares. For example:

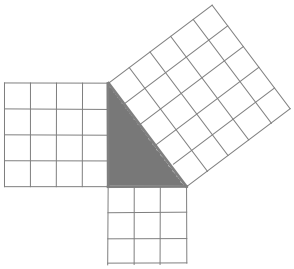


General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Provide groups of students with a variety of right triangles which have whole number sides, such as the 3 cm – 4 cm – 5 cm triangle, the 6 cm – 8 cm – 10 cm triangle, or the 5 cm – 12 cm –13 cm triangle (or ask students to draw such triangles). Have students cut out squares from centimetre grid paper so the sides of each square are the same as the side lengths for each triangle. Place the squares on the sides of the triangle as shown. Find the area of each square. Ask students what they notice.



(8SS1.1)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 1.5: The Pythagorean Theorem

Technology: Verifying the Pythagorean Theorem

ProGuide: pp. 29-36

SB: pp. 31-38

Shape and Space

Specific Outcomes

Students will be expected to

8SS1 Continued...

Achievement Indicators:

8SS1.1 *(Continued) Model and explain the Pythagorean theorem concretely, pictorially or using technology.*

Suggestions for Teaching and Learning

Encourage students to identify which areas are provided to avoid the common error of always adding the two given areas. They should recognize that, in this case, the areas of the larger square and one of the smaller squares are provided. To determine the missing area students may think "36 + ___ = 61". Others may suggest subtracting 36 from 61.

Discuss with students the conventions associated with right triangles:

- The two shorter sides that form the right angle are called legs. They are often denoted by the letters a and b.
- The longest side, opposite the right angle, is called the hypotenuse. It is denoted by the letter h.

Using this notation, students should relate the areas of the two smaller squares to the expressions a^2 and b^2. The area of the larger square can be represented by c^2. The Pythagorean theorem can be summarized as: $a^2 + b^2 = c^2$. Alternatively, it can be expressed as: $(\text{leg})^2 + (\text{leg})^2 = (\text{hyp})^2$. Students should work with triangles that are labelled using variables other than a, b, and c.

8SS1.2 *Determine the measure of the third side of a right triangle, given the measures of the other two sides, to solve a given problem.*

Students should solve a variety of problems in which they have to determine either the hypotenuse or one of the legs of a right triangle. Present diagrams of right triangles in various orientations to strengthen student understanding of the Pythagorean theorem. Encourage them to identify which sides are provided and substitute them into the Pythagorean theorem. To determine the measure of a missing leg, they should apply the preservation of equality to solve the resulting equation. Alternatively, students could rearrange the Pythagorean theorem.

$$(\text{leg}_1)^2 + (\text{leg}_2)^2 = (\text{hyp})^2$$

$$(\text{leg}_1)^2 + (\text{leg}_2)^2 - (\text{leg}_2)^2 = (\text{hyp})^2 - (\text{leg}_2)^2$$

$$(\text{leg}_1)^2 = (\text{hyp})^2 - (\text{leg}_2)^2$$

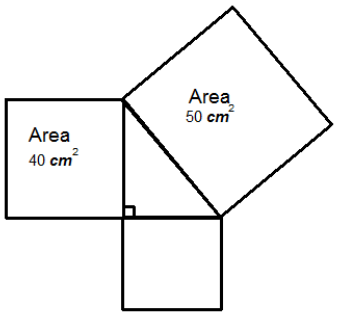
Whether formula rearrangement is used first, or side lengths are substituted into the Pythagorean theorem immediately, the procedure reiterates the concept of preservation of equality that students were introduced to in Mathematics 6.

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Paper and Pencil

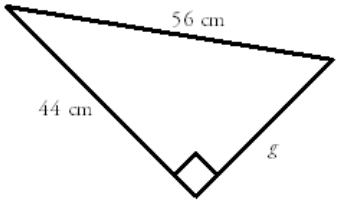
- Ask students to determine the area of the remaining square in the diagram below:

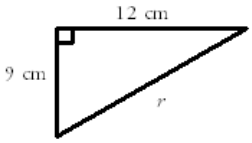


They should explain how they determined their answer.

(8SS1.1)

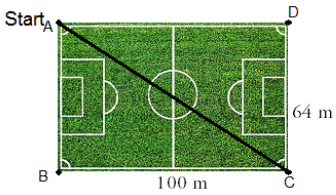
- Ask students to determine the missing side lengths in each of the right triangles shown:





(8SS1.2)

- Rebecca and Julia have to run one lap around the soccer field during practice.



They begin at the corner of the field, as shown in the above diagram. They both run from A to B to C. However, while running Rebecca gets tired and decides to cut across the soccer field (from C to A), while Julia completes her lap around the field. Ask students to determine how much further Julia runs.

(8SS1.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 1.5: The Pythagorean Theorem

Lesson 1.6: Exploring the Pythagorean Theorem

Lesson 1.7: Applying the Pythagorean Theorem

ProGuide: pp. 29-34, 44-49

Master 1.19, 1.21

CD-ROM: Masters 1.28, 1.30

SB: pp. 31-36, 39-51

PB: pp. 13-14, 18-20

Shape and Space

Specific Outcomes

Students will be expected to

8SS1 Continued...

Achievement Indicators:

8SS1.2 *(Continued) Determine the measure of the third side of a right triangle, given the measures of the other two sides, to solve a given problem.*

Suggestions for Teaching and Learning

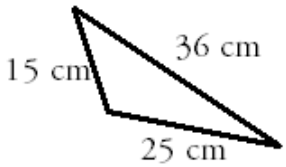
Students should solve a variety of contextual problems requiring the use of the Pythagorean theorem including, but not limited to:

- determining the distance between two points on a coordinate plane
- determining how how high a ladder will reach up a wall
- determining the length of the diagonal of a square or rectangle
- determining the distance from 1st base to 3rd base on a baseball field
- construction problems: ensuring a corner is square, staircase construction, roof rafters

8SS1.3 *Explain, using examples, that the Pythagorean theorem applies only to right triangles.*

8SS1.4 *Determine whether or not a given triangle is a right triangle by applying the Pythagorean theorem.*

It is important for students to understand that the Pythagorean theorem only applies to right triangles. Provide them with a non-right triangle to reinforce this idea.



Through exploration, students should recognize that $15^2 + 25^2 = 850$, while $36^2=1296$.

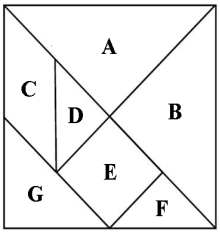
If the sides of a triangle have lengths a, b and c such that $a^2 + b^2 = c^2$, then the triangle is a right triangle. Provide students with various triangles and problem solving situations that require them to determine if a triangle is a right triangle.

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Paper and Pencil

- Ask students to solve the following problems:
  - (i) An airplane is flying at an elevation of 5000 m. The airport is 3 kilometers away from a point directly below the airplane on the ground. How far is the airplane from the airport? (8SS1.2)
  - (ii) Ross has a rectangular garden in his backyard. He measures one side of the garden as 7 m and the diagonal as 11 m. What is the length of the other side of his garden? (8SS1.2)
  - (iii) The dimensions of a rectangular frame are 30 cm × 50 cm . A carpenter wants to put a diagonal brace between two opposite corners of the frame. How long should the brace be? (8SS1.2)
  - (iv) Designate the side length of the square made of the seven Tangram pieces as 1 unit. Using the Pythagorean theorem, determine the lengths of all sides of each of the seven Tangram pieces.



(8SS1.2)

- Ask students to solve the following problem:

  A triangular flower garden is created where two walkways intersect at right angles. The flower garden extends 2 m along one walkway and 1.5 m along the other.

  (i) Steve wants to put a border around the whole garden. What length of border will be required?

  (ii) Steve wishes to spray the area for pests. He needs to know the area of the garden to determine the amount of pesticide to purchase. What is the area of the garden? (8SS1.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 1.5: The Pythagorean Theorem

Lesson 1.6: Exploring the Pythagorean Theorem

Lesson 1.7: Applying the Pythagorean Theorem

ProGuide: pp. 29-34, 44-49

Master 1.19, 1.21

CD-ROM: 1.29

SB: pp. 31-36, 39-51

PB: pp. 13-14, 18-20

Shape and Space

Specific Outcomes

Students will be expected to

8SS1 Continued...
Achievement Indicator:

8SS1.5 *Solve a given problem that involves Pythagorean triples, e.g. 3, 4, 5 or 5, 12, 13.*

Suggestions for Teaching and Learning

A Pythagorean triple consists of three positive integers *a*, *b*, and *c*, such that $a^2 + b^2 = c^2$. Pythagorean triples are commonly written as (*a*, *b*, *c*) or *a*–*b*–*c*. (3, 4, 5), for example, is a Pythgaorean triple. If (*a*, *b*, *c*) is a Pythagorean triple, then (*ka, kb, kc*) is also a Pythagorean triple, where *k* is a positive integer. Since (3, 4, 5) is a triple, for example, (6, 8, 10) and (9, 12, 15) are Pythagorean triples.

Right triangles with non-integer sides do not form Pythagorean triples. For instance, the triangle with sides $a = b = 1$ and $c = \sqrt{2}$ is right, but $\left(1,1,\sqrt{2}\right)$ is not a Pythagorean triple because $\sqrt{2}$ is not a positive integer.

Consider using *What's Your Angle, Pythagoras?* as a culminating activity. In this book, a curious boy named Pythagoras listens to his father and other village people describe challenges they are having. While on a trip with his father, Nef, a local builder, shows Pythagoras a special rope he uses to ensure that the corners in his building are square. Pythagoras' curiosity leads him to reproduce the special rope. He then uses this knowledge to help the village people and his father solve their problems. After reading this book with the class, small groups could complete various tasks such as the following:

- Using the triangles on page 14, show that the Pythagorean theorem can only be used for right triangles.
- Enlarge the triangle in the courtyard by a factor of 3. Show that this triangle is also a right triangle. Write the Pythgorean triple representing the triangle.
- Suppose the temple wall was 15 m high and Pythagoras wanted the bottom of the ladder to be 5 m from the wall. How long would the ladder have to be?
- Suppose the distance from Samos to Rhodes was 160 km and the distance from Rhodes to Crete was 256 km. What would be the distance from Samos to Crete. How much shorter is this distance for Pythagoras' dad?

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Paper and Pencil

- The side lengths of various triangles are given below. Ask students to determine whether each triangle is a right triangle.

  (i) 9 cm, 12 cm, 15 cm
  (ii) 16 mm, 29 mm, 18 mm
  (iii) 9 m, 7 m, 13 m

  (8SS1.4)

- Ask students to make a list of as many Pythagorean triples as they can think of in two minutes. After the time is up, ask them to pass their list to another student. Then, one at a time, ask them to read one Pythagorean triple from the list in front of them. Everybody who has that triple on their list will cross it off. At the end, the list with the most remaining entries is the winner. Students could be paired up for this activity.

  (8SS1.5)

Performance

- Provide students with various triangles. Ask them to measure the side lengths and then determine whether the triangle is a right trianlge. Students should explain their reasoning.

  (8SS1.3, 8SS1.4)

Journal

- Carpenters often use a 3-4-5 triangle to determine if corners are square (90°). Ask students to explain why this strategy works.

  (8SS1.5)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 1.6: Exploring the Pythagorean Theorem

ProGuide: pp. 37-43

CD-ROM: Master 1.29

SB: pp. 39-45

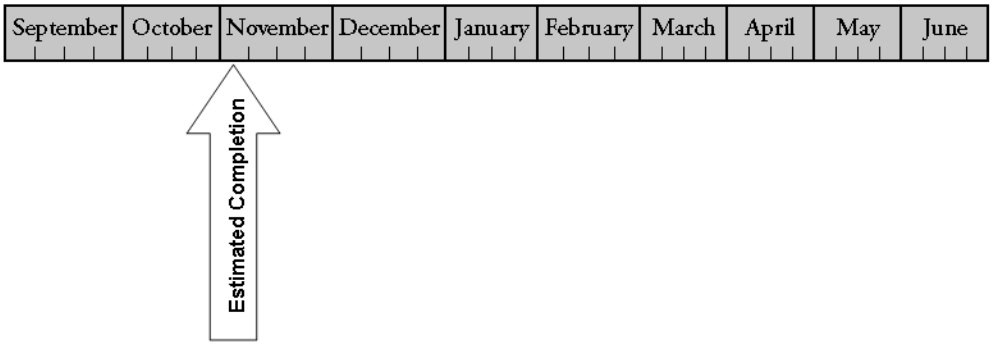
PB: pp. 15-17

Supplementary Resource

What's Your Angle, Pythagoras? – Julie Ellis

Integers

Suggested Time: 3 Weeks



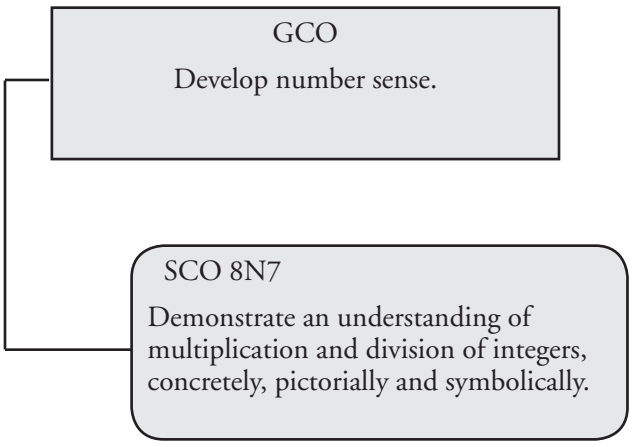
Unit Overview

Focus and Context

In this unit, students will explore multiplication and division of integers. They will model these processes concretely, pictorially and symbolically. They will generalize and apply rules for determining the signs of products and quotients and use these new understandings to solve problems that involve multiplication and divison of integers. Combining these new skills with integer addition and subtraction from Mathematics 7, students will solve problems involving integers and the order of operations.

Integers are used in fields such as science, engineering, and finance. They are used to describe rates of change, position, elevation, energy, temperature and profit/loss. Proficiency with integers will allow students to interpret these situations in a meaningful way and provides the fundamental building blocks for operations involving rational numbers in Mathematics 9.

Outcomes Framework



SCO Continuum

Grade 7	Grade 8	Grade 9
Measurement		
7N6 Demonstrate an understanding of addition and subtraction of integers, concretely, pictorially and symbolically. [C, CN, PS, R, V]	8N7 Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically. [C, CN, PS, R, V]	9N3 Demonstrate an understanding of rational numbers by: • comparing and ordering rational numbers • solving problems that involve arithmetic operations on rational numbers. [C, CN, PS, R, T, V]

Mathematical Processes

[C] **Communication**	[PS] **Problem Solving**
[CN] **Connections**	[R] **Reasoning**
[ME] **Mental Mathematics and Estimation**	[T] **Technology**
	[V] **Visualization**

Number

Specific Outcomes

Students will be expected to

8N7 Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically.

[C, CN, PS, R, V]

Achievement Indicator:

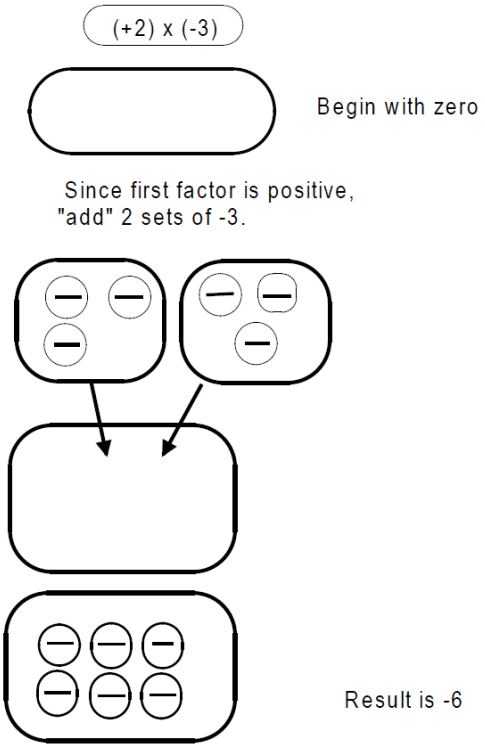
8N7.1 *Model the process of multiplying two integers using concrete materials or pictorial representations and record the process.*

Suggestions for Teaching and Learning

In Mathematics 6, students represented integers concretely, pictorially and symbolically. They also placed integers on a number line, compared two integers and ordered them. In Mathematics 7, students explored the addition and subtraction of integers concretely, pictorially, and symbolically. They will now explore the multiplication and division of integers.

Although the rules for multiplying integers are easy for students to learn, explaining why these rules make sense is a greater challenge. Two models that can assist with this are integer tiles and number lines. Students should continue to make the connection between multiplication and repeated addition. (+3) × (-5), for example, can also be expressed as 3 groups of -5 or (-5) + (-5) + (-5). Students should explore repeated addition using integer counters. Modelling multiplication of two positive integers is straightforward. (+3) × (+2), for example, can be modelled by creating 3 groups with each group containing 2 positive integer tiles. Multiplication involving negative integers can be more challenging for students.

One strategy that can be used to model multiplication of a positive and a negative integer, commonly known as the bank model, is shown below.



General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Ask students to express each repeated addition as a multiplication.
  (i) (-6) + (-6) + (-6) + (-6) + (-6)
  (ii) (+4) + (+4) + (+4) + (+4)

  (8N7.1)

- Ask students to express each multiplication as repeated addition.
  (i) (+7) × (+2)
  (ii) (+7) × (-2)

  (8N7.1)

- Kyle borrowed $6 from each of his two friends, Dillan and Jayden. Ask students to model this situation using integer counters or a pictoral representation.

  (8N7.1)

Performance

- Provide students with a blank sheet of paper and a collection of integer counters.
  (i) Ask students to model 4 groups of -2 using their counters. They should sketch the associated pictorial representation and write the appropriate number sentence.

  (ii) Ask students to model (-3) × 2 using their integer counters. They should stetch a pictorial representation.

  (8N7.1)

Journal

- Ask students to respond to the following:

  Your friend, Ben, missed class the day you learned how to model multiplication of integers. Explain to Ben how to determine (2) × (-5) and (-2) × (+5).

  (8N7.1)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 2.1: Use Models to Multiply Integers

ProGuide: pp. 4-9

CD-ROM: Master 2.18

Student Book (SB): pp. 64-69

Practice and Homework Book (PB): pp. 29-31

Number

Specific Outcomes

Students will be expected to

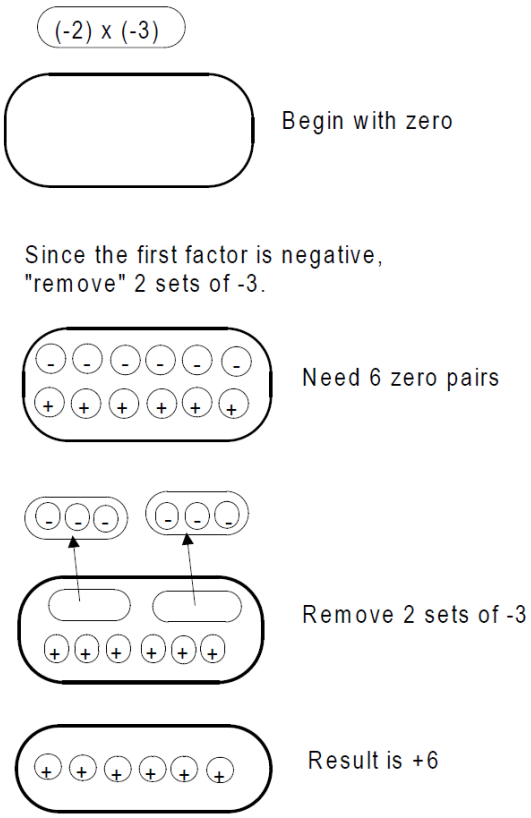
8N7 Continued ...

Achievement Indicator:

8N7.1 *(Continued) Model the process of multiplying two integers using concrete materials or pictorial representations and record the process.*

Suggestions for Teaching and Learning

Students may have more difficulty modelling multiplication of two integers in which the first integer is negative. It does not make sense to have a negative number of groups. For this situation, the use of zero pairs is necessary. Consider the following:



Students may have difficulty determining the number of zero pairs to add when using a counter model. To help them make this decision, relate the number of zero pairs to the number of counters that have to be removed.

It may be helpful to organize exploration with the bank model using a table such as the one below.

Multiplication Statement	# of Sets to Deposit or Withdraw	How much in each set	Result
2 × 3	deposit 2 sets	3	+6
2 × -3	deposit 2 sets	-3	-6
-2 × 3	withdraw 2 sets	3	-6
-2 × -3	withdraw 2 sets	-3	+6

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Ask students to model the following situations:
  (i) Johnny deposits $4 every day for 3 days.
  (ii) Sally withdraws $6 every day for 4 days.

  (8N7.1)

- Ask students to use a diagram to model each of the following situations. They should write a number sentence to represent the solution to the problem.
  (i) Brittany lost 3 points in each round (hand) of cards that was played. If she played 4 rounds, what was her score at the end of the game?
  (ii) Jeffrey owed $5 to each of 3 friends. What integer could be used to represent Jeffrey's total debt?

  (8N7.1)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 2.1: Use Models to Multiply Integers

ProGuide: pp. 4-9

CD-ROM: Master 2.18

SB: pp. 64-69

PB: pp. 29-31

Number

Specific Outcomes

Students will be expected to

8N7 Continued ...

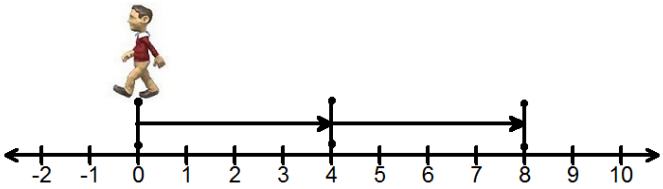
Achievement Indicator:

8N7.1 *(Continued) Model the process of multiplying two integers using concrete materials or pictorial representations and record the process.*

Suggestions for Teaching and Learning

Another model for multiplying integers is the number line. The first integer indicates which direction to face and how many steps to take, while the second integer indicates which direction to move and the size of the steps.

To model (-2) × (-4), for example, start at zero and face the negative end of the line. Take 2 steps of size 4 backward to stop at +8.



Students may benefit from organizing results from working with the number line using a table such as the one shown below.

Multiplication Statement	Direction to Face	# of Steps	Direction to Walk	Step Size	Result
2 × 4	positive	2	forward	4	+8
2 × -4	positive	2	backward	4	-8
-2 × 4	negative	2	forward	4	-8
-2 × -4	negative	2	backward	4	+8

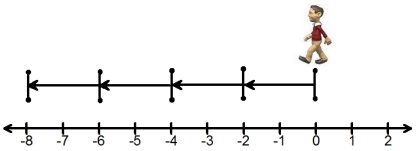
General Outcome: Develop Number Sense

Suggested Assessment Strategies

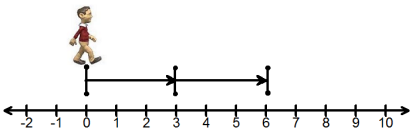
Performance

- Using a number line on the classroom floor, ask students to demonstrate and describe the multiplication statement represented by each diagram.

(i)



(ii)



(8N7.1)

Paper and Pencil

- Ask students to write a number sentence for each of the following:
  - (i) Roxanne is facing the positive end of the line and takes 2 steps of size 5 backwards.
  - (ii) Corbin is facing west and walks backwards 9 steps of size 2.

(8N7.1)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 2.1: Use Models to Multiply Integers

ProGuide: pp. 4-9

CD-ROM: 2.18

SB: pp. 64-69

PB: pp. 29-31

Number

Specific Outcomes

Students will be expected to

8N7 Continued ...

Achievement Indicator:

8N7.2 *Generalize and apply a rule for determining the sign of the product of integers.*

Suggestions for Teaching and Learning

Building upon the models that have been used, students should develop the sign rules for multiplication of integers. They should analyze products such as:

(+2) × (+3) = +6	(+2) × (+4) = +8
(+2) × (-3) = -6	(+2) × (-4) = -8
(-2) × (+3) = -6	(-2) × (+4) = -8
(-2) × (-3) = +6	(-2) × (-4) = +8

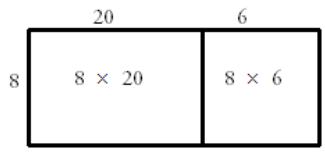
Ask students questions such as:

- What do you notice about the product of two positive integers?
- What do you notice about the product of a positive integer and a negative integer?

They should realize that:

- when the integers have the same sign, the product is positive.
- when the integers have different signs, the product is negative.

The strategies used to multiply whole numbers with two or more digits can also be used to multiply integers with two or more digits. The sign rules can be applied before or after the multiplication is completed. When asked to determine -8 × 26, for example, students should recognize that the product must be negative and then determine 8 × 26.

Multiplying Left to Right	Area Model	Standard Algorithm
26 ×8 160 +48 208	 160 + 58 = 208	4 26 ×8 208

Students should conclude -8 × 26 = -208.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:
  - (i) Write -20 as the product of 2 integers in as many ways as possible. Repeat for +20.
  - (ii) The sum of two integers is -2. The product of the same two integers is -24. What are the two integers? Explain your reasoning.
  - (iii) Without evaluating the products, identify the smallest product. Explain your reasoning.
    (-199) × (+87)
    (-199) × (-87)
    (+199) × (+87)

(8N7.2)

Observation

- Place students in groups and ask them to write a multiplication sentence for each of the following situations:
  - (i) The product of two integers equals one of the integers.
  - (ii) The product of two integers equals the opposite of one of the integers.
  - (iii) The product of two integers is less than both integers.
  - (iv) The product of two integers is greater than both integers.

(8N7.2)

Interview

- Ask the student to explain why the product of two negative integers has to be greater than their sum.

(8N7.2)

Journal

- Ask students to respond to the following:

  Suppose a friend knows how to multiply positive integers but has never multiplied negative integers.

  (i) How could you use the following pattern to show your friend how to calculate (+6) × (-4)?

  (+6) × (+2) = +12
  (+6) × (+1) = +6
  (+6) × (0) = 0
  (+6) × (-1) = -6
  (+6) × (-2) = ?
  (+6) × (-3) = ?
  (+6) × (-4) = ?

  (ii) Make up a pattern to show your friend how to calculate (+5) × (-3).

(8N7.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 2.2: Developing Rules to Multiply Integers

Game: What's My Product?

ProGuide: pp.10-15

CD-ROM: Master 2.19

SB: pp.70-76

PB: pp. 32-33

Number

Specific Outcomes

Students will be expected to

8N7 Continued ...

Achievement Indicators:

8N7.3 *Provide a context that requires multiplying two integers.*

8N7.4 *Solve a given problem involving the multiplication of integers.*

Suggestions for Teaching and Learning

It is important that students make meaningful connections between real-world contexts and integers. Some examples include:

- a thermometer - temperatures above/below 0 degrees
- in an elevator - floors above/below ground level
- golf scores - above/below par
- elevation - above/below sea level
- money - being in debt/having money; deposits/withdrawls
- hockey - a player's plus/minus statistic

To successfully solve problems involving integer multiplication, students must understand the use of positive and negative integers to represent the quantities that are multiplied. When solving problems, students should communicate their answer in a statement to explain the meaning of the integer product. Consider the following:

Matthew has committed his support to a local charity for 5 years. If he has $25 deducted automatically from his bank account each year, what will be his total deductions?

First, students must decide which integers to multiply.

- -25 represents the yearly $25 deduction
- +5 represents the number of years
- $(-25) \times (+5) = -125$

Students should recognize that the negative product in this case indicates the deduction. Matthew's total deductions will be $125.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

- Ask students to respond to the following:

  Marcy has $16 and spends $3 each day. Johnny has $20 and spends $4 each day. Who will have more money or less debt at the end of 7 days? They should support their solution using pictures, numbers, and words.

(8N7.1, 8N7.2, 8N7.4)

Paper and Pencil

- Ask students to answer the following:

  You borrow $2 each day for three days. What is your total debt at the end of the third day?

(8N7.2, 8N7.4)

Performance

- Ask stududents to play the game: "Operation Integers"

  Players: 2 to 4

  Materials: A deck of cards (no face cards)

  Description:

  Deal all the cards face down on the table. Black suits are positive and red suits are negative. Each player turns over two cards and decides whether to add, subtract, multiply or divide the two numbers on the cards. The player who has the greatest result wins all the cards that are face up.

  Goal: The play continues until one person (the winner) has all the cards.

  Variations:

  - Use fewer cards or cards with only certain numbers.
  - Use fewer operations (limit to multiplication and division).
  - Turn over three or four cards instead of two cards for each player.
  - The player who has the least sum, difference, product or quotient wins all the cards that are face up.
  - Each player rolls two (or more) dice with integers on each face rather than using playing cards. The player with the greatest (or least) number resulting from the operations scores one point. The winner is the player with the most points.

Teachers may prefer to wait until division of integers is complete before using this activity with students.

(8N7.2, 8N7.4)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 2.2: Developing Rules to Multiply Integers

ProGuide: pp.10-15

CD-ROM: Master 2.19

SB: pp.70-76

PB: pp. 32-33

Number

Specific Outcomes

Students will be expected to

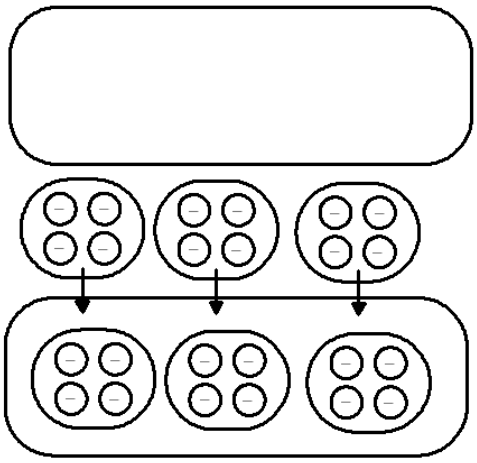
8N7 Continued ...

Achievement Indicators:

8N7.5 *Model the process of dividing an integer by an integer using concrete materials or pictorial representations and record the process.*

Suggestions for Teaching and Learning

The models used to develop an understanding of the multiplication of integers should also be used to develop an understanding of division of integers. The diagram shown below illustrates how to model (-12) ÷ (-4) using integers counters.



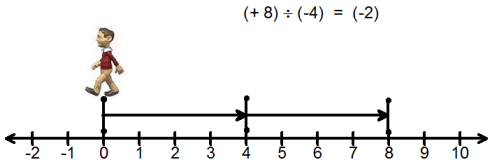
Start with 0 in the bank. Ask students how to get −12 in the bank using groups of −4?

3 groups of −4 must be deposited.

(-12) ÷ (-4) = +3

Some students may have difficulty modeling division when one integer is positive and one is negative. Emphasize that the dividend represents the tiles in the bank at the end. The divisor identifies which tiles to deposit or withdraw. Make the connection between the dividend and the number of zero pairs for this case.

Number lines should also be used to model the division of integers. The dividend identifies the final location on the number line. The divisor indicates which direction to move and the step size. To determine the quotient of (+8) ÷ (-4), for example, students should recognize that -4 indicates the step size and that the direction of motion is backwards. Starting at 0, to end up at +8, they would take 2 steps backward. They are facing the negative end of the number line. Students should conclude that (+8) ÷ (-4) = −2.



General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Ask students to work in pairs to model each situation.
  (i) Chris and his three friends together owe $12. They agree to share the debt equally. What is each person's share of the debt?
  (ii) The temperature in Nain was falling 2°C each hour. How many hours did it take for the temperature to fall 10°C?

  (8N7.5)

- Give students a blank sheet of paper and a collection of integer counters. Ask them to model (–10)÷(2) and record the process. They should sketch a diagram to represent the situation.

  (8N7.5)

- Ask students to model the following:

  Tyler modelled (+18) ÷ (+6) by separating 18 positive chips into groups of 6. Dillan separated 18 positive chips into 6 equal groups. Explain how they each determined the correct quotient.

  (8N7.5)

- Using only 20 counters, ask students to develop a pattern to evaluate (-2000) ÷ (-500).

  (8N7.5)

Resources/Notes

Math Makes Sense 8

Lesson 2.3: Using Models to Divide Integers

ProGuide: pp. 17-22

CD-ROM: Master 2.20

SB: pp. 77-82

PB: pp. 34-36

Number

Specific Outcomes

Students will be expected to

8N7 Continued ...

Achievement Indicator:

8N7.6 *Generalize and apply a rule for determining the sign of the quotient of integers.*

Suggestions for Teaching and Learning

As with multiplication, the models that have been used should lead to the sign rules for division of integers. Students should look for patterns in division sentences, such as:

(+12) ÷ (+4) = +3 (+8) ÷ (+4)=+2
(+12) ÷ (-4) = -3 (+8) ÷ (-4) = -2
(-12) ÷ (+4) = -3 (-8) ÷ (+4) = -2
(-12) ÷ (-4) = +3 (-8) ÷ (-4) = +2

Ask them questions such as:

- What do you notice about the quotient of two positive integers?
- What do you notice about the quotient of a positive integer and a negative integer?

They should recognize that:

- When the integers have the same sign, the quotient is positive
- When the integers have different signs, the quotient is negative.

Students should be provided with the opportunity to apply these rules to determine integer quotients.

To develop this rule, students could also explore the connection between multiplication and division. Teachers could write several multiplication statements and ask students to write the related division statements, as shown in the following table:

Multiplication	Related Division	
(+2) × (+4) = (+8)	(+8) ÷ (+2) = (+4)	(+8) ÷ (+4) = (+2)
(+2) × (-4) = (-8)	(-8) ÷ (+2) = (-4)	(-8) ÷ (-4) = (+2)
(-2) × (+4) = (-8)	(-8) ÷ (-2) = (+4)	(-8) ÷ (+4) = (-2)
(-2) × (-4) = (+8)	(+8) ÷ (-2) = (-4)	(+8) ÷ (-4) = (-2)

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- The sum of two integers is +15. Dividing the larger integer by the smaller integer gives a quotient of -4. Ask students to determine the two integers and explain their reasoning.

  (8N7.5, 8N7.6)

- Ask students to answer the following:

  If 14 times an integer is -84, what is the integer?

  (8N7.6)

Interview

- Without evaluating the quotients, ask the student which quotient will have the lowest value. The student should explain the reasoning.

  (-1428) ÷ (+84)

  (+1428) ÷ (+84)

  (-1428) ÷ (-84) (8N7.5, 8N7.6)

- Ask the student to explain why the quotient of two negative integers has to be greater than their sum.

  (8N7.6)

- Without performing any calculations, ask the student to explain why the quotients(-468) ÷ (-26) and (+468) ÷ (+26) must be the same.

  (8N7.6)

Resources/Notes

Math Makes Sense 8

Lesson 2.3: Using Models to Divide Integers

Lesson 2.4: Developing Rules to Divide Integers

ProGuide: pp. 17-22, 24-29

CD-ROM: Masters 2.20. 2.21

SB: pp. 77-82, 84-89

PB: pp. 34-36, 37-38

Number

Specific Outcomes

Students will be expected to

8N7 Continued ...

Achievement Indicators:

8N7.7 *Provide a context that requires dividing two integers.*

8N7.8 *Solve a given problem involving the division of integers (2-digit by 1-digit) without the use of technology.*

8N7.9 *Solve a given problem involving the division of integers (2-digit by 2-digit) with the use of technology.*

Suggestions for Teaching and Learning

The division of integers can occur in many contexts. Give students a problem such as the following:

- From Monday to Friday, Alex spent $20 on lunches. On average, how much did he spend each day?

Ask students to discuss the calculations needed and to create other situations for which dividing integers represents a meaningful calculation. Encourage students to describe situations that can be solved by dividing both positive and negative numbers. Prompt them with questions such as:

- What are some examples of measurements that can be positive?
- What are some examples of measurements that can be negative?
- What types of situations can be solved using division?

The use of appropriate terminology, such as dividend, divisor and quotient, is important. Students should be exposed to different forms of division notation. For example, a division statement can be written as (-6)÷(-3), $-3\overline{)-6}$ or $\frac{-6}{-3}$.

To divide 2-digit by 1-digit problems, students should apply the strategies from Mathematics 4 (using related multipication facts, repeated subtraction, renaming the dividend, sharing, standard algorithm) to determine the quotient and apply the appropriate sign rule. When asked to solve a division problem involving a 2-digit divisor, calculators are appropriate.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:
  - (i) A football team was penalized 30 yards in 3 plays. Suppose the team was penalized an equal number of yards on each play. Write a division statement using integers and solve it to find the number of yards in each penalty.

    (8N7.7, 8N7.8)
  - (ii) Anna and Sarah ran 5 laps of a race. When Anna finished, Sara was 15 meters behind her. Suppose Sara fell behind the same number of meters during each lap. Write a division statement using integers, and use it to determine how far Sara fell behind in each lap.

    (8N7.7, 8N7.8)

Journal

- Michael said, "When I divide +12 by +4, +3, +2, or +1, the quotient is less than or equal to +12. If I divide -12 by +4, +3, +2, or +1, I think the quotient should be less than or equal to -12." Ask students to respond to Michael's statement by agreeing or disagreeing with him.

  (8N7.6, 8N7.8)
- Ask students to respond to three reflective prompts that describe what they learned about integers.

3 new things I learned 1. 2. 3.
2 things I am still struggling with 1. 2.
1 thing that will help me tomorrow 1.

  Provide students with a copy of the reflection sheet and time to complete their reflections. They could also be paired up to share their 3-2-1 reflections.

  (8N7)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 2.4: Developing Rules to Divide Integers

ProGuide: pp. 24-29

CD-ROM: 2.21

SB: pp. 84-89

PB: pp. 37-38

Number

Specific Outcomes

Students will be expected to

8N7 Continued ...

Achievement Indicator:

8N7.10 *Identify the operation required to solve a given problem involving integers.*

Suggestions for Teaching and Learning

After exploring multiplication and division independently, students should solve a variety of problems involving integers. Prior to determining the answer, students should be given the opportunity to identify the operation required to solve a problem involving data such as, temperature, height above and below sea level, net worth, and sporting events. Often there are key words present in a given problem that indicate the operation necessary. These key words include, but are not limited to, those shown in the table.

Addition	*Subtraction*	*Multiplication*	*Division*
increase	decrease	double	share
more	less	triple	group
sum	difference	product	quotient

As students are exposed to a variety of word problems and identify the operation necessary to solve them, encourage them to record their list of key words.

Students should be aware that contextual information plays an important role in determining the correct operation, and they should not solely rely on keywords, as they can sometimes be misleading. Discuss with students how the same key words have different meanings in the contexts below.

- Jean has 6 muffins. Deanne has two times as many muffins. How many muffins does Deanne have?
- Deanne has two times as many muffins as Jean. If Deanne has 6 muffins, how many muffins does Jean have?

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Ask students to identify which operation is necessary to solve the following problems. They should write an expression that could be used to solve the following problems:
  - (i) An oil rig is drilling a well at 2 m per minute. How deep is the well after the first 8 minutes?
  - (ii) The temperature decreased 16°C over a 4 hour period. Assuming that the temperature decreased at a constant rate, how much did the temperature decrease each hour?

(8N7.10)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 2.5: Order of Operations with Integers

ProGuide: pp. 30-35

SB: pp. 90-93

PB: pp. 39-40

Number

Specific Outcomes

Students will be expected to

8N7 Continued ...

Achievement Indicator:

8N7.11 *Solve a given problem involving integers taking into consideration order of operations.*

Suggestions for Teaching and Learning

Students have already used the order of operations, excluding exponents, but limited to whole numbers and decimals. This will now be extended to calculations with integers. It is important to note that work with exponents is not a specific outcome until Grade 9. For this unit, students will complete calculations based on the following order of operations:

- Brackets
- Division/Multiplication (in the order they appear)
- Addition/Subtraction (in the order they appear)

Rules for order of operations are necessary in order to maintain consistency of results. Provide students with situations, such as the following, in which they can recognize the need for the order of operations.

- Emily spends $10 each week for 7 weeks and then spends $5 each week for 3 weeks. Determine how much money Emily spends in total.

Students should write an expression to represent this situation: 7(−10)+3(−5). They should recognize that multiplication must be performed first, followed by addition, to get the correct amount spent. Ask them why ignoring the order of operations and calculating from left to right would produce an incorrect answer to this question.

Teachers should discuss the appropriate use of brackets:

- Brackets can be used to show integers as positive or negative such as (-3) or (+4). These brackets do not involve an operation.
- There is a need for brackets around (-4) in the expression -5 – (-4), but they are not necessary for (-5).
- For positive integers the positive sign is understood. For example, 4 is understood to be the same as (+4), so brackets are not necessary in this case.

Students should use the order of operations to evaluate a variety of expressions involving integers and should be exposed to problem solving contexts that require the use of the order of operations.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Ask students to evaluate the following.
  (i) (-4) - (+8) × (-2) - (+15)
  (ii) (+3) × [(+14) + (-18)] - (+8) ÷ (-4)
  (iii) $\frac{[6+(-38)]\div 4(-2)}{(-2+4)(5-6)}$

  (8N7.11)
- Ask students to insert one pair of brackets in the expression -4 × 6 - 3 × 4 - 5 so that it simplifies to -53.

  (8N7.11)
- The formula for converting temperatures from Fahrenheit (F) to Celsius (C) is C = (F - 32) × 5 ÷ 9. Ask students to use the formula to convert 23°F to degrees Celsius.

  (8N7.11)
- The daily low temperatures in La Scie for 5 days for November were -4°C, +1°C, -2°C, +1°C and -6°C. Ask students to determine the mean of these temperatures.

  (8N7.11)
- Ask students to identify the error(s) in this solution and make the necessary corrections.

  3 × (-8) ÷ (-2 - 4)
  = -24 ÷ (-2 - 4)
  = 12 - 4
  = 8

  (8N7.11)

Journal

- To win a free trip, you must answer the following skill-testing question correctly: -3 × -4 + (-18) ÷ 6 - (-5). The contest organizers claim that the answer is +4. Ask students to write a letter to the organizers explaining there is an error in their solution. They should identify the error and provide the contest organizers with the correct solution.

  (8N7.11)

Performance

- Pass the Pen: Write a multistep problem such as, (-12) ÷ (-2) + 7 × [(4 – (-3)] on the board and call on one student to complete the first step. They should explain how to complete this step to the class and then call on another student to complete the next step. This continues until the problem is finished. When a question arises, the student holding the pen must answer the question, call on another student to help, or pass the pen to a different student.

  (8N7.11)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 2.5: Order of Operations with Integers

Understanding the Problem

ProGuide: pp. 30-35

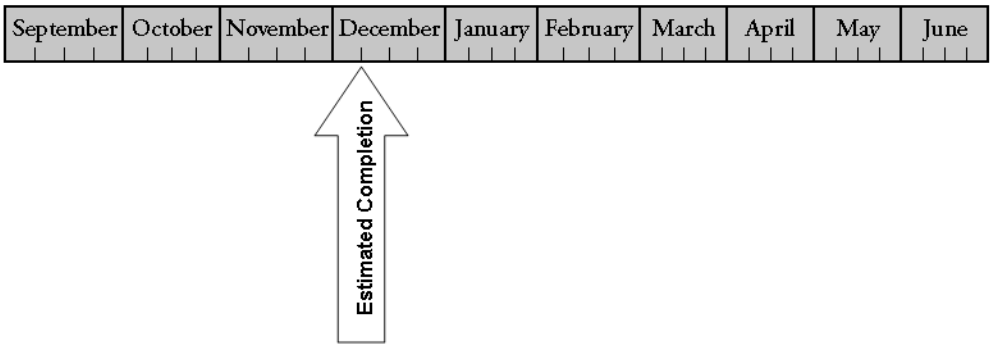
CD-ROM: Master 2.22

SB: pp. 90-93, 94-95

PB: pp. 39-40

Operations with Fractions

Suggested Time: 4 Weeks



Unit Overview

Focus and Context

In this unit, students will extend their knowledge of fractions and whole number operations to develop an understanding of multiplication and division of positive fractions and mixed numbers. They will begin by modeling fraction multiplication using manipulatives such as fraction strips, pattern blocks, number lines and area models. Through modeling students will generalize and apply a rule for multiplying fractions. Students will then explore division in a similar manner, beginning with modelling and moving to the symbolic level. Estimating with benchmarks of zero, one-half, and one whole is encouraged throughout the unit to help students determine the reasonableness of answers. Finally, students will consolidate the four operations with fractions by applying the order of operations.

Whether purchasing floor covering or material for four bridesmaids' dresses, modifying recipes, or figuring out the amount and size of lumber for a particular project, there are many daily experiences that require multiplication and division of fractions. Having a strong understanding of these concepts allow students to analyze, interpret, and solve such problems. It also provides the fundamental building blocks for future work with rational expressions, algebra and trionometry.

Outcomes Framework

GCO
Develop Number Sense

SCO 8N5

Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

SCO Continuum

Grade 7	Grade 8	Grade 9
Strand: Number		
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
7N5 Demonstrate an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike denominators, concretely, pictorially and symbolically (limited to positive sums and differences). [C, CN, ME, PS, R, V]	8N6 Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]	9N3 Demonstrate an understanding of rational numbers by: • comparing and ordering rational numbers • solving problems that involve arithmetic operations on rational numbers. [C, CN, PS, R, T, V]

Mathematical Processes

[C] Communication	**[PS] Problem Solving**
[CN] Connections	**[R] Reasoning**
[ME] Mental Mathematics and Estimation	**[T] Technology**
	[V] Visualization

Number

Specific Outcomes

Students will be expected to

8N6 Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically.

[C, CN, ME, PS]

Achievement Indicator:

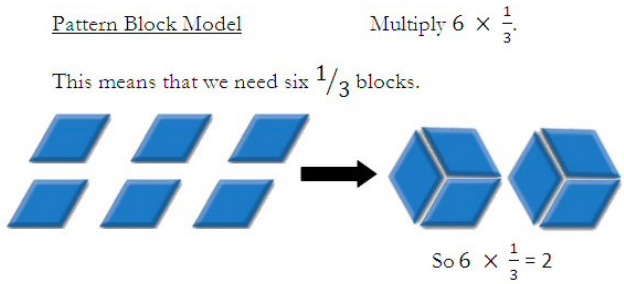
8N6.1 *Model multiplication of a positive fraction by a whole number concretely or pictorially and record the process.*

Suggestions for Teaching and Learning

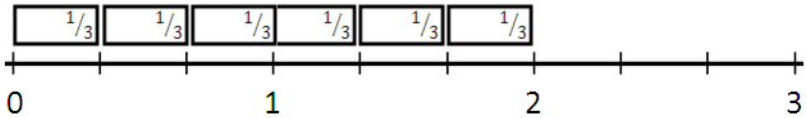
In Grade 7, students demonstrated an understanding of adding and subtracting positive fractions and mixed numbers, with like and unlike demonimators, concretely, pictorially and symbolically. In this unit, students will extend this knowledge to explore multiplication and division of fractions (limited to positive products and quotients). It is important that students understand that the meaning of multiplication and division has not changed just because they are working with fractions.

Research indicates that the teaching of fractions through memorizing rules has significant dangers. The rules do not help students think in any way about the meanings of the operations or why they work and the mastery observed in the short term is often quickly lost (Van de Walle 2001, p.228). Exploring operations with fractions through the use of models such as number lines, the area model, counters, fraction circles and strips helps solidify understanding of these concepts.

When multiplying a fraction by a whole number, encourage students to make connections with previous work involving multiplication. When asked to multiply $6 \times \frac{1}{3}$, for example, they should recognize that they have 6 groups of $\frac{1}{3}$. The following models illustrates this multiplication:



When using a number line, remind students that the denominator identifies the number of equal parts that make up the whole and suggests the divisions to use on their number lines.



General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Ask students to use a number line to demonstrate why each of the following statements is true:

  (i) $\frac{1}{3} \times 3 = 1$

  (ii) $3 \times \frac{1}{3} = 1$

  (iii) Use a different model to verify the above.

  (8N6.1)

- Wayne filled 5 glasses with $\frac{7}{8}$ of a litre of soda in each glass. Ask students to use a model to determine how much soda Wayne used. (8N6.1)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.1: Using Models to Multiply Fractions and Whole Numbers

ProGuide: pp. 4-9

Master 3.16

CD-ROM: Master 3.27

Student Book (SB): pp. 104-109

Practice and Homework Book (PB): pp. 50-51

Number

Specific Outcomes

Students will be expected to

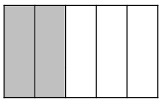
8N6 Continued...

Achievement Indicators:

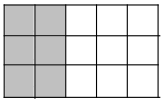
8N6.2 *Model multiplication of a positive fraction by a positive fraction concretely or pictorially using an area model and record the process.*

Suggestions for Teaching and Learning

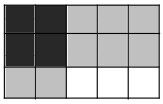
Students should use an area model to demonstrate an understanding of multiplication of 2 positive fractions. To model, $\frac{2}{3}\times\frac{2}{5}$, for example, they should create a rectangle and divide the rectangle into 5 equal parts vertically. They should shade 2 parts to represent $\frac{2}{5}$:



Next, to determine two-thirds of the shaded two-fifths, they should divide the rectangle into thirds horizontally:



Finally, students should shade two-thirds horizontally. The product will be the area that is double shaded (four pieces out of fifteen).



Therefore, $\frac{2}{3}\times\frac{2}{5}=\frac{4}{15}$.

Emphasize to students that the part where the shading overlaps represents the numerator in the product. The total number of parts represents the denominator.

8N6.3 *Provide a context that requires the multiplication of two given positive fractions.*

Relating multiplication of fractions to real-life situations helps solidify student understanding. Encourage students to provide a context that requires the multiplication of two given positive fractions. They may suggest adjusting a recipe or sharing the remaining pizza left from a birthday party. Encourage them to share their problems with their classmates. As students explore problem solving contexts, they should recognize that the word "of" indicates multiplication. For example, $\frac{1}{2}$ of 6 is equivalent to $\frac{1}{2}\times 6$.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

- Lisa has $\frac{3}{4}$ of a large candy bar. She gave $\frac{1}{3}$ of it to Shannon. Ask students to complete the following:
  (i) Demonstrate that Shannon got less than $\frac{1}{3}$ of an entire bar.
  (ii) What fraction of the whole bar does Shannon receive?
  (iii) What fraction of the whole bar does Lisa have left?

  (8N6.2)
- Ask students to create a word problem to accompany each of the following expressions:
  (i) $5 \times \frac{2}{3}$
  (ii) $\frac{1}{2} \times \frac{3}{4}$

  They should exchange questions with a classmate and solve the word problem by modeling the multiplication.

  (8N6.1, 8N6.2, 8N6.3)
- Ask students to explain how they could use a diagram to determine $\frac{3}{4} \times \frac{2}{5}$.

  (8N6.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.2: Using Models to Multiply Fractions

ProGuide: pp. 10-14

Master 3.16, 3.17

CD-ROM: Master 3.28

SB: pp. 110-114

PB: pp. 52-53

Number

Specific Outcomes

Students will be expected to

8N6 Continued...

Achievement Indicator:

8N6.4 *Estimate the product of two given positive proper fractions to determine if the product will be closer to 0, $\frac{1}{2}$, or 1.*

Suggestions for Teaching and Learning

Estimating the product of two given positive proper fractions will help students develop number sense and it allows them the opportunity to check the reasonableness of their answers.

Teachers could begin estimation of products by discussing the following properties with students:

$0 \times n = 0$, where n is any number

$1 \times n = n$, where n is any number

$1 \times 1 = 1$

Applying these properties and using benchmarks of 0, $\frac{1}{2}$, and 1 for given factors, students should be able to estimate the product of any two positive proper fractions.

To estimate the product of $\frac{1}{9}$ and $\frac{8}{9}$, for example, students should recognize that $\frac{1}{9}$ is close to 0. Since $0 \times \frac{8}{9} = 0$, $\frac{1}{9} \times \frac{8}{9}$ would be close to 0. Similarly, the following products can be estimated using benchmarks:

	Determine Benchmarks	Multiply using Benchmarks	Estimate Product
$\frac{8}{9} \times \frac{4}{9}$	$\frac{8}{9} \doteq 1$, $\frac{4}{9} \doteq \frac{1}{2}$	$1 \times \frac{1}{2} = \frac{1}{2}$	$\frac{8}{9} \times \frac{4}{9} \doteq \frac{1}{2}$
$\frac{8}{9} \times \frac{8}{9}$	$\frac{8}{9} \doteq 1$	$1 \times 1 = 1$	$\frac{8}{9} \times \frac{8}{9} \doteq 1$

Estimation helps fraction computations make sense. It should play a significant role in the development of multiplication strategies.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Students could play the ***Spinner Game***. Use a four section spinner. Label each section with fractions such as: $\frac{1}{9}, \frac{9}{10}, \frac{11}{12}, \frac{5}{11}$.
  Spin twice and estimate the product. Students score no points if the closest benchmark is zero, one point if the closest benchmark is $\frac{1}{2}$, and two points if the closest benchmark is 1. The student who scores 20 points first wins the game.

(8N6.4)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.3: Multiplying Fractions

ProGuide: pp. 15-20

CD-ROM: Master 3.29

SB: pp. 115-120

Number

Specific Outcomes

Students will be expected to

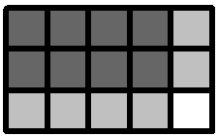
8N6 Continued...

Achievement Indicator:

8N6.5 *Generalize and apply rules for multiplying positive proper fractions, including mixed numbers.*

Suggestions for Teaching and Learning

Students should reflect on the models they have used to multiply positive proper fractions. Consider the following area model to multiply $\frac{2}{3} \times \frac{4}{5}$:



$$\frac{2}{3} \times \frac{4}{5} = \frac{8}{15}$$

Teachers could ask students questions such as the following:

- What is the relationship between the numerators of the factors and the numerator in the product?
- What is the relationship between the denominators of the factors and the denominator in the product?

After working with models, students should recognize that when two fractions are multiplied, the numerator is the product of the numerators, and the denominator is the product of the denominators:
$\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5} = \frac{8}{15}$.

Students should apply the rule for multiplying positive proper fractions to a variety of problems. They should express their answers in simplest form. Encourage students to continue to use their estimatation skills to check the reasonableness of their solution. In the above example, they should recognize that $\frac{2}{3}$ is close to $\frac{1}{2}$ and $\frac{4}{5}$ is close to 1. Since $\frac{1}{2} \times 1 = \frac{1}{2}$, the product should be close to $\frac{1}{2}$. The product, $\frac{8}{15}$, is close to $\frac{1}{2}$, so it is a reasonable answer.

When multiplying a positive fraction by a whole number, a common error occurs when students multiply both the numerator and denominator by the whole number. Remind students that any whole number can be written as a fraction having a denominator of 1.

A common misconception is that multiplying always results in a product which is greater than the two factors. When one of the factors is between zero and one, this is not the case. The use of models, as well as estimation, should help students make these connections.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Ask students to solve each of the following problems:
  - (i) The last time that Ms. Martinez ordered pizza, there was $\frac{2}{3}$ of a 12 slice pizza left. Bobby came in and ate $\frac{1}{2}$ of what was left. The other students were mad that Bobby ate $\frac{1}{2}$ of it. Bobby said "I only ate 2 pieces." Was he right? How many pieces did he eat? What fraction of the whole pizza did he eat? (8N6.5)
  - (ii) $1\frac{3}{5}$ m of fabric is needed to sew one blouse. How many metres of fabric are needed to sew 12 such blouses? (8N6.5)
  - (iii) In your job as a gardener, you must decide how to use your garden. You mark $\frac{1}{2}$ of the garden for potatoes. You use $\frac{1}{4}$ of the remaining area for corn. Then you plant cucumbers in $\frac{1}{3}$ of what is left. The rest of your garden is used for carrots. What fraction of your garden is used for carrots? (8N6.5)

Journal

- Ask students to respond to the following:

  Jared calculated $\frac{3}{5}\times\frac{2}{5}$ as follows: $\frac{3}{5}\times\frac{2}{5}=\frac{6}{5}$.
  - (i) What mistake did Jared make?
  - (ii) How could you use estimation to show Jared that he made a mistake?
  - (iii) What is the correct answer? (8N6.4, 8N6.5)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.3: Multiplying Fractions

Lesson 3.4: Multiplying Mixed Numbers

ProGuide: pp. 15-20, 21-26

Master 3.19

CD-ROM: Master 3.29

SB: pp. 115-120, 121-126

PB: pp. 54-55, 56-57

Number

Specific Outcomes

Students will be expected to

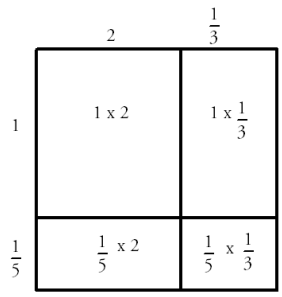
8N6 Continued...

Achievement Indicator:

8N6.5 *(Continued) Generalize and apply rules for multiplying positive proper fractions, including mixed numbers.*

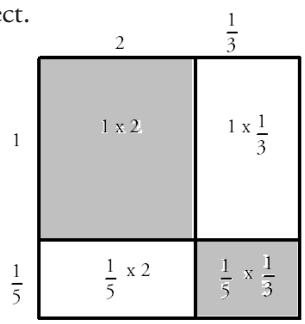
Suggestions for Teaching and Learning

Modelling multiplication of mixed numbers should be done prior to multiplying the equivalent improper fractions. No reference to improper fractions is necessary when using the models. Consider $1\frac{1}{5} \times 2\frac{1}{5}$, for example. An area model to multiply $1\frac{1}{5}$ by $2\frac{1}{5}$ is shown below.



$$(1\times 2)+\left(1\times\frac{1}{3}\right)+\left(\frac{1}{5}\times 2\right)+\left(\frac{1}{5}\times\frac{1}{3}\right)$$
$$=2+\frac{1}{3}+\frac{2}{5}+\frac{1}{15}$$
$$=2+\frac{5}{15}+\frac{6}{15}+\frac{1}{15}$$
$$=2\frac{12}{15}$$
$$=2\frac{4}{5}$$

A common error when finding the product of mixed numbers is to multiply the whole numbers together and multiply the fractions together. Use of the area model should help students understand that this is incorrect.



Since the product is the area of the entire rectangle, multiplying only the whole numbers together and the fractions together omits the two unshaded pieces. Continued use of the area model should help students avoid this common error. Eventually, students should be able to perform these calculations without having to draw the area model.

Another strategy that should be used to multiply two mixed numbers is rewriting each mixed number as an improper fraction. Students expressed mixed numbers as improper fractions in grade 6, and they revisited this concept in grade 7. As with multiplying proper fractions, they should be encouraged to check the reasonableness of their answer using estimation.

Students worked with equivalent fractions in grade 7. As with adding and subtracting, they should be encouraged to reduce fractions to simplest form when multiplying as well.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

- Ask students to respond to the following:
  Joanne gave the following answer on her homework assignment.

$$2\tfrac{1}{3}\times 1\tfrac{3}{4}=3\tfrac{1}{12}$$

  (i) Use an area model to show why this answer is incorrect.
  (ii) What mistake did Joanne make?
  (iii) What is the correct answer?

(8N6.5)

- Ask students to respond to the following:
  Jane multiplied $2\frac{1}{3}\times 2\frac{1}{2}$ as follows:

$$\begin{aligned} 2\tfrac{1}{3}\times 2\tfrac{1}{2} &= \tfrac{7}{3}\times\tfrac{5}{2} \\ &= \tfrac{14}{6}\times\tfrac{15}{6} \\ &= \tfrac{210}{36} \\ &= \tfrac{35}{6} \\ &= 5\tfrac{5}{6} \end{aligned}$$

  (i) Was Jane's final answer correct?
  (ii) How did Jane make the calculations longer than necessary?

(8N6.5)

Interview

- Ask students to estimate each of the following and to explain their thinking:

  (i) $5\frac{1}{6}\times 8$

  (ii) $4\times 8\frac{3}{8}$

(8N6.5)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.3: Multiplying Fractions

Lesson 3.4: Multiplying Mixed Numbers

ProGuide: pp. 15-20, 21-26

Master 3.19

CD-ROM: Master 3.29

SB: pp. 115-120, 121-126

PB: pp. 54-55, 56-57

Note

To activate prior knowledge on relating mixed numbers and improper fractions teachers could use the PB: pp. 48-49 and CD-ROM: Master 3.37b

Number

Specific Outcomes

Students will be expected to

8N6 Continued...

Achievement Indicator:

8N6.6 *Model division of a whole number and a positive proper fraction, concretely or pictorially and record the process.*

Suggestions for Teaching and Learning

Work with concrete and pictorial models is necessary when students are first introduced to dividing fractions. It is not enough for students' knowledge of the division of fractions to be limited to the traditional invert-and-multiply algorithm. To develop students' conceptual understanding of division of fractions, teachers should move them from the concrete to pictorial to symbolic representations.

Students were introduced to division of whole numbers in two ways: sharing and grouping. This idea can be extended to division of fractions. It is appropriate to think of dividing a fraction by a whole number as equal sharing. Students should work with examples such as:

Janelle has $\frac{2}{3}$ of a pizza to divide evenly among 3 people. How much pizza would each person receive?

Students can think about this as sharing $\frac{2}{3}$ into 3 equal parts. They could use fraction strips to model this division, beginning by representing $\frac{2}{3}$:

Next, they should identify the fraction strip that represents each one third cut into three equal pieces:

They should recognize that $\frac{2}{3}$ is equivalent to $\frac{6}{9}$. Dividing $\frac{6}{9}$ into three equal groups results in $\frac{2}{9}$. Each person would receive $\frac{2}{9}$ of the pizza.

Similiarly, fraction circles could be used to represent division.

General Outcome: Develop Number Sense

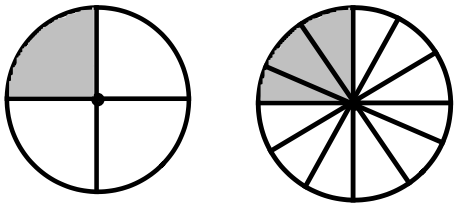
Suggested Assessment Strategies

Journal

- Ask students to explain the difference between "six divided by one half" and "six divided in half". They should write a division statement for each phrase and find each quotient.

(8N6.6)

- Ask students to explain how the following diagram can be used to calculate $\frac{1}{4} \div 3$.



Ask them to model or share other diagrams that illustrate this division.

(8N6.6)

Paper and Pencil

- Tell students they have $\frac{3}{4}$ of a pizza to divide equally between 2 people. Ask them to use a model to determine how much pizza each person would receive.

(8N6.6)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.5: Dividing Whole Numbers and Fractions

ProGuide: pp. 29-34

Master 3.20

CD-ROM: Master 3.31

SB: pp. 129-134

PB: pp. 58-59

Number

Specific Outcomes

Students will be expected to

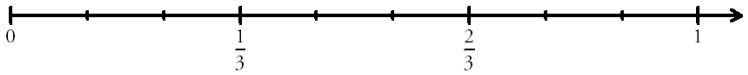
8N6 Continued...

Achievement Indicator:

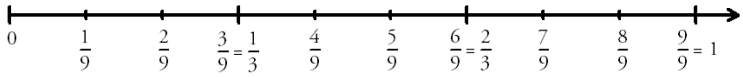
8N6.6 *(Continued) Model division of a whole number and a positive proper fraction, concretely or pictorially and record the process.*

Suggestions for Teaching and Learning

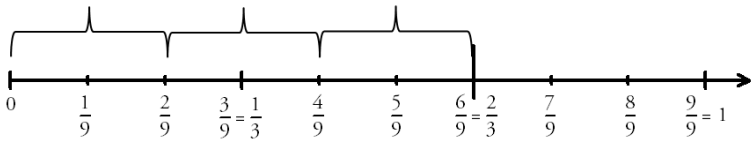
Number lines can also be used to model division. To model $\frac{2}{3} \div 3$, students should begin by drawing and labeling a number line snowing thirds, and dividing each third into three equal pieces.



Ask students to identify what fraction each part represents. They should recognize that the whole has 9 pieces and, therefore, each piece represents $\frac{1}{9}$. Students should label the number line accordingly:



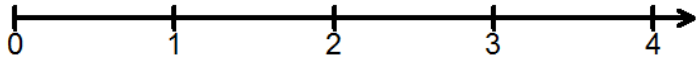
They should recognize that $\frac{2}{3}$ is equivalent to $\frac{6}{9}$. Dividing $\frac{6}{9}$ into three equal groups results in $\frac{2}{9}$.



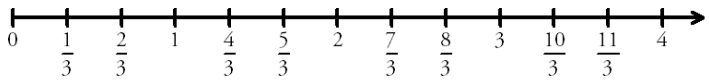
Therefore, $\frac{2}{3} \div 3 = \frac{2}{9}$.

When dividing a whole number by a fraction, students should determine how many groups can be made. This could be modelled with fraction cirles or strips.

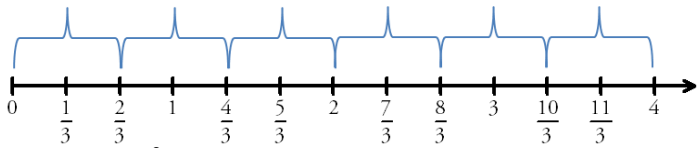
Students could also use a number line to model $4 \div \frac{2}{3}$. They would begin by drawing and labeling a number line to show 4.



Next, they should divide each whole into thirds:



Students should recognize that 4 is equivalent to $\frac{12}{3}$. Arranging $\frac{12}{3}$ into groups of $\frac{2}{3}$ results in 6 groups.



Therefore, $4 \div \frac{2}{3} = 6$.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Sandra pays \$3 for $\frac{3}{4}$ kg of nuts. Ask students to use a model to determine how much 1 kg of these nuts would cost.

(8N6.6)

Performance

- Ask students to demonstrate the following by using concrete or pictorial representations:
  (i) $2 \div \frac{1}{4} = 8$
  (ii) $\frac{1}{2} \div 2 = \frac{1}{4}$

(8N6.6)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.5: Dividing Whole Numbers and Fractions

ProGuide: pp. 29-34

Master 3.20

CD-ROM: Master 3.31

SB: pp.129-134

PB: pp. 58-59

Number

Specific Outcomes

Students will be expected to

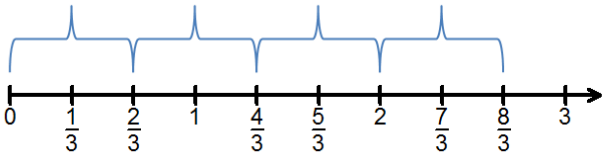
8N6 Continued...

Achievement Indicator:

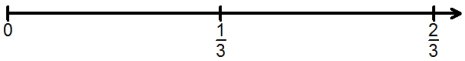
8N6.6 *(Continued) Model division of a whole number and a positive proper fraction, concretely or pictorially and record the process.*

Suggestions for Teaching and Learning

Some students may experience difficulty using models and number lines when the quotient is not a whole number. They should model $3 \div \frac{2}{3}$ on a number line.



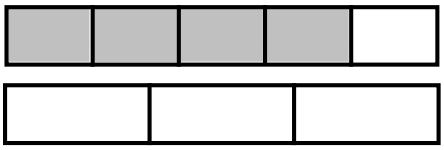
4 groups of $\frac{2}{3}$ can be made, with $\frac{1}{3}$ remaining. Students should identify what fraction of $\frac{2}{3}$ is $\frac{1}{3}$. A number line might help them visualize this relationship:



Students should recognize that $\frac{1}{3}$ is $\frac{1}{2}$ of $\frac{2}{3}$. Therefore, $3 \div \frac{2}{3} = 4\frac{1}{2}$.

A common misconception students have about division is that the quotient is always smaller than the dividend. Emphasize that this is not always the case by exposing them to examples such as $3 \div \frac{2}{3}$. When a whole number is divided by a proper fraction, the quotient is greater than the dividend.

Modelling division of a whole number and proper fraction should provide a smooth transition to dividing positive proper fractions. When dividing $\frac{4}{5}$ by $\frac{1}{3}$, for example, students should use fraction strips to determine how many groups of $\frac{1}{3}$ are in $\frac{4}{5}$. The diagram below shows that the number of groups of $\frac{1}{3}$ in $\frac{4}{5}$ is between 2 and 3.



It is difficult to determine precisely how many groups there are. When dividing two proper fractions, students will have to identify a common denominator. In this case, the common denominator for 5 and 3 is 15. Using a rectangle divided into fifteenths will help students determine the exact number of groups.

In $\frac{12}{15}$ there are 2 whole groups of $\frac{5}{15}$, plus $\frac{2}{5}$ of another group.

Therefore, $\frac{4}{5} \div \frac{1}{3} = 2\frac{2}{5}$.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Ask students to use a diagram to determine the following quotients:
  (i) $4 \div \frac{1}{3}$

  (ii) $3 \div \frac{1}{2}$

  (iii) $2 \div \frac{1}{5}$

  (8N6.6)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.5: Dividing Whole Numbers and Fractions

ProGuide: pp. 29-34

Master 3.20

CD-ROM: Master 3.31

SB: pp. 129-134

PB: pp. 58-59

Number

Specific Outcomes

Students will be expected to

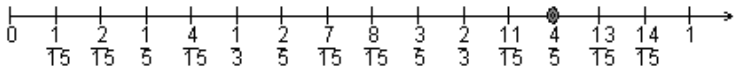
8N6 Continued...

Achievement Indicator:

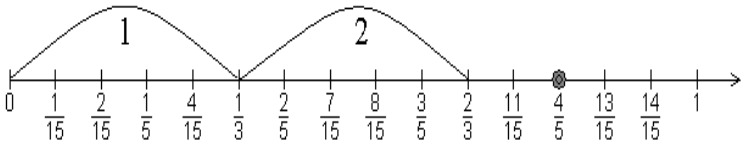
8N6.7 *Model division of a positive proper fraction by a positive proper fraction pictorially and record the process.*

Suggestions for Teaching and Learning

Modeling division of a fraction by a fraction using a number line follows the same pattern as the fraction strip model. Students should rewrite each fraction with a common denominator to help them identify how many groups can be made. To model $\frac{4}{5} \div \frac{1}{3}$, for example, students would use a number line with fifteenths.

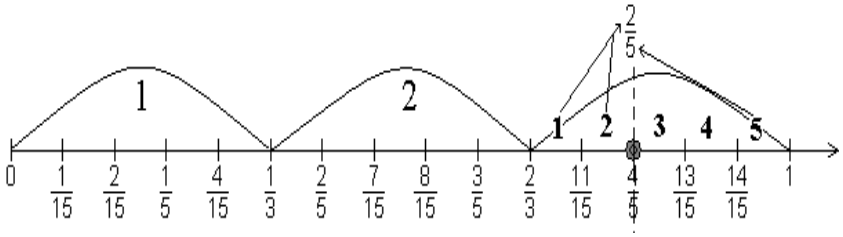


Starting at zero, students would mark off groups of $\frac{5}{15}\left(\frac{1}{3} = \frac{5}{15}\right)$ as shown below:



Two groups of $\frac{4}{15}$ are formed.

Five fifteenths make up one whole and two fifteenths remain. In otherwords, 2 pieces out of 5, or $\frac{2}{5}$, remain.



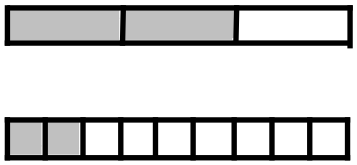
Therefore, $\frac{4}{5} \div \frac{1}{3} = 2\frac{2}{5}$; the same result as with the fraction strips.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Ask students to write the division sentence represented by the diagram shown.



(8N6.7)

- Ask students to use a fraction strip model to determine $\frac{7}{8} \div \frac{1}{4}$. (8N6.7)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.6: Dividing Fractions

ProGuide: pp. 35-40

CD-ROM: Master 3.32

SB: pp. 135-139

PB: pp. 60-61

Number

Specific Outcomes

Students will be expected to

8N6 Continued...

Achievement Indicator:

8N6.8 *Estimate the quotient of two given positive fractions and compare the estimate to whole number benchmarks.*

Suggestions for Teaching and Learning

Students should be encouraged to use estimation to determine the reasonableness of their answers. When estimating quotients close to whole numbers, they should consider the following:

$0 \div n = 0$, where n is any number and $n \neq 0$

$n \div 1 = n$, where n is any number

$1 \div n = \frac{1}{n}$, where n is any number and $n \neq 0$

$n \div n = 1$, where n is any number and $n \neq 0$

Students may need to be reminded that division by zero is undefined. Applying these properties and using whole number benchmarks, students can estimate a quotient, as shown in the table below.

	Determine Benchmarks	Divide Using Benchmarks	Estimate Quotient
$\frac{1}{9} \div \frac{8}{9}$	$\frac{1}{9} \doteq 0,\ \frac{8}{9} \doteq 1$	$0 \div 1 = 0$	$\frac{1}{9} \div \frac{8}{9} \doteq 0$
$\frac{4}{5} \div 2\frac{1}{3}$	$\frac{4}{5} \doteq 1,\ 2\frac{1}{3} \doteq 2$	$1 \div 2 = \frac{1}{2}$	$\frac{4}{5} \div 2\frac{1}{3} \doteq \frac{1}{2}$
$4\frac{1}{5} \div 1\frac{8}{9}$	$4\frac{1}{5} \doteq 4,\ 1\frac{8}{9} \doteq 2$	$4 \div 2 = 2$	$4\frac{1}{5} \div 1\frac{8}{9} \doteq 2$
$2\frac{8}{9} \div 3\frac{1}{10}$	$2\frac{8}{9} \doteq 3,\ 3\frac{1}{10} \doteq 3$	$3 \div 3 = 1$	$2\frac{8}{9} \div 3\frac{1}{10} \doteq 1$

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Anne has $\frac{5}{6}$ of a litre of ice cream. Ask students to estimate and then calculate how many $\frac{1}{2}$ litre cartons could be filled with the ice cream. They should include a diagram with their solution.

(8N6.7, 8N6.8)

Interview

- Ask students to estimate each of the following and explain their thinking.

(i) $24 \div 4\frac{1}{4}$

(ii) $32 \div 7\frac{3}{4}$

(8N6.8)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.6: Dividing Fractions

ProGuide: pp. 35-40

CD-ROM: Master 3.32

SB: pp. 135-139

PB: pp. 60-61

Number

Specific Outcomes

Students will be expected to

8N6 Continued...

Achievement Indicator:

8N6.9 *Generalize and apply rules for dividing positive proper fractions.*

Suggestions for Teaching and Learning

One approach to generalizing rules for dividing fractions involves using previous models. When dividing $\frac{4}{5}$ by $\frac{1}{3}$, for example, students identified a common denominator of 15. Using a rectangle divided into fifteenths allowed students to determine the exact number of groups.

$\frac{4}{5} \div \frac{1}{3} = 2\frac{2}{5}$

Using common denominators, the division can be written as $\frac{12}{15} \div \frac{5}{15}$. Ask students to complete the following:

- Express $2\frac{2}{5}$ as an improper fraction.
- Compare this result to $\frac{12}{15} \div \frac{5}{15}$. What do you notice?

Students should recognize that if the two fractions have a common denominator, the quotient can be obtained by dividing the numerators.

Teachers should also generalize rules for dividing fractions by exploring the connection between division and the related multiplication.

Division Modeled Using Number Lines	Related Multiplication	Conclusion
$\frac{2}{3} \div 3 = \frac{2}{9}$	$\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$	$\frac{2}{3} \div 3 = \frac{2}{3} \times \frac{1}{3}$
$4 \div \frac{2}{3} = 6$	$4 \times \frac{3}{2} = 6$	$4 \div \frac{2}{3} = 4 \times \frac{3}{2}$
$3 \div \frac{2}{3} = 4\frac{1}{2} = \frac{9}{2}$	$3 \times \frac{3}{2} = \frac{9}{2} = 4\frac{1}{2}$	$3 \div \frac{2}{3} = 3 \times \frac{3}{2}$
$\frac{4}{5} \div \frac{1}{3} = 2\frac{2}{5} = \frac{12}{5}$	$\frac{4}{5} \times \frac{3}{1} = \frac{12}{5} = 2\frac{2}{5}$	$\frac{4}{5} \div \frac{1}{3} = \frac{4}{5} \times \frac{3}{1}$

Using the patterns in the table, students should conclude that when dividing two positive proper fractions (or a whole number and a proper fraction), they can write the reciprocal of the divisor and multiply.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

- Sarah carried out the division $\frac{3}{4} \div \frac{2}{3}$ as follows:

$$\frac{3}{4} \div \frac{2}{3} = \frac{4}{3} \times \frac{2}{3}$$
$$= \frac{8}{9}$$

  Ask students whether they agree with Sarah's method and answer. Explain.

  (8N6.9)

- Ask students to explain why $\frac{15}{16} \div \frac{5}{8}$ is half of $\frac{15}{16} \div \frac{5}{16}$.

  (8N6.9)

Performance

- For Five-Minute Review, allow teams five minutes to review how to divide fractions. Students in their groups can ask a clarifying question to the other members or answer questions of others.

  (8N6)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.6: Dividing Fractions

ProGuide: pp. 35-40

Master: 3.21

CD-ROM: Master 3.32

SB: pp.135-140

PB: pp. 60-61

Number

Specific Outcomes

Students will be expected to

8N6 Continued...

Achievement Indicator:

8N6.9 *(Continued) Generalize and apply rules for dividing positive proper fractions.*

Suggestions for Teaching and Learning

The traditional invert-and-multiply algorithm introduces students to the concept of reciprocal. Reciprocals are two numbers whose product is 1. For example, $\frac{3}{4}$ and $\frac{4}{3}$ are reciprocals because $\frac{3}{4}\times\frac{4}{3}=\frac{12}{12}=1$.
A reciprocal is the result of switching the numerator and the denominator in a fraction. Teachers should reinforce that any whole number can be written in fractional form with a denominator of 1.

This algorithm is probably one of the most poorly understood procedures in intermediate mathematics. For the benefit of teachers, a mathematical justification for this approach is provided here.

Why Multiplying by the Reciprocal Works (Example)	Explanation of Steps
$\frac{2}{3}\div\frac{4}{5}=\blacklozenge$	Division in fractional form.
$\dfrac{\frac{2}{3}}{\frac{4}{5}}=\blacklozenge$	
$\left(\dfrac{\frac{2}{3}}{\frac{4}{5}}\right)\left(\frac{4}{5}\right)=\blacklozenge\left(\frac{4}{5}\right)$	Multiply each side of the equation by the denominator: $\frac{4}{5}$
$\left(\dfrac{\left(\frac{2}{3}\right)\left(\frac{4}{5}\right)}{\left(\frac{4}{5}\right)}\right)=\blacklozenge\left(\frac{4}{5}\right)$	Simplify.
$\frac{2}{3}=\blacklozenge\times\frac{4}{5}$	
$\frac{2}{3}\times\frac{5}{4}=\blacklozenge\times\frac{4}{5}\times\frac{5}{4}$	Isolate $\blacklozenge$ by multiplying both sides of the equation by $\frac{5}{4}$, the reciprocal of $\frac{4}{5}$.
$\frac{2}{3}\times\frac{5}{4}=\blacklozenge\times\left(\frac{4}{5}\times\frac{5}{4}\right)$	1 is the product of the reciprocals.
$\frac{2}{3}\times\frac{5}{4}=\blacklozenge\times(1)$	
$\frac{2}{3}\times\frac{5}{4}=\blacklozenge$	
$\therefore\ \frac{2}{3}\div\frac{4}{5}=\frac{2}{3}\times\frac{5}{4}$	This is true because $\frac{2}{3}\div\frac{4}{5}=\blacklozenge$ and $\frac{2}{3}\times\frac{5}{4}=\blacklozenge$.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Students could play a game of Bingo to reinforce division of positive proper fractions. Teachers should select a division statement for students to calculate. If students have the quotient on their card they should cross it off. The first students having a straight line wins.

(8N6.9)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.6: Dividing Fractions

ProGuide: pp. 35-40

Master 3.21

CD-ROM: Master 3.32

SB: pp. 135-140

PB: pp. 60-61

Number

Specific Outcomes

Students will be expected to

8N6 Continued...

Achievement Indicator:

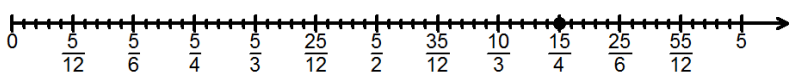
8N6.10 *Model, generalize and apply rules for dividing fractions with mixed numbers.*

Suggestions for Teaching and Learning

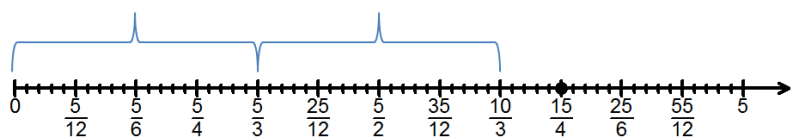
Work with mixed numbers is a natural extension of the modelling and rules applied to dividing proper fractions. To model $3\frac{3}{4} \div 1\frac{2}{3}$ on a number line, for example, students should begin by writing equivalent improper fractions with a common denominator.

$$3\frac{3}{4} \div 1\frac{2}{3}$$
$$= \frac{15}{4} \div \frac{5}{3}$$
$$= \frac{45}{12} \div \frac{20}{12}$$

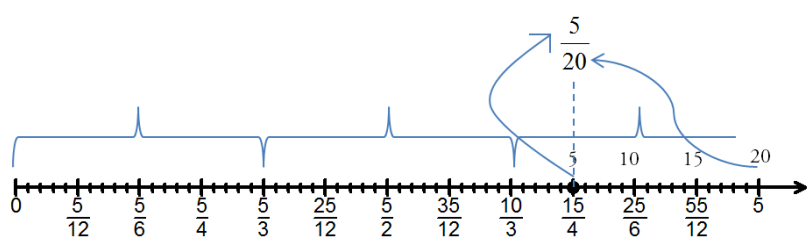
They should construct a number line with twelfths and identify $\frac{45}{12}\left(\frac{45}{12} = \frac{15}{4}\right)$, the first fraction in the operation.



Starting at zero, they would indicate groups of $\frac{20}{12}\left(\frac{20}{12} = \frac{5}{3}\right)$:



Two groups of $\frac{20}{12}$ are formed, with 5 twelfths remaining. Since 20 twelfths make up one whole, $\frac{5}{20} = \frac{1}{4}$, remain.



Therefore, $3\frac{3}{4} \div 1\frac{2}{3} = 2\frac{1}{4}$. Students should be encouraged to check the reasonableness of their answer: $3\frac{3}{4} \doteq 4$, $1\frac{2}{3} \doteq 2$ and $4 \div 2 = 2$. The answer $2\frac{1}{4}$ is reasonable.

Both strategies (determining common denominators and multiplying by the reciprocal) can be used to determine the quotient of fractions with mixed numbers. Students should rewrite each mixed number as an equivalent improper fractions and determine the quotient as they did with proper fractions.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Portfolio

- Ask students to answer the following question:

  Caitlin decides to make muffins for the school picnic. Her recipe requires $2\frac{1}{4}$ cups of flour to make 12 muffins. There were exactly 18 cups of flour in the canister, so she decided to use all of it.

  (i) How many muffins can Caitlin expect to get?

  (ii) The principal of the school liked Caitlin's muffins and asked her to cater the school picnic next year, providing enough muffins for all 400 students. How many cups of flour will Caitlin require? (8N6.10)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.7: Dividing Mixed Numbers

ProGuide: pp. 41-46
Master 3.22

CD-ROM: Master 3.33

SB: pp. 141-146

PB: pp. 62-63

Number

Specific Outcomes

Students will be expected to

8N6 Continued...

Achievement Indicators:

8N6.11 *Provide a context that requires the dividing of two given positive fractions.*

Suggestions for Teaching and Learning

Brainstorm with students situations that involve the division of two fractions. Students might suggest some of the following:

- sharing leftover pizza
- adjusting a recipe
- construction

Making connections between the division of fractions and their lives will strengthen student understanding. Students should write word problems that fit a given division expression. Encourage them to share their problems with their classmates.

8N6.12 *Identify the operation required to solve a given problem involving positive fractions.*

Provide groups of students with a variety of problems and ask them to determine the operations required to solve each. Ask them to share with their classmates how they made their decisions. In some cases a key word will help them identify which operation is necessary to solve a given problem. Encourage students to record these key words in a table such as the following:

Addition	*Subtraction*	*Multiplication*	*Division*
Sum	Difference	Product	Quotient
Total	Exceed	Multiply	Equal Shares
Altogether	Subtract	Times	Equal Groups
	How much greater than?		Divide
	How much less than?		

Students could continue to add to this table as they encounter new contexts. Remind students that they should read each question carefully and consider the entire context to ensure the operation they select makes sense.

To emphasize the importance of reading each question carefully, they should consider the following:

- Jack can usually drive home at an average speed of 50 km/h. One day, a winter storm reduced his speed ***by*** three-fifths of his usual speed. What was his average speed on his drive home that day?
- Jack can usually drive home at an average speed of 50 km/h. One day, a winter storm reduced his speed ***to*** three-fifths of his usual speed. What was his average speed on his drive home that day?

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Shelley's salsa recipe is very popular. This ingredient list makes enough salsa for 6 people.

  $2\frac{1}{4}$ cups of diced tomatoes
  $\frac{1}{2}$ cup of onions
  $\frac{3}{4}$ teaspoon of salt
  $\frac{1}{8}$ teaspoon of sugar
  $\frac{2}{3}$ cup of green pepper

  (i) Shelly is having a party and will have 17 guests. Ask students how she should change the ingredient list to ensure she has enough salsa for her party. They should write out the new ingredient list.
  (ii) Ask students: If Shelly is having a movie night and will only have 2 people to share her salsa, how much smaller will the batch be? They should write out the new ingredient list.

  (8N6.4 and 8N6.11)

- Ask students to create a problem that could be solved using the following operations:

  (i) 3 divided by $\frac{1}{4}$
  (ii) $1\frac{2}{3}$ divided by $\frac{1}{6}$
  (iii) $\frac{3}{4}$ divided by $\frac{1}{6}$

  They should solve their problem. Encourage students to share their problems with the class.

  (8N6.9, 8N6.10, 8N6.11)

- Give students a set of word problems. Ask them to identify the operation involved in solving the problem and explain how they know.

  (8N6.12)

Performance

- As a class activity, have students participate in *Fractions in Everyday Life,* in which they must adjust and prepare a cookie recipe for their class.

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.6: Dividing Fractions

Lesson 3.7: Dividing Mixed Numbers

Lesson 3.8: Solving Problems with Fractions

ProGuide: pp. 39, 45,52

Master 3.9a, 3.9b

SB: pp. 140, 146, 152

PB: pp. 60-61, 62- 63, 64-66

Suggested Resource

https://www.k12pl.nl.ca/curr/7-9/math/grade8/links/unit3.html

- Link to *Fractions in Everyday Life* Activity

Number

Specific Outcomes

Students will be expected to

8N6 Continued...

Achievement Indicator:

8N6.13 *Solve a given problem involving positive fractions taking into consideration order of operations (limited to problems with positive solutions).*

Suggestions for Teaching and Learning

In Grade 6, students applied the order of operations to solve problems involving whole numbers. In Grade 7, they built upon this knowledge by applying the order of operations to include decimal numbers. In this course, they explored the order of operations with integers and will now extend this to fractions. Students are working with positive fractions only and questions must be limited to those that have positive solutions.

The mnemonic BEDMAS is often used to remember the order of operations. Since exponents are not included as part of this outcome, students could be encouraged to create their own mnemonic. Remind them that division and multiplication are completed in the order they appear from left to right, as are addition and subtraction.

Students should be asked to evaluate expressions such as the following:

- $\frac{1}{4}\times\left(\frac{7}{8}-\frac{1}{2}\right)$
- $3\frac{1}{4}\div\left(2\frac{1}{2}-\frac{1}{4}\right)$
- $\left(\frac{7}{15}-\frac{1}{5}\right)\div\frac{2}{3}$
- $\frac{5}{8}+\frac{1}{3}\times\frac{3}{4}\div\frac{1}{2}-\frac{2}{5}$

While concrete and pictorial representations may continue to be helpful for students who are having difficulty with addition, subtraction, multiplication and division of fractions, the expectation is that students be able to solve problems that involve the order of operations symbolically.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

- Margie is entering a competition to win a cell phone. She must answer the following skill-testing question.

  What is the value of $10 - 2 \times \frac{1}{2}$? Ask students to answer the following questions:
  (i) How could Margie determine a possible answer of 4?
  (ii) How could Margie determine a possible answer of 9?
  (iii) What is the correct answer? Explain.

  (8N6.13)

- Ask students to respond to the following:

  How does knowing the order of operations help ensure that you get the same answer to $\frac{3}{4} + \frac{1}{4} \times \frac{5}{12}$ as other students in the class?

  (8N6.13)

Paper and Pencil

- Ask students to insert one set of brackets to make the following statements true. They should justify their answer.

  (i) $\frac{1}{2} + \frac{1}{4} \times \frac{2}{3} = \frac{1}{2}$

  (ii) $\frac{3}{4} \times \frac{1}{5} + \frac{2}{3} \times \frac{5}{3} = 1\frac{1}{12}$

  (8N6.13)

- Ask students to evaluate each of the following expressions:

  (i) $\frac{7}{4} \times \frac{2}{3} - \frac{5}{6}$

  (ii) $\frac{5}{6} \times \frac{1}{2} - \frac{3}{4} \div \frac{2}{3}$

  (iii) $\left(4\frac{1}{2} + \frac{3}{4}\right) \div 2\frac{1}{3}$

  (8N6.13)

- Jay has soccer practice every day from Monday to Friday for $1\frac{1}{2}$ h. On Saturday, he stayed to the soccer field for an additional $3\frac{3}{4}$ h. Ask students to determine the total number of hours Jay practiced soccer for the week.

  (8N6.13)

- There are 20 Grade 8 students attending a Winter Trip to Ski Mountain. $\frac{1}{4}$ of them plan on cross country skiing, $\frac{2}{5}$ plan on snowboarding, and the rest plan on downhill skiing. Ask students to determine the number of students who plan on downhill skiing.

  (8N6.13)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 3.8: Solving Problems with Fractions

Lesson 3.9: Order of Operations with Fractions

ProGuide: pp. 47-52, 53-55

Master 3.24

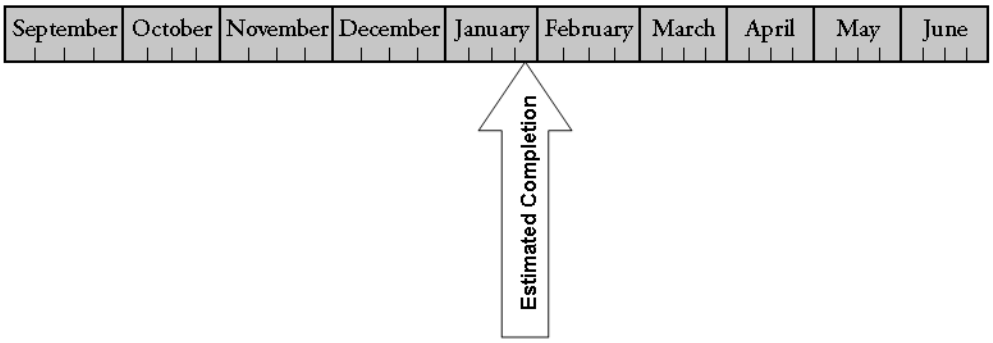
CD-ROM: Master 3.34, 3.35

SB: pp. 147-152, 153-155

PB: pp. 64-66, 67-68

Measuring Prisms and Cylinders

Suggested Time: 5 Weeks



Unit Overview

Focus and Context

In this unit, students will use nets to create three-dimensional solids. They will explore the faces of various nets to make connections between the area of the 2-D shape and the surface area of a given 3-D object. Through such exploration, students will develop and apply formulas for determining the surface area of right prisms and right cylinder. They will then explore the amount of space enclosed within prisms and cylinders, and develop and apply formulas for determining the volume of these solids. Throughout the unit, students will be encouraged to draw diagrams and models to help them visualize the 3-D objects that are described.

Developing a good understanding of surface area and volume of 3-D objects will help students analyze situations in their own lives. Activities such as painting a room, wrapping a present, filling a water bottle, putting siding on a house, tracking waste management, and determining the amount of concrete required for a project all require an understanding of surface area and volume. Students will gain an appreciation of these concepts through relevant problem solving activities.

Outcomes Framework

GCO
Use direct and indirect measurement to solve problems.

SCO 8SS2

Draw and construct nets for 3-D objects.

SCO 8SS3

Determine the surface area of:

- right rectangular prisms
- right triangular prisms
- right cylinders

to solve problems.

SCO 8SS4

Develop and apply formulas for determining the volume of right prisms and right cylinders.

SCO Continuum

Grade 7	Grade 8	Grade 9
Strand: Shape and Space (Measurement)		
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
7SS1 Demonstrate an understanding of circles by: • describing the relationships among radius, diameter and circumference • relating circumference to pi • determining the sum of the central angles • constructing circles with a given radius or diameter • solving problems involving the radii, diameters and circumferences of circles.	8SS2 Draw and construct nets for 3-D objects. 8SS3 Determine the surface area of: • right rectangular prisms • right triangular prisms • right cylinders to solve problems. 8SS4 Develop and apply formulas for determining the volume of right prisms and right cylinders.	9SS1 Solve problems and justify the solution strategy, using the following circle properties: • the perpendicular from the center of a circle to a chord bisects the chord • the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc • the inscribed angles subtended by the same arc are congruent • a tangent to a circle is perpendicular to the radius at the point of tangency 9SS2 Determine the surface area of composite 3-D objects to solve problems. 9SS3 Demonstrate an understanding of similarity of polygons.

Mathematical Processes

[C] **Communication**	[PS] **Problem Solving**
[CN] **Connections**	[R] **Reasoning**
[ME] **Mental Mathematics and Estimation**	[T] **Technology**
	[V] **Visualization**

Shape and Space (Measurement)

Specific Outcomes

Students will be expected to

8SS2 Draw and construct nets for 3-D objects. [C, CN, PS, V]

Achievement Indicators:

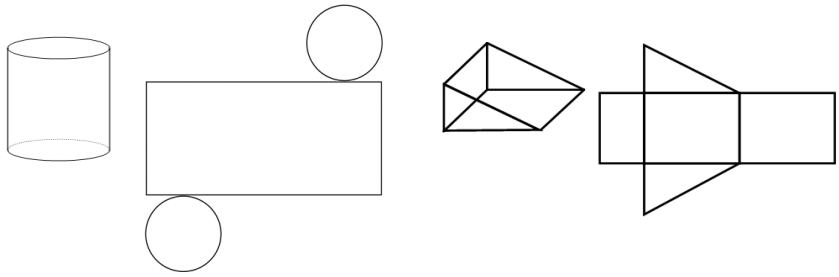
8SS2.1 *Match a given net to the 3-D object it represents.*

8SS2.2 *Draw nets for a given right circular cylinder, right rectangular prism, and verify by constructing the 3-D objects from the nets.*

Suggestions for Teaching and Learning

In previous grades, the focus has been on the study of two-dimensional shapes. In this unit, students will explore 3-D objects. They will draw nets, determine the correct nets for different objects, and build 3-D objects from nets before extending this knowledge to explore the surface area and volume of cylinders and prisms.

This will be students' first exposure to the use of nets to investigate and create 3-D solids. A net is a two-dimensional figure that can be cut out and folded up to make a three-dimensional solid. The nets of various 3-D objects are shown below:



A strong understanding of the nets for 3-D shapes will strengthen student understanding of surface area and volume later in the unit. Understanding concrete models allows students to visualize the object and encourages them to use reasoning rather than merely attempt to follow procedures or formulae.

As students explore various nets, they should become comfortable with the use of the following terms:

Net	Solid	Cylinder	Regular prism
Right prism	Area	Volume	Right Rectangular Prism
Cube	Surface Area	Right Triangular Prism	Polyhedron

When students draw nets, their focus should be on the faces, and how they fit together to form the solid. Encourage them to visualize what the 3-D objects would look like if they were taken apart. Students should be reminded that the pieces must be the correct size to fit together, especially the circles on a cylinder. Ensure that there are no overlapping pieces or that there is no hole where there should be a side. They must also be reminded to connect the shapes in the net. They may have all the pieces, but still have difficulty drawing the net.

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

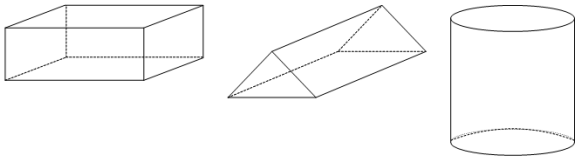
Performance

- Give students a deck of cards. Half the cards should contain nets and the other half should contain pictorial representations of the associated 3-D objects. Ask students to match each net to the 3-D object it represents.

(8SS2.1)

Paper and Pencil

- Provide students with a set of 3-D objects, such as the ones shown below.



Ask them to construct the net for each solid. They should share their nets with their classmates.

(8SS2.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 4.1: Exploring Nets

ProGuide: pp. 4-10

Master 4.6

CD-ROM: Master 4.36

Student Book (SB): pp. 170-176

Practice and Homework Book (PB): pp. 76-77

Shape and Space (Measurement)

Specific Outcomes

Students will be expected to

8SS2 Continued...

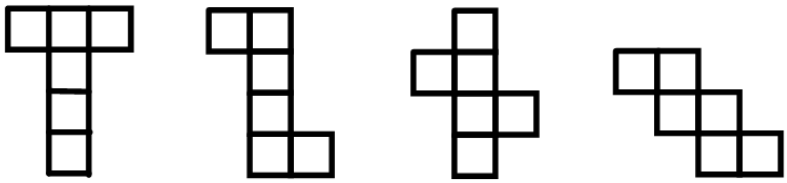
Achievement Indicators:

8SS2.3 *Predict 3-D objects that can be created from a given net and verify the prediction.*

8SS2.4 *Construct a 3-D object from a given net.*

Suggestions for Teaching and Learning

Students should understand that a given 3-D object can be created from more than one net. A cube, for example, can be created from the following nets:



Through investigation, students may begin to develop strategies to predict the object that will be formed from a given net:

- The net of a rectangular prism should have six rectangular sides.
- The net of a triangular prism has five sides, two of which are congruent triangles and three of which are rectangles.
- The net of a cylinder should have two circles and a rectangle.

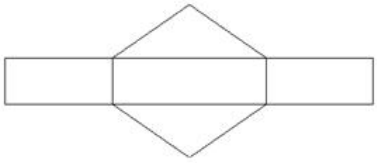
Students should always be encouraged to predict before actually folding the nets to construct the 3-D objects.

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Performance

- Give students a net, such as the one shown below, and ask them to predict the 3-D object that can be created from it.



After making their prediction, encourage students to fold the net to construct the 3-D object.

(8SS2.3, 8SS2.4)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 4.2: Creating Objects from Nets

ProGuide: pp. 11-16

CD-ROM: Master 4.37

SB: pp. 177-182

PB: pp. 78-80

Shape and Space (Measurement)

Specific Outcomes

Students will be expected to

8SS3 Determine the surface area of:

- **right rectangular prisms**
- **right triangular prisms**
- **right cylinders**

to solve problems.

[C, CN, PS, R, V]

Achievement Indicators:

8SS3.1 *Identify all the faces of a given prism, including right rectangular and right triangular prisms.*

8SS3.2 *Explain, using examples, the relationship between the area of a 2-D shape and the surface area of a given 3-D object.*

Suggestions for Teaching and Learning

In Grade 6, students developed the formula for determining the area of a rectangle and the volume of right rectangular prisms. In Grade 7, they extended this knowledge to include the area of triangles, parallelograms and circles. They will now extend this to determine the surface area and volume of cylinders and prisms. The use of the corresponding nets is essential in developing and strengthening student understanding of the relationship between the area of 2-D shapes and the surface area of 3-D shapes.

In Mathematics 3 students identified the faces, edges and vertices of given 3-D objects including cubes, spheres, cones, cylinders, pyramids and prisms. Students may benefit from a review of these terms.

Teachers should present various 3-D objects, including right rectangular prisms and right triangular prisms. Ask students questions such as:

- How many faces does the object have?
- How many edges does the object have?
- How many vertices does the object have?

Ask students to identify the faces, edges, and vertices of each 3-D object. They should also identify congruent faces in a given prism. In a rectangular prism, for example, students should recognize that the top and bottom rectangular faces are congruent, the left and right rectangular faces are congruent, and the front and back rectangular faces are congruent. Similarly, they should identify the two triangular faces in a right triangular prism as being congruent.

Students should make connections between the area of a 2-D shape and the surface area of a 3-D object. Provide students with a 3-D object such as a cereal bar box, a Tolberone box, or a poster tube. Ask students to decompose their object to construct its net. Ask them questions such as the following:

- What shape is each face of your net?
- How could you find the area of each face?
- How could you find the total area of your net?

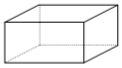
Introduce students to the term surface area as the sum of the areas of all faces or surfaces of a solid. Having this understanding of surface area will allow students to determine the surface area of **any** 3-D object. Students should identify the faces, determine the necessary dimensions of each face, and apply appropriate formulas to calculate area.

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Paper and Pencil

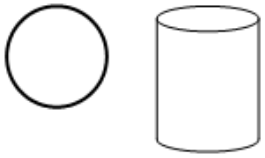
- Ask students to complete the following table:

3-D Object	Identify the object	Total number of faces	Total number of triangular faces	Total number of rectangular faces
				
				

(8SS3.1)

Journal

- Ask students to respond to the following:
  How does the notion of area apply to these two objects?



(8SS3.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 4.3: Surface Area of a Right Rectangular Prism

Lesson 4.4: Surface Area of a Right Triangular Prism

ProGuide: pp. 17-21, 22-27

CD-ROM: Master 4.38, 4.39

SB: pp. 183-187, 188-193

PB: pp. 81-82, 83-84

Shape and Space (Measurement)

Specific Outcomes

Students will be expected to

8SS3 Continued...

Achievement Indicators:

8SS3.3 *Describe and apply strategies for determining the surface area of a given right rectangular or right triangular prism.*

8SS3.4 *Solve a given problem involving surface area.*

Suggestions for Teaching and Learning

Students should work with a variety of right rectangular prisms and right triangular prisms so that they become proficient at recognizing the faces and analyzing the corresponding nets. Consider the following:

The surface area of each prism could be determined by using the net. Working from the net also allows for easy identification of congruent faces, which will allow for a more efficient determination of the surface area. Since the triangular faces in a right triangular prism are congruent, for example, students could determine the area of one of the triangular faces and then multiply it by 2 instead of replicating the same calculation.

Through various explorations, students should begin to notice patterns in determining the surface area of a right rectangular prism or a right triangular prism. For a right rectangular prism, for example, there are three pairs of congruent rectangles. Students could determine the area of each different rectangle and multiply by two (since there are two of each). To ensure students have gained the conceptual understanding of surface area, such strategies should be explored prior to introducing the formula: $SA = 2lw + 2lh + 2wh$ or $SA = 2(lw + lh + wh)$.

Similarly, the net of a triangular prism shows the two congruent triangular faces and three rectangular faces making up the prism. Students should recognize that the surface area of a right triangular prism can be found by determining the sum of the areas of the two triangular faces and the areas of each rectangular face. Encourage students to investigate the relationships that exist if the triangular faces are isosceles or equilateral.

When determining the surface area of a 3-D object, students should express their answer using square units (usually cm^2 or m^2).

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Journal

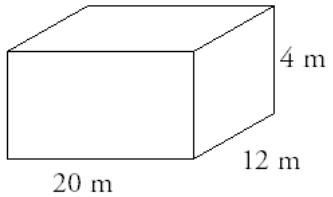
- Ask students to describe how to determine the surface area of a rectangular prism. Ask if there is anything that would shorten the process.

(SS3.3)

Paper and Pencil

- Ask students to answer the following:

  Your are renovating your home (shown below) and will be replacing the siding. Siding is on sale for $15.00 per square metre. How much will this project cost (not considering windows and doors)?



(SS3.5)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 4.3: Surface Area of a Right Rectangular Prism

Lesson 4.4: Surface Area of a Right Triangular Prism

ProGuide: pp. 17-21, 22-27

CD-ROM: Master 4.38, 4.39

SB: pp. 183-187, 188-193

PB: pp. 81-82, 83-84

Shape and Space (Measurement)

Specific Outcomes

Students will be expected to

8SS3 Continued...

Achievement Indicators:

8SS3.5 *Describe and apply strategies for determining the surface area of a given right cylinder.*

8SS3.4 *(Continued) Solve a given problem involving surface area.*

Suggestions for Teaching and Learning

Teachers should use objects such as paper towel rolls, Pringles cans, poster tubes or soup cans to develop the surface area of right cylinders. When using a can of soup, for example, the label can be removed to represent the curved surface (rectangle) and will allow students to see the relationship between the circumference of the circular faces and the length of the corresponding curved surface (rectangle). Ask students questions such as:

- What shape is the top and bottom of your cylinder?
- How can you determine the area of the two circles?
- What shape is the curved surface of your cylinder?
- How can you determine the area of the rectangle?
- What is the length of the rectangle?
- How can you determine the circumference of the circle?
- What is the width of the rectangle?

Students should recognize that the top and bottom of the cylinder are circles. They know that the area of a circle is $A = \pi r^2$ and they should recognize that the two circles are congruent. Students may experience more difficulty with determining the length of the rectangle. The use of a cylinder that can be unrolled will allow them to see that the length of the rectangle is the circumference of the circle, which can be determined by using $C = \pi d$ or $C = \pi(2r) = 2\pi r$. The width of the rectangle is the height of the cylinder. The area of the rectangle (curved surface) of the cylinder can be determined using $A = \pi dh$ or $A = 2\pi rh$. Some students may conclude that the area of a cylinder can be determined using

$$SA = 2A_{\text{circle}} + A_{\text{curved surface (rectangle)}}$$
$$SA = 2(\pi r^2) + lw$$
$$SA = 2\pi r^2 + C_{\text{circle}} \times h_{\text{cylinder}}$$
$$SA = 2\pi r^2 + 2\pi rh$$

or

$$SA = 2A_{\text{circle}} + A_{\text{curved surface (rectangle)}}$$
$$SA = 2(\pi r^2) + lw$$
$$SA = 2\pi r^2 + C_{\text{circle}} \times h_{\text{cylinder}}$$
$$SA = 2\pi r^2 + \pi dh$$

As with prisms, students should develop a conceptual understanding of the surface area of a cylinder before being introduced to the formula.

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Journal

- Ask students to respond to the following:

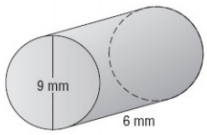
  Using an example, demonstrate how the area of the curved surface of a cylinder is related to the area of a rectangle.

  (8SS3.4)

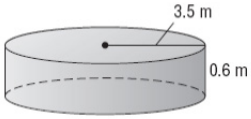
Paper and Pencil

- Ask students to determine the surface area of each of the following cylinders:

  (i)



  (ii)



  (8SS3.4)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 4.7: Surface Area of a Right Cylinder

ProGuide: pp. 43-48

CD-ROM: Master 4.42

SB: pp. 209-214

PB: pp. 90-92

Shape and Space (Measurement)

Specific Outcomes

Students will be expected to

8SS4 Develop and apply formulas for determining the volume of right prisms and right cylinders. [C, CN, PS, R, V]

Achievement Indicators:

8SS4.1 *Determine the volume of a given right prism, given the area of the base.*

8SS4.2 *Explain the connection between the area of the base of a given right 3-D objects and the formula for the volume of the object.*

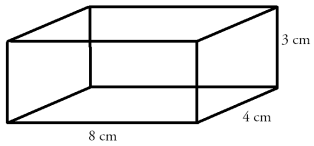
Suggestions for Teaching and Learning

Volume refers to the amount of space filled by three-dimensional objects. It is measured in cubic units. In Mathematics 6, students developed and applied the formula for determining the volume of right rectangular prisms. They will now extend this knowledge to include right triangular prisms and right cylinders.

To activate prior knowledge, give students a rectangular prism made out of linking cubes to students and ask the following questions:

- How can you determine the volume of this rectangular prism?
- How is the volume of the rectangular prism related to the area of the base?

Some students may suggest counting the number of cubes to determine the volume. Others may suggest multiplying the number of cubes in each layer (the area of the base) by the number of layers (the height). Since the base of a rectangular prism is a rectangle, this can also be written as $V = l \times w \times h$. Consider the prism below:



V = (number of cubes in one layer) × (number of layers)
V = (Area of the base) × (height)
V = (8 cm × 4 cm) × 3 cm
V = 72 cm^3

Teachers should present a triangular prism and ask students how they could determine its volume. They may suggest using the same method: V = (Area of the Base) × Height. Students should begin exploring triangular prisms in which the area of the base is given.

Ask students how they could determine the volume of a triangular prism if the area of the base is not given. Since the base of a triangular prism is a triangle, it follows that the volume can be determined by calculating the area of the triangular face and multiplying it by the height of the prism. Remind students that the area of a triangle is

$A = \frac{\text{base} \times \text{height}}{2} = \frac{bh}{2}$ or $A = \frac{1}{2}bh$

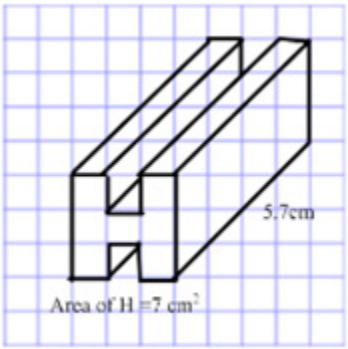
A common misconception is that the base of the prism is the bottom face (the face the object is lying on). Students need to recognize that the base of a cylinder is one of the circular faces, the base of a triangular prism is one of the triangular faces, and the base of a rectangular prism is one of the rectangles. Exploring prisms in varying positions should emphasize the term base as it relates to volume calculations.

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Portfolio

- Ask students to draw a right prism that has a base in the shape of their first initial using 1 cm grid paper. They will have to draw their first initial as a block letter and extend the vertices to form a 3-D prism. They should complete the following:
  (i) Count the squares to find the area of your initial.
  (ii) Use a ruler to find the height of the prism, and multiply it by the area of the initial to find the volume.



  (iii) How is the area of your initial and the height of your prism related to the volume?
  (iv) How does the shape of the base affect the volume of the prism?
  (8SS4.1)

Performance

- Ask students to construct small gift boxes from greeting cards by following the steps outlined in *Foldable Greeting Card Gift Boxes*. Once the rectangular prism has been created, ask students to complete the following:
  (i) Determine the surface area of the gift box.
  (ii) Determine the volume of the gift box.
  (iii) You would like to fill the gift box with Hershey Kisses. Each Hershey Kiss has a volume of 0.29 cubic centimeters. Determine the number of Hershey Kisses you could fit in your gift box.
  (8SS2.2, 8SS2.4, 8SS3.3, 8SS3.5, 8SS4.5)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 4.5: Volume of a Right Rectangular Prism

Lesson 4.6: Volume of a Right Triangular Prism

ProGuide: pp. 29-35, 36-42

Master 4.31

SB: pp. 195-200, 202-208

PB: pp. 85-86, 87-89

Supplementary Resource

Resource Link: https://www.k12pl/nl.ca/curr/k-6/math/grade-4/links/unit4.html

- *Foldable Greeting Card Gift Boxes* Activity

Shape and Space (Measurement)

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to **8SS4 Continued...** **Achievement Indicators:**	
8SS4.3 *Generalize and apply a rule for determining the volume of right cylinders.* 8SS4.2 *(Continued) Explain the connection between the area of the base of a given right 3-D objects and the formula for the volume of the object.*	Students should make connections between calculating the volume of a prism and calculating the volume of a cylinder. The volume of a rectangular prism can be determined using V = (Area of the Base) × Height. Since the base of a cylinder is a circle, and the area of a circle is $A = \pi r^2$, they should conclude the volume of a cylinder can be determined using $V_{\text{cylinder}} = \pi r^2 h$. Developing formulas in meaningful ways should eliminate the need for students to memorize them as isolated pieces of mathematical facts. Rather, they can derive them from what they already know. As students explore various problem solving situations involving volume, they should be introduced to the relationship between a cubic centimeter and a milliliter: 1 cm^3 = 1 ml.
8SS4.4 *Demonstrate that the position of a given 3-D object does not affect its volume.*	Students should understand that the position of a given 3-D object does not affect its volume. Teachers could present a 3-D object, such as a soup can, and ask students to determine its volume. Stand the soup can on its end and ask for the volume. Then tip the can on its side and ask the class for the volume. Discuss why the volume is the same in each case. Students should recognize that the volume does not change as a result of the cylinder's position, since its dimensions (the radius and height) stay the same. Similarly, when a prism is placed on a different face, the dimensions do not change. The volume does not change.

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

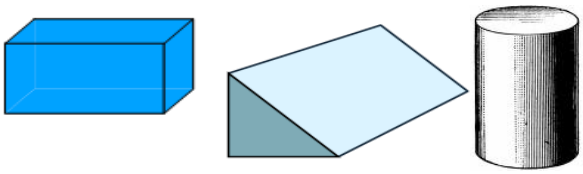
Paper and Pencil

- Ask students to answer the following questions:
  (i) The area of one CD is 11 304 mm^2. Each CD has a height of 1 mm. Sarah has 30 of her CDs on a CD stack. How could you use this information to find the volume?
  (8SS4.2, 8SS4.3)
  (ii) What is the volume of a cylinder having a radius of 14 cm and a height of 12 cm?
  (8SS4.2)

Journal

- Ask students to respond to the following prompt:

  John was absent from class when you learned how to determine the volume of each of the 3-D objects shown below. Explain to John how he can determine the volume of each.



(8SS4.1, 8SS4.2, 8SS4.3)

Portfolio

- Ask students to complete the following task:

  Our class is having a fundraiser by selling popcorn. You must create your own container to help save on expenses. You have sheets of cardboard with dimensions of 27 cm by 43 cm. Would you have a larger volume if you folded the sheets to make cylindrical containers with a height of 27 cm or with a height of 43 cm? (You will add a circular base once the cardboard sheet is used for the sides). Justify your decision mathematically.
  (8SS4.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 4.8: Volume of a Right Cylinder

ProGuide: pp. 49-53

Master 4.33

CD-ROM: Master 4.43

SB: pp. 215-219

PB: pp. 93-94

Shape and Space (Measurement)

Specific Outcomes

Students will be expected to

8SS4 Continued...

Achievement Indicator:

8SS4.5 *Apply a formula to solve a given problem involving the volume of a right cylinder or a right prism.*

Suggestions for Teaching and Learning

Students should solve a variety of problems that involve the volume of a right cylinder or a right prism. Encourage them to draw diagrams (when applicable) or models to help them visualize the shapes described in the problems. Remind students that the volume of any prism or cylinder can be determined by using V = (Area of the Base) × Height. Students should answer questions such as the following:

- The solid wooden dowel used to make a paper towel holder has a radius of 1 *cm* and a height of 37 *cm*. How much wood is in the dowel?
- A triangular prism has a volume of 105 cm^3. It's height is 7 cm. What is the area of its base?
- An aquarium has the following dimensions: length 80 cm, width 35 cm, and height 50 cm. You must fill the aquarium up to 4 cm from the top. How much water will you put in the aquarium?
- Mrs. Hicks is making hot chocolate for her class. The cylindrical hot water urn she is using has a diameter of 25 cm and a height of 50 cm.
  - (i) How much hot chocolate can she make in the urn?
  - (ii) Mrs. Hicks has 30 students in her class. The cups she is using holds 235 ml of hot chocolate each. Is there enough hot chocolate in the urn for her class?

General Outcome: Use Direct or Indirect Measurement to Solve Problems

Suggested Assessment Strategies

Paper and Pencil

- Ask students to determine the volume of a cube having a surface area of 96 cm^2.

(8SS4.1, 8SS4.5)

Performance

- Ask students to create a foldable to summarize what they learned about the surface area and volume of prisms and cylinders.

(8SS2, 8SS4)

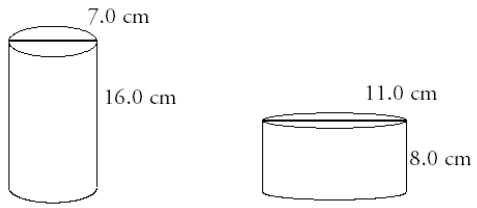
- Ask students to locate one cylindrical object at home. They should create a sketch of the object, including the measures of the diameter and height. Using these dimensions, students should determine the surface area and volume of their chosen cylinder. Alternatively, ask students to locate a right prism at home, record its dimensions and determine the volume and surface area.

(8SS4.3, 8SS4.5)

Journal

- Ask students to respond to the following:

  Joan has a choice between two different ice cream containers at Mac's ice cream shop:



  Both containers cost the same amount.

  Which should Joan choose if she wants to get more ice cream for her money?

(8SS4.3, 8SS4.5)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 4.5: Volume of a Right Rectangular Prism

Lesson 4.6: Volume of a Right Triangular Prism

Lesson 4.8: Volume of a Right Cylinder

ProGuide: pp. 29-34, 36-42, 49-53

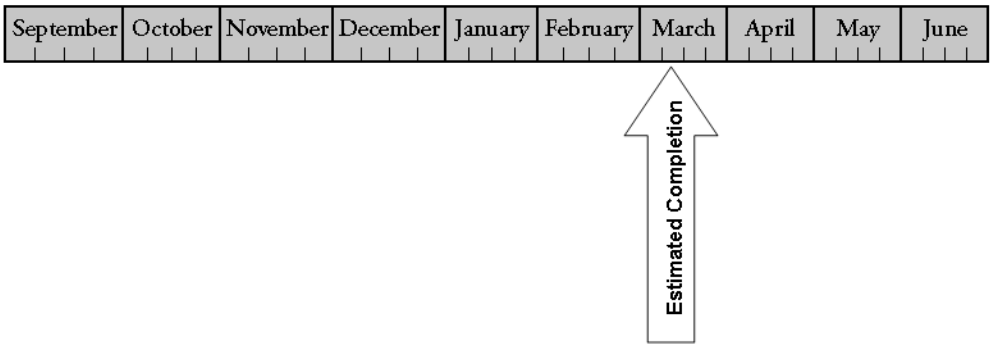
CD-ROM: Master 4.40, 4.41, 4.43

SB: pp. 195-200, 202-208, 215-219

PB: pp. 85-86, 87-89, 93-94

Percent, Ratio, and Rate

Suggested Time: 5 Weeks



Unit Overview

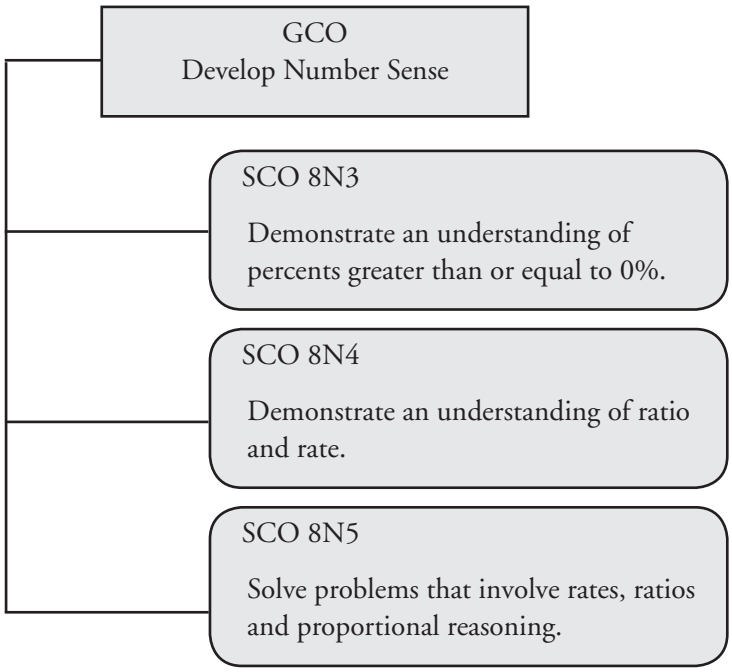
Focus and Context

In this unit, students will extend their knowledge of percents to include percents between 0% and 1%, greater than 100%, and fractional percents. They will represent such percents using grid paper and will translate between percents, decimals, and fractions. Students will continue their previous work with part-to-part and part-to-whole ratios to express ratios as fractions and percents. They will also explore three term ratios. They will express ratios as fractions and percents.

Students will be introduced to rates. They will express rates using words or symbols and identify rates from real life contexts, recording them symbolically.

Students will solve a variety of problems involving percents, ratios, rates and proportional reasoning. Sales tax and discounts, test scores, sports statistics, weather reports, public opinion surveys, nutritional labels, currency conversions, interest charges, commission, speed, fuel consumption, heart rate, and determining the better buy all involve an understanding of percents, ratios, rates and proportional reasoning. A strong understanding of these concepts is necessary for students to make sense of these situations in their daily lives.

Outcomes Framework



SCO Continuum

Grade 7	Grade 8	Grade 9
Strand: Number		
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
7N2 Solve problems involving percents from 1% to 100%. [C, CN, R, T]	8N3 Demonstrate an understanding of percents greater than or equal to 0%. [CN, PS, R, V] 8N4 Demonstrate an understanding of ratio and rate. [C, CN, V] 8N5 Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]	

Mathematical Processes

[C] Communication	**[PS] Problem Solving**
[CN] Connections	**[R] Reasoning**
[ME] Mental Mathematics and Estimation	**[T] Technology**
	[V] Visualization

Number

Specific Outcomes

Students will be expected to

8N3 Demonstrate an understanding of percents greater than or equal to 0%.

[CN, PS, R, V]

Suggestions for Teaching and Learning

In Mathematics 7, students solved problems involving percents from 1% to 100%. More specifically, they converted between percent, fractional and decimal forms, and solved problems involving finding a percent of a number. In this unit, students will solve problems involving percents between 0% and 1% and percents greater than 100%. They will determine the whole number when a percent of it is known, and solve problems involving percent increase and decrease, combined percents, and finding the percent of a percent.

Achievement Indicator:

8N3.1 *Provide a context where a percent may be more than 100% or between 0% and 1%.*

Teachers could activate prior knowledge by asking students to discuss real life contexts involving percents. They may suggest some of the following:

- test marks (78% on a science test)
- sales tax (13% tax on a purchase)
- discount (25% off the regular price)
- probability (10% chance of rain)
- sports statistics (25% of shots on goal were by Jared)

Discuss situations where a percent may be more than 100% or between 0% and 1%. Students may suggest some of the following:

- a test that contains a bonus question may result in a percentage more than 100
- the percent increase of a product from 1970 to 2015
- the chances of a particular team winning the Stanley Cup (as a percent)
- the chances of it snowing in August (as a percent)
- percent of recommended daily intake on nutritional labels

Students should be able to place a given percent in the correct location on a number line. Teachers could set up a string with percent benchmarks such as 0%, 50%, 100%, 150%, and 200%. Provide each students with a percent card and ask them to place their card in the appropriate location on the number line.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Interview

- Ask students to respond to the following statements:
  - (i) When your coach tells you to "give 110%", what does he mean? (8N3.1)
  - (ii) What are the chances of the principal giving you the day off school because of your smile? (8N3.1)
  - (iii) A newspaper article includes 200% in its headline. Provide a context to which the article may be referring. (8N3.1)

Journal

- Ask students to respond to the following:
  - (i) Paul bragged that he received a 105% on his math test. Is this mark possible? Explain. (8N3.1)
  - (ii) Jill predicted that the chance of Maple Academy winning the championship game against Evergreen Collegiate is 0.50%. Which school do you think Jill attends? Explain your choice. (8N3.1)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 5.1: Relating Fractions, Decimals, and Percents

Lesson 5.2: Calculating Percents

ProGuide: pp. 4-11, 12-17

Student Book (SB): pp. 234-241, 242-247

Number

Specific Outcomes

Students will be expected to

8N3 Continued...

Achievement Indicators:

8N3.2 *Represent a given fractional percent using grid paper.*

8N3.3 *Represent a given percent greater than 100 using grid paper.*

8N3.4 *Determine the percent represented by a given shaded region on a grid and record it in decimal, fractional and percent form.*

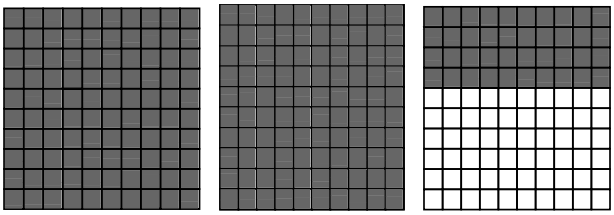
Suggestions for Teaching and Learning

In Mathematics 6, students represented percents from 1% to 100% using a hundreds grid, whereby each small square represented one hundredth or 1%. They will extend this to represent percents between 0% and 1%, percents greater than 100%, as well as other fractional percents.

To activate prior knowledge, begin with using a hundreds grid to represent a whole number percent between 1 and 100, such as 84%. Ask them questions such as:

- How could you represent 110% using a hundreds grid?
- How could you represent 0.5% using a hundreds grid?
- How could you represent 28.25% using a hundreds grid?

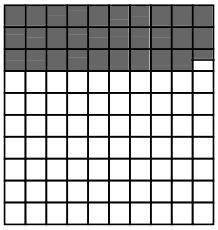
To represent percents greater than 100%, students will have to use more than one hundreds grid chart. To represent 240%, for example, they should completely shade 2 hundreds grids and 40 squares of the third hundreds grid, as shown below.



In this diagram, two full hundreds charts and 40 blocks of another hundred chart are shaded.

To represent fractional percents, students should recognize that they must shade the corresponding fractional part of a square. To represent 0.5%, for example, they would have to shade ½ of one small square. To represent 0.25% they would have to shade ¼ of one small square.

To represent 29.5% using a hundreds grid, students should shade 29 full squares and ½ of another square:



In this diagram, out of the 100 blocks, 29 full blocks and half of another block are shaded. This would represent 29.5%

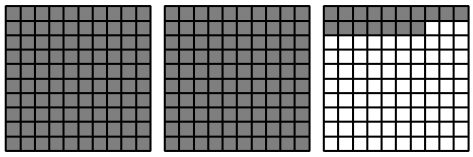
General Outcome: Develop Number Sense

Suggested Assessment Strategies

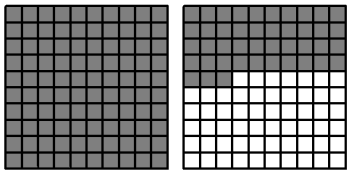
Paper and Pencil

- Ask students to identify the percent represented by each of the following diagrams:

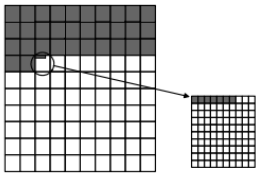
(i)



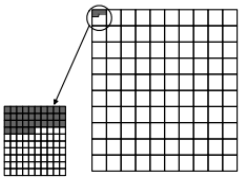
(ii)



(iii)



(iv)



(8N3.4)

- Ask students to represent each of the following percents using grid paper:

  (i) 140%

  (ii) 71.42%

  (iii) 0.64%

(8N3.2, 8N3.3)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 5.1: Relating Fractions, Decimals, and Percents

Lesson 5.2: Calculating Percents

ProGuide: pp. 4-11, 12-17

CD-ROM: Master 5.21, 5.22

SB: pp. 234-241, 242-247

Practice and Homework Book (PB): pp. 102-104, 105-106

Number

Specific Outcomes

Students will be expected to

8N3 Continued...

Achievement Indicators:

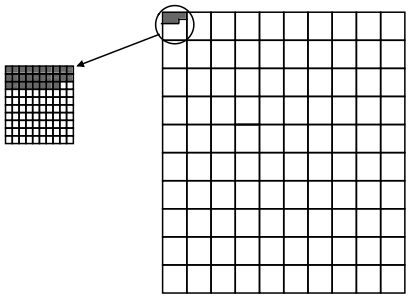
8N3.2 *(Continued) Represent a given fractional percent using grid paper.*

8N3.3 *(Continued) Represent a given percent greater than 100 using grid paper.*

8N3.4 *(Continued) Determine the percent represented by a given shaded region on a grid and record it in decimal, fractional and percent form.*

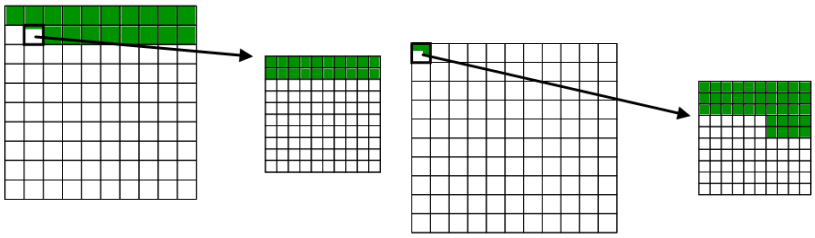
Suggestions for Teaching and Learning

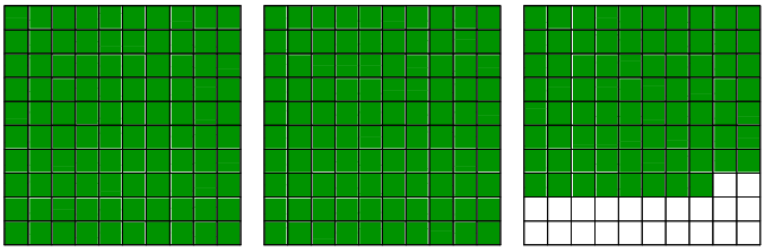
Other percents, such 0.28%, are more difficult to represent using the hundreds grid. Such percentages require the use of a second hundreds grid (hundredths chart), where each small square represents 0.01%. Students should use their hundreds grid to shade an approximation of 0.28 and then use the hundredths grid to shade 28 out of the 100 squares, as shown below.



In this diagram, part of a block is shaded. The hundredths chart is used and 28 blocks out of 100 are shaded.

When given a shaded region on a grid, such as those shown below, students should determine the percent it represents.





General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Students could play *Find Your Partner.* Create a deck of cards where half of the cards illustrate a pictorial representation of a percent and the other half of the cards illustrate the symbolic representation of the percent. Distribute one card to each student in the class. Ask students to find their partner. If one student has 154%, for example, they must find their classmate who holds the card with the pictorial representation of 154%.

  (8N3.2, 8N3.3, 8N3.4)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 5.1: Relating Fractions, Decimals, and Percents

Lesson 5.2: Calculating Percents

ProGuide: pp. 4-11, 12-17

CD-ROM: Master 5.21, 5.22

SB: pp. 234-241, 242-247

PB: pp. 102-104, 105-106

Number

Specific Outcomes

Students will be expected to

8N3 Continued...

Achievement Indicators:

8N3.5 *Express a given percent in decimal or fractional form.*

8N3.6 *Express a given decimal in percent or fractional form.*

8N3.7 *Express a given fraction in decimal or percent form.*

8N3.4 *(Continued) Determine the percent represented by a given shaded region on a grid and record it in decimal, fractional and percent form.*

Suggestions for Teaching and Learning

In Mathematics 6, students expressed a given percent as a fraction and a decimal, limited to whole number percents between 1% and 100%. They will now express percents between 0% and 1%, percents greater than 100% and other fractional percents in decimal and fractional form.

Teachers could activate prior knowledge by asking students to express 56% as a fraction and then a decimal. Remind them that percents mean 'out of 100.' They should recognize that 56%, for example, is 56 out of 100 or $\frac{56}{100}$. Using their knowledge of place value, students should recognize this as 56 hundredths, or 0.56. Next, ask students how they would write a fractional percent such as 46.7% as a fraction and a decimal. Many students would write $\frac{46.7}{100}$ which is equivalent to $\frac{467}{1000}$ and 0.467.

A common error occurs when students equate 0.1% and 0.1. Similarly, students may confuse $\frac{3}{4}$% with 75%. The use of the hundreds and hundredths grid charts should help them understand the difference between the two.

Students should use their knowledge of place value to convert a given decimal to fractional form. When asked to convert 0.365 to a fraction, for example, students should recognize that this is three hundred sixty five **thousandths**. This understanding should allow them to connect 0.365 to the fraction $\frac{365}{1000}$. Since percent means out of 100, students should create an equivalent fraction having a denominator of 100: $\frac{36.5}{100}$ = 36.5%.

When converting a fraction to a percent or a decimal, students could use proportional reasoning. Consider $\frac{2}{5}$, for example. They could write the proportion:

$$\frac{2}{5} = \frac{?}{100} \xrightarrow{\times 20} \frac{2}{5} = \frac{?}{100} \xrightarrow{\times 20} \frac{2}{5} = \frac{40}{100}$$

Once they have created an equivalent fraction having a denominator of 100, students should easily be able to express the given fraction as 40% or 0.40.

Students should be able to express the shaded region of a grid in fraction, decimal and percent form.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Ask students to copy and complete the table shown below.

Percent	Decimal	Fraction
148%		
$\frac{7}{20}$%		
26.4%		
	2.65	
	0.003	
	0.254	
		$\frac{8}{5}$
		$\frac{1}{250}$
		$\frac{3}{8}$

(8N3.5, 8N3.6, 8N3.7)

Performance

- Create a deck of cards containing a variety of percents, decimals, fractions, and pictorial pairs. Pairs of students can play a game of concentration. Students should place all the cards face down and take turns trying to match the various representations.

(8N3.5, 8N3.6, 8N3.7)

Journal

- Ask students to respond to the following prompt:

  Your friend was absent from school when your teacher explained fractional percents. When he was studying for his test, he said that $\frac{1}{2}$% was 0.5 as a decimal. How would you help him to understand the mistake he made?

(8N3.5, 8N3.6, 8N3.7)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 5.1: Relating Fractions, Decimals, and Percents

Lesson 5.2: Calculating Percents

ProGuide: pp.4-11, 12-17

Master 5.6a, 5.6b

CD-ROM: Master 5.21, 5.22

SB: pp. 234-241, 242-247

PB: pp. 102-104, 105-106

Number

Specific Outcomes

Students will be expected to

8N3 Continued ...

Achievement Indicator:

8N3.8 *Solve a given problem involving percents.*

Suggestions for Teaching and Learning

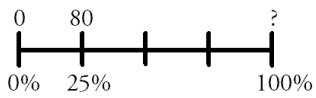
Students should solve a variety of problems involving percents, including:

- determining the percent of a number
- determining the whole number when the percent is given
- determining the percent increase or percent decrease

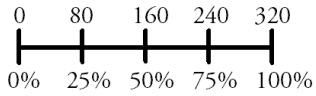
As students explore various problem solving situations, encourage them to use a variety of strategies.

- 25% of a number is 80. What is the number?

  Students could use a number line and benchmarks to determine the solution.



Place 80 above the 25% mark on a number line that runs from 0% to 100%.



Write appropriate multiples of 80 above the appropriate multiples of 25% until you reach 100%.

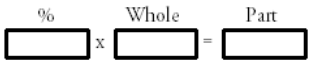
The matching multiples, 320 and 100%, are equivalent.

- 5% of a number is 20. What is the number?

  Students should recognize that since 5% of a number is 20, then 1% of the number must be 4 (20 ÷ 5 = 4). It follows that 100% of the number must be 400 (4 × 100).

  Alternatively, they may recognize that if 5% of the number is 20, then 10% of the number is 40, and 100% of the number would be 400.

  Students could also write an equation to determine the number:



  5% of a number = 20

  0.05 × ? = 20 or $0.05x = 20$

  Dividing both sides of the equation by 0.05 results in ? = 400 or $x = 400$.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Ask students to respond to the following questions:
  - (i) Trina received an 80% on a recent math test. If she answered 48 questions correctly, how many questions were on the test? (8N3.8)
  - (ii) Adam increased his song list by 40%. If he had 300 songs originally, how many songs does he now have? (8N3.8)
  - (iii) Shawn earned $85 and spent $15. What percent of his money did he spend? (8N3.8)
  - (iv) Last week the canteen sold 60 sandwiches. This week they sold 48 sandwiches. Calculate the percent change. How can you check your answer? (8N3.8)

Journal

- Ask students to respond to the following:

  Catherine said that her amount of homework increased 400% when it went from one half hour of work to two hours of work. Do you agree? Explain. (8N3.8)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 5.2: Calculating Percents

Lesson 5.3: Solving Percent Problems

Lesson 5.4: Sales Tax and Discount

ProGuide: pp. 12-17, 18-25, 26-33

CD-ROM: Master 5.22, 5.23, 5.24

SB: pp. 242-247, 248-255, 256-262

PB: pp. 105-106, 107-109, 110-111

Number

Specific Outcomes

Students will be expected to

8N3 Continued...

Achievement Indicator:

8N3.8 *(Continued) Solve a given problem involving percents.*

Suggestions for Teaching and Learning

- The enrolment in junior high last year was 120 students. This year enrolment increased by 15%. What is the enrolment this year?

  A common error occurs when students simply determine the given percent of the number. In this case, for example, students may simply multiply 0.15 by 120 to get a result of 18. Remind students that the enrollment is increasing - it cannot be less than 120. This should help students make the connection that they must add 18 to 120. Alternatively, students may choose to add 100% (enrollment from last year) to 15% (increase) and then determine 115% of 120 students.

Another application of percent increase or decrease is to determine the amount of change as a percentage rather than the final or initial amounts. Students should solve problems such as:

- A tree, which was 3.7 m high last year, is measured and found to now be 4.8 m tall. What is the percent change in the height of the tree?
- A large bag of potato chips used to cost $2.99. The store offered the chips at the new price of $2.65 during the Christmas season. What is the percent change in the price of the chips over the Christmas season?

Students should recognize that the percent increase or decrease can be determining by calculating $\frac{\text{Amount of Increase or Decrease}}{\text{Original Amount}} \times 100\%$.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Students could play a game of *Beat that Percent*. Each group will need a standard deck of cards.

  Goal

  The goal of the game is to obtain 10 points before your opponent(s).

  How to Play

  1. Shuffle the cards. Deal four cards to each player.
  2. The aces count as 1, the face cards count as 0 and numbered cards count as their face values.
  3. Each player chooses two of the cards to form a two digit number that represents a percent. The remaining two cards form a two digit number.
  4. Calculate the percent of the number.
  5. Compare results with your opponent(s). The one with the greatest value gets a point.

  (8N3.8)

Paper and Pencil

- In 1970 a loaf of bread cost $0.25. Today, a loaf of bread costs $2.69. Ask students to determine the percent change in the price of a loaf of bread.

  (8N3.8)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 5.2: Calculating Percents

Lesson 5.3: Solving Percent Problems

Lesson 5.4: Sales Tax and Discount

ProGuide: pp. 12-17, 18-25, 26-33

CD-ROM: Master 5.22, 5.23, 5.24

SB: pp. 242-247, 248-255, 256-262

PB: pp. 105-106, 107-109, 110-111

Number

Specific Outcomes

Students will be expected to

8N3 Continued...

Achievement Indicator:

8N3.9 *Solve a given problem involving combined percents.*

Suggestions for Teaching and Learning

A relevant example of combined percents is addition of percents, such as Goods & Service Tax (GST) and Provincial Sales Tax (PST). Students encounter combined percentages everyday when they buy items at stores. Tax is charged in Newfoundland and Labrador by both the federal and provincial government. Currently, the Federal government charges 5% GST and the Provincial government charges 8% PST. A total of 13% tax is charged on purchases in Newfoundland and Labrador. This is called Harmonized Sales Tax (HST). Consider the following:

- Jason purchases a hockey stick that has a price of $74.99.
  - (i) How much GST will Jason have to pay?
  - (ii) How much PST will Jason have to pay?
  - (iii) What is the total tax on the hockey stick?
  - (v) How much will Jason have to pay for the hockey stick?

Some students may recognize that the price for the hockey stick including taxes can be determined by calculating 113% of $74.99 (100% of the price on the tag plus the 13% taxes).

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Provide students with a table of provincial tax rates and ask them to use it to complete the following question:

  Sheri is required to travel across Canada as part of her job. She plans on purchasing a new laptop. The laptop sells for $1850 in NL and sells for $1925 on Alberta, before tax. How much will the laptop cost in each province. In which province should Sheri purchase the laptop?

  (8N3.9)

Journal

- Ask students to respond to the following:

  Your friend lives in Ontario. You plan a trip together to Quebec City and want to wear matching jackets during the trip. The jacket costs $59.90 in each province. Write an email to your friend to convince her in which of the three provinces, Ontario, Newfoundland and Labrador, or Quebec, the jackets should be purchased, and why.

  (8N3.9)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 5.4: Sales Tax and Discount

ProGuide: pp. 26-32

CD-ROM: Master 5.24

SB: pp. 256-262

PB: pp. 110-111

Number

Specific Outcomes

Students will be expected to

8N3 Continued...

Achievement Indicator:

8N3.10 *Solve a given problem that involves finding the percent of a percent.*

Suggestions for Teaching and Learning

Students should solve a variety of problems that involve determining the percent of a percent.

- A store discounted clothing by 20% off the regular price one week. The next week an additional 40% was taken off the already discounted price.
  (i) If the regular price of a pair of jeans was $59.99, what is the reduced price in the second week before tax?
  (ii) What will be the total cost of the jeans?
  (iii) Could you determine the reduced price by adding 20% to 40% and then calculating 60% of $59.99? Explain.

A common error occurs when students add 20% to 40% and conclude that the overall discount is 60%. Remind students that the 40% is taken off **the already discounted price.** Guiding students through such an exploration will allow them to see the difference in the price and reinforce the proper sequence of calculations.

- Carey Price won 67% of the games he played during the 2014-2015 regular season. Of the games he won, 20% were shutouts. If he played 66 regular season games, how many shutouts did Carey Price record?
- 96% of Science 8 students passed their last unit test. Of those that passed, 87.5% received an A. If there are 25 students in Science 8, how many students got an A on the last test?
- A flat screen TV has a regular price of $599. The store is offering 25% sale. How much, including tax, will the TV cost?

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Journal

- Ask students to respond to the following prompts:
  (i) Two stores offer different discount rates as follows:

  Store A: 50% off one day only.

  Store B: 25% off one day followed by 25% off the reduced price the second day.

  Which store has the better sale?

  (8N3.10)

  (ii) A jacket cost $100. The discount on the jacket is 15%. However you must also pay 15% sales tax. Would the jacket cost you $100, less than $100 or more than $100? Explain your reasoning.

  (8N3.10)

  (iii) Charlie works part-time at a local fast food restaurant. On his next pay check, he will receive a 5% increase in pay. Six months later he will receive another 10% raise. Charlie tells his friends he is receiving a 15% raise in pay. Is he correct? Explain.

  (8N3.10)

Paper and Pencil

- Cyril collects hockey cards. He had 150 cards in his collection. For his birthday his friends gave him hockey cards which increased his collection by 20%. At Christmas his hockey card collection increased by another 15%. Ask students to determine the total number of cards in his collection at Christmas.

  (8N3.10)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 5.4: Sales Tax and Discount

ProGuide: pp. 26-32

CD-ROM: Master 5.24

SB: pp. 256-262

PB: pp. 110-111

Number

Specific Outcomes

Students will be expected to

8N4 Demonstrate an understanding of ratio and rate.

[C, CN, V]

Suggestions for Teaching and Learning

In Mathematics 6, students learned that a ratio is a comparison of two quantities having the same unit. They represented two-term ratios concretely, pictorially and symbolically. They also explored equivalent ratios and solved problems involving ratios. In this unit, they will extend this knowledge to three-term ratios. Students will, for the first time, explore the concept of rates. They will describe and record rates using real-life examples and solve a variety of problems involving rates, such as unit prices. Solving problems using rates and unit prices will allow students to make connections to their own lives and will strengthen their understanding.

Achievement Indicators:

8N4.1 *Express a two-term ratio from a given context in the forms 3:5 or 3 to 5.*

To activate prior knowledge, present a problem such as:

Mr. Johnson is taking an inventory of the sports equipment in the gymnasium. He counts 24 volleyballs, 10 soccer balls, and 5 baseballs.

(i) What is the ratio of volleyballs to baseballs? What type of ratio is this?

(ii) What is the ratio of soccer balls to the total number of balls? What type of ratio is this?

Students should recognize that a part-to-part ratio compares part of a set with another part of the set while a part-to-whole ratio compares part of a set with the entire set. Students should work with ratios in multiple forms as they explore this unit. They should express a ratio in these forms. The ratio of 24:5, for example, can also be written as 24 to 5.

8N4.2 *Express a three-term ratio from a given context in the forms 4:7:3 or 4 to 7 to 3.*

Using the previous example, ask students to write the ratio of volleyballs to soccer balls to baseballs. Many students will easily make the transition from two-term ratios to three-term ratios and recognize that the ratio of vollyballs to soccer balls to baseballs can be written as 24:10:5 or 24 to 10 to 5. A three term ratio also compares three quantities having the same unit.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Interview

- Ask students to use their classroom environment to determine the following ratios:
  (i) Boys to girls
  (ii) Girls to boys
  (iii) Boys to total students
  (iv) Boys to girls to total students
  (v) Window to doors
  (vi) Desks to chairs

(8N4.1, 8N4.2)

Paper and Pencil

- Ask students to write a part:part and a part:part:whole ratio for each of the following situations:
  (i) A bag contains 3 jujubes and 5 lollipops.
  (ii) A fishing basket holds 6 trout and 5 smelt.
  (iii) In the harbour there are two types of boats: dories and longliners. There are 40 boats in total and 7 of them are longliners.
  (iv) prime and composite numbers less than or equal to 20.

(8N4.1, 8N4.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 5.5: Exploring Ratios

ProGuide: pp. 34-38

CD-ROM: Master 5.25

SB: pp. 264-268

PB: pp. 112-114

Number

Specific Outcomes

Students will be expected to

8N4 Continued...

8N5 Solve problems that involve rates, ratios and proportional reasoning.

[C, CN, PS, R]

Achievement Indicators:

8N4.3 *Express a part-to-part ratio as a part-to-whole fraction.*

8N4.4 *Express a given ratio as a percent.*

8N5.1 *Explain the meaning of* $\frac{a}{b}$ *within a given context.*

8N5.2 *Provide a context in which* $\frac{a}{b}$ *represents a:*

- *fraction*
- *rate*
- ***ratio***
- *quotient*
- *probability.*

Suggestions for Teaching and Learning

Since fractions represent part of a whole or set, part-to-whole ratios can be written as fractions. Suppose, for example, the ratio of bananas to total pieces of fruit in a basket is 6:20. This can be represented by the fraction $\frac{6}{20}$ and can be interpreted as 6 out of 20 pieces of fruit are bananas.

To express a part-to-part ratio as a part-to-whole fraction, students will need to rewrite the ratio as a part to whole ratio, as in the following example:

- The ratio of girls to boys in the school band is 13:7.
  - (i) Describe the ratio of girls to total number of students.
  - (iv) Write the corresponding part-to-whole fraction.

Since the ratio 13:7 does not compare one part to the whole, it cannot be directly written as a fraction. The ratio of girls to total number of students is 13:20, which can be expressed as a fraction, $\frac{13}{20}$. There are 13 girls out of 20 students.

Ealier in this course, students expressed fractions as percents. They should build upon this knowledge to express a given ratio as a percent. Expressing a given ratio as a percent involves one extra step - converting the ratio to a fraction.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

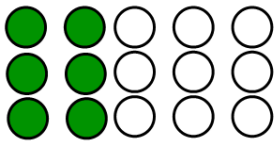
Paper and Pencil

- Ask students to write each of the following part-to-part ratios as a part-to-whole fraction in simplest form:

  (i) 14 to 6
  (ii) 4:22
  (iii) 18:12
  (iv) 25 to 20
  (v) 18:21
  (vi) 18:3
  (vii) 7:21
  (viii) 20 to 9
  (ix) 4:10
  (x) 84 to 16

  (8N4.3)

- Ask students to use the diagram below to answer the questions which follow:



  (i) Explain how the diagram illustrates the ratios 2:3 and 2:5.

  (ii) Convert each ratio to a fraction and a percent.

  (8N4.3, 8N4.4)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 5.5: Exploring Ratios

ProGuide: pp.34-38

CD-ROM: Master 5.25

SB: pp. 264-268

PB: pp. 112-114

Number

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to **8N4 Continued...** **Achievement Indicators:** 8N4.5 *Identify and describe ratios from real life examples, and record them symbolically.*	Through exploration and making meaningful conections, ratios can be related to everyday situations. Brainstorm with students where they have encountered ratios in their everyday lives. Students might suggest some of the following: • Ratios are used to mix frozen orange juice. three cups of water must be added for every can (i.e., the ratio of water to concentrate to make orange juice is 3:1 or "3 to 1"). • Ratios are used on maps. A scale of 1:100 on a map, for example, means that 1 cm on the map represents an actual distance of 100 km. Students should see the necessity of this scale, or ratio - it is impossible to show the actual size and/or distances on a map. • Mixing gas and oil for chainsaws, snowblowers and snowmobiles. The gas:oil ratio for some machines is 50:1. This means that for every 50 L of gas there would be 1 L of oil needed. As students explore a variety of real life examples, they should identify the ratio and record it in symbolic form. Encourage students to describe the ratio in words first. This may help them write the terms of the ratio in the correct order of comparison when expressing them in number form.
8N5.3 *Solve a given problem involving ratio.*	When solving problems involving ratios, a variety of strategies are appropriate including, but not limited to, drawing a picture, using equivalent ratios, expressing one term of the ratio as 1, and using percents. Students should solve problems such as: • The ratio of indoor basketballs to outdoor basketballs at the recreation centre is 6:3. If the recreation centre has 45 basketballs, how many of them are indoor basketballs? • The ratio of boys to girls in your class is 3:5. What percent of the class are boys? • The ratio of wins to losses for the girls soccer team was 7:3. The ratio of wins to losses for the boys soccer team was 5:2. Each team played 12 games. Which team had a more successful season? • A homemade strawberry ice cream recipe calls for 400 mL of milk and 2 cups of strawberries. Emily only has 150 mL of milk. How many cups of strawberries should she use? As students explore various problems, encourage them to communicate their strategies to the class. These conversations will strengthen their understanding of ratios and allow students to see a variety of approaches to the same problem.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Performance

- Ask students to explore newspapers, magazines or the Internet to locate a real-life example of a ratio. Students should:
  (i) print or copy the ratio and the context in which it was used
  (ii) identify the ratio as a part-to-part or a part-to-whole ratio
  (iii) express the ratio as a fraction and a percent

  They should share their findings with the class.

  (8N4.3, 8N4.4, 8N4.5)

Journal

- To make pancakes the ratio of pancake mix to water is 4:3. Ask students to discuss what this ratio means and to describe another example of ratios used in cooking.

  (8N4.5)

- Ask students to discuss whether or not the following could be solved using a proportion:

  David is 6 years old and Ellen is 2 years old. How old will Ellen be when David is 12 years old?

  (8N5.3)

Paper and Pencil

- A statue of John Cabot was made from a model. The height of the model was 25 cm. Ask students to determine the height in metres of the statue if it was made using a scale of 1:15 (scale represents ratio of model to actual height).

  (8N5.3)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 5.5: Exploring Ratios

Lesson 5.6: Equivalent Ratios

Lesson 5.7: Comparing Ratios

Lesson 5.8: Solving Ratio Problems

ProGuide: pp. 34-38, 39-45, 49-56, 57-63

Master 5.6a, 5.6b

CD-ROM: Master 5.25, 5.26, 5.27, 5.28

SB: pp. 264-268, 269-275, 279-286, 287-293

PB: pp. 112-114, 115-117, 118-121, 122-123

Number

Specific Outcomes

Students will be expected to

8N4 Continued...
8N5 Continued...

Suggestions for Teaching and Learning

Students have worked with ratios, which compare quantities with the same unit. They will now work with rates, which compare quantities with different units. Students will express given rates using symbols and words, identify rates from real-life examples and solve problems involving rates

Achievement Indicators:

8N4.6 *Express a given rate using words or symbols.*

8N4.7 *Identify and describe rates from real life examples, and record them symbolically.*

8N5.1 *(Continued) Explain the meaning of* $\frac{a}{b}$ *within a given context.*

8N5.2 *(Continued)Provide a context in which* $\frac{a}{b}$ *represents a:*

- *fraction*
- ***rate***
- *ratio*
- *quotient*
- *probability.*

8N4.8 *Explain why a rate cannot be represented as a percent.*

The following questions could be used to activate prior knowledge:

- What is a rate?
- What examples of rates are you familiar with?

Students are already familiar with rates, even if they have not been classified as rates. They may suggest some of the following:

- speed - Jay walked 1.2 km in 15 minutes (1.2km/15min)
- fuel consumption - 20 L per 100 km (20L/100km)
- heart rate - Mike's heart rate was 80 beats per minute (80 beats/min)
- prices - Turkey costs $1.97 per pound ($1.97/lb)
  - Sliced chicken costs $2.79 per 100 g ($2.79/100g)
  - 24 bottles of water cost $3.49 ($3.49/24 bottles)
- text messaging - Debbie sends 350 texts per 7 days (350 texts/7 days)
- money earned - Edward got paid $120 for working 4 hours ($120/4hr)
- school schedules - Stephen has 10 Math classes in 14 days (10 classes/14 days)

Students should include the units when writing a rate, as it identifies the two quantities being compared.

Students should understand that rate cannot be expressed as a percent since the quantities have different units. A percent compares quantities having the same unit.

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Ask students to identify the rates in the following situations, and express them using words and symbols:
  (i) When Denise bought gasoline, she paid $27.44 for 11.2 litres. Find the price of gasoline per litre.
  (ii) Jacob filled his 60-gallon bathtub in 5 minutes. How fast was the water flowing?
  (iii) On her vacation, Charmaine's flight lasted 4.5 hours. She traveled 954 miles. Find the average speed of the plane.

(8N4.6)

Interview

- Ask students to describe and provide an example of a rate for each of the following:
  (i) the speed you travel on the highway
  (ii) how many eggs a family uses in:
  - a day
  - a week
  - a month

  (iii) hockey statistics

(8N4.6, 8N4.7)

Journal

- Ask students to respond to the following:

  How are ratios and rates alike? How are ratios and rates different? Use examples to support your explanation. Students could compare ratios and rates using a Venn diagram.

(8N4.8, 8N5.1, 8N5.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 5.9: Exploring Rates

Lesson 5.10: Comparing Rates

ProGuide: pp. 64-69, 70-76

SB: pp. 294-299, 300-306

PB: pp. 124-126

Number

Specific Outcomes

Students will be expected to

8N5 Continued...

Achievement Indicator:

8N5.4 *Solve a given problem involving rate.*

Suggestions for Teaching and Learning

Problem solving with rates often involves making a comparison. It is important for students to be aware that when they are comparing rates, the units should be consistent. If comparing one quantity measured in grams with another measured in kilograms, for example, students should express both measurements in grams or both measurements in kilograms. A review of conversion from one unit of measurement to another may be necessary.

When writing equivalent rates, students should ensure the positioning of the units is consistent. A rate equivalent to 100 km/h, for example, should be written with the measurement of distance in the numerator and the measurement of time in the denominator.

To solve problems involving distance, time and average speed, or to determine the better buy in consumer situations, it is often beneficial to use unit rates. A unit rate illustrates two measurements that are directly proportional, where one term is 1. Students can then quickly determine which is the better buy or faster object. As students explore various problem solving contexts, encourage them to communicate their reasoning. Students should solve a variety of contextual problems involving rates, such as:

- The local drugstore is advertising cases of macaroni and cheese at a sale price of $8.99. There are 12 boxes in a case. The grocery store across the street is selling the same macaroni and cheese at a price of $5 for 6 boxes. Which is the better buy?
- Fred walked 6.4 km in 80 minutes. If he continues this pace, how far will he walk in 2 hours?
- At Thomas Amusements it costs $1.50 per coupon or you can purchase a book of 20 coupons for $25.00.
  - (i) What is the unit price for a coupon if you buy the booklet?
  - (ii) Each ride requires 3 coupons. Jared plans on getting on 6 rides. Which option should he choose?

General Outcome: Develop Number Sense

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following questions:
  - (i) Jane found a good deal on soft drinks. She could buy 12 cans for $2.99. She needs 72 cans for her party. Explain how she can calculate the cost. (8N5.4)
  - (ii) Which is the better buy: 1.2 L of orange juice for $2.50, or 0.75 L of orange juice for $1.40? Explain your answer. (8N5.4)

Interview

- When making lemonade Sue uses 5 scoops of powder for 6 cups of water, and Sarah uses 4 scoops of powder for 5 cups of water. Ask students to respond to the following:
  - (i) Are the rates of powder to water equivalent? Explain.
  - (ii) In which situation is it likely the lemonade will be more flavourful? What assumptions did you make?

(8N5.4)

Presentation

- Ask students to research a local or national long-distance running event, and compare the performance of winners from different years. Studnets should compare the distance travelled to the time it takes to complete the race. (8N5.4)
- Ask students to determine the fuel economy for their family vehicle. They could use a log such as the one below to track fuel purchases, kilometres driven, and fuel economy over several weeks.

Amount of Gas Purchased (*L*)	Beginning Odometer Reading (*km*)	Ending Odometer Reading (*km*)	Total Distance Travelled	Fuel Efficiency

(8N5.4)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 5.9: Exploring Rates

Lesson 5.10: Comparing Rates

ProGuide: pp. 64-69, 70-76,

Master 5.7a, 5.7c

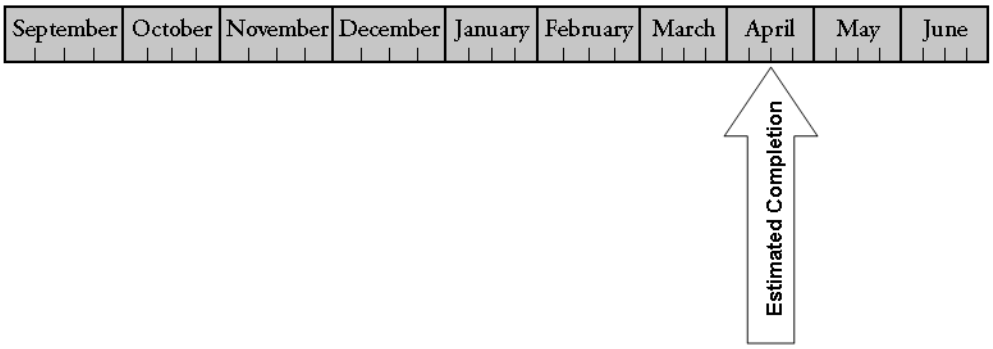
CD-ROM: Master 5.29

SB: pp. 294-299, 300-306

PB: pp. 124-126, 127-128

Linear Equations and Graphing

Suggested Time: 4 Weeks



Unit Overview

Focus and Context

In this unit, students will continue to build upon their algebra skills by solving problems using one-step and multi-step linear equations involving integers and the distributive property. There should be a progression from the use of concrete materials to solving equations symbolically. Students will also verify solutions to linear equations and analyze solutions to identify and correct errors. They will apply their knowledge of linear equations to solve contextual problems.

Students will create tables of values for linear relations by substituting values into the equation. They will also determine missing values in ordered pairs. Students will graph two-variable linear relations by plotting the ordered pairs created from the table of values. They will describe the relationship between the variables in a given graph. Exploring relevant contexts, such as financial applications, will strengthen student understanding of these concepts.

The study of algebra assists with the development of logical thinking and problem solving skills. Algebraic skills are needed in occupations such as computer science, electronics, engineering, medicine, and commerce.

Outcomes Framework

GCO
Represent algebraic expressions in multiple ways.

SCO 8PR1
Graph and analyze two-variable linear relations.

SCO 8PR2
Model and solve problems using linear equations of the form:

- $ax = b$
- $\frac{x}{a} = b,\ a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c,\ a \neq 0$
- $a(x + b) = c$

  where a, b, and c are integers.

SCO Continuum

Grade 7	Grade 8	Grade 9
Strand: Patterns and Relations (Patterns)		
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
7PR1 Demonstrate an understanding of oral and written patterns and their equaivalent linear relations. [C, CN, R] 7PR2 Create a table of values from a linear relation, graph the table of values, and analyze the graph to draw conclusions and solve problems. [C, CN, PS, R, V]	8PR1 Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V]	9PR1 Generalize a pattern arising from a problem-solving context, using linear equations, and verify by substitution. [C, CN, PS, R, V] 9PR2 Graph a linear relation, analyze the graph, and interpolate or extrapolate to solve problems. [C, CN, PS, R, T, V]
Strand: Patterns and Relations (Variables and Equations)		
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
7PR3 Demonstrate an understanding of preservation of equality by: • modelling preservation of equality, concretely, pictorially and symbolically • applying preservation of equality to solve equations. [C, CN, PS, R, V] 7PR4 Explain the difference between an expression and an equation. [C, CN] 7PR5 Evaluate an expression, given the value of the variable(s). [CN, R]	8PR2 Model and solve problems using linear equations of the form: • $ax = b$ • $\frac{x}{a} = b,\ a \neq 0$ • $ax + b = c$ • $\frac{x}{a} + b = c,\ a \neq 0$ • $a(x + b) = c$ where a, b, and c are integers. [C, CN, PS, V]	9PR3 Model and solve problems using linear equations of the form: • $ax = b$ • $\frac{x}{a} = b,\ a \neq 0$ • $ax + b = c$ • $\frac{x}{a} + b = c,\ a \neq 0$ • $ax = b + cx$ • $a(x + b) = c$ • $ax + b = cx + d$ • $a(bx + c) = d(ex + f)$ • $\frac{a}{x} = b, x \neq 0$ where a, b, c, d, e, and f are rational numbers. [C, CN, PS, V]

SCO Continuum

Strand: Patterns and Relations (Variables and Equations)		
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
7PR6 Model and solve, concretely, pictorially and symbolically, problems that can be represented by one-step linear equations of the form $x + a = b$, where a and b are integers. [CN, PS, R, V] 7PR7 Model and solve, concretely, pictorially and symbolically, problems that can be represented by linear equations of the form: • $ax + b = c$ • $ax - b = c$ • $ax = b$ • $\frac{x}{a} = b,\ a \neq 0$ where a, b and c are whole numbers. [CN, PS, R, V]		9PR4 Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context. 9PR5 Demonstrate an understnading of polynomials (limited to polynomials of degree less than or equal to 2). [C. CN, R, V] 9PR6 Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2). [C, CN, PS, R, V] 9PR7 Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically. [C, CN, R, V]

Mathematical Processes

[C] Communication	**[PS] Problem Solving**
[CN] Connections	**[R] Reasoning**
[ME] Mental Mathematics and Estimation	**[T] Technology**
	[V] Visualization

Strand: Patterns and Relations

Specific Outcomes

Students will be expected to

8PR2 Model and solve problems, concretely, pictorially and symbolically, using linear equations of the form:

- $ax = b$
- $\frac{x}{a} = b,\ a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c,\ a \neq 0$
- $a(x + b) = c$

where *a*, *b* and *c* are integers.

[C, CN, PS, V]

Achievement Indicators:

8PR2.1 *Model a given problem with a linear equation and solve the equation using concrete models.*

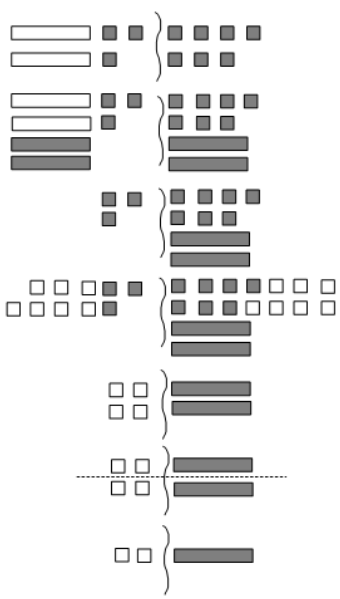
8PR2.2 *Draw a visual representation of the steps used to solve a given linear equation and record each step symbolically.*

Suggestions for Teaching and Learning

In Mathematics 7, students modelled and solved probelms that could be represented by one-step linear equations of the form $x+a=b$, where a and b were integers. They also modelled and solved, concretely, pictorially and symbolically, problems that could be represented by linear equations of the forms: $ax + b = c$, $ax - b = c$, $ax = b$, $\frac{x}{a} = b,\ a \neq 0$, where a, b and c were whole numbers. Students will extend this knowledge to solve these equation types where a, b and c are integers. They will also solve equations of the form $a(x + b) = c$ and $\frac{x}{a} + b = c,\ a \neq 0$. Students should begin with concrete materials and pictorial models, and then move to solving equations symbolically. The ultimate goal is that students solve one and two-step equations without concrete or pictorial support.

Two different colour algebra tiles are needed for students to model equations. Regardless of the colour tiles you have available, decide which colour will represent positive and which will represent negative. Throughout this curriculum guide shaded tiles represent positive values and unshaded tiles represent negative values.

In Mathematics 7, students used pan balances and algebra tiles to model and solve linear equations. They should continue to use these models to represent and solve linear equations and they should record the steps symbolically, as well as pictorially. Consider $-2x + 3 = 7$, for example.

Pictorial	Symbolic
	$-2x + 3 = 7$
	$-2x + 3 + 2x = 7 + 2x$
	$3 = 7 + 2x$
	$3 + (-7) = 7 + 2x + (-7)$
	$-4 = 2x$
	$-2 = x$

General Outcome: Represent Algebraic Expressions in Multiple Ways

Suggested Assessment Strategies

Performance

- Ask students to work in pairs to model and solve the following equations:

  (i) $3x = -6$

  (ii) $6x = 4x - 4$

  Taking turns, the first person should model and solve the given equation, explaining each step. The second student should draw a visual representation and record each step symbolically.

  (8PR2.1, 8PR2.2)

- Students are digitally photographed while using algebra tiles to solve a linear equation. Give students their photograph and ask them to write about what they are doing under the photograph. They should describe what they learned as a result.

  (8PR2.1, 8PR2.2)

Journal

- Ask students to explain how they can use a model, such as algebra tiles or a pan balance, to solve a linear equation.

  (8PR2.1, 8PR2.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 6.1: Solving Equations Using Models

ProGuide: pp. 4-12, Master 6.11

CD-ROM: Master 6.20

Student Book (SB): pp. 318-326

Practice and Homework Book (PB): pp.138-141

Strand: Patterns and Relations

Specific Outcomes

Students will be expected to

8PR2 Continued...

Achievement Indicators:

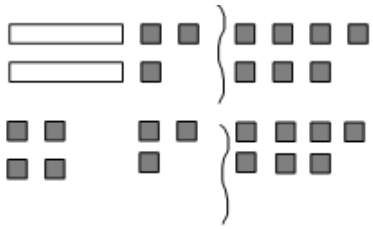
8PR2.1 *(Continued) Model a given problem with a linear equation and solve the equation using concrete models.*

8PR2.2 *(Continued) Draw a visual representation of the steps used to solve a given linear equation and record each step*

8PR2.3 *Verify the solution to a given linear equation using a variety of methods, including concrete materials, diagrams and substitution.*

Suggestions for Teaching and Learning

Once students have solved a linear equation, they should verify the solution. This can be done by redrawing the diagram if a two-pan balance was used. When algebra tiles are used, students should replace each variable tile with the value in the solution to determine if both sides of the equation remain equal. In the previous example, there are negative variable tiles in the equation. It is important that students recognize that if $x = -2$, then $-x$ is the "opposite" of x. Students should conclude that if $x = -2$, then $-x = 2$. To verify their solution they must replace each of the negative variable tiles with 2 positive unit tiles:



Since both sides have the same value, the solution is correct.

Students are not expected to model and solve equations of the form $\frac{x}{a} + b = c,\ a \neq 0$ concretely or pictorially.

8PR2.4 *Solve a given linear equation symbolically.*

Once students have developed an understanding of solving equations concretely and pictorially, they should solve linear equations symbolically, by applying the preservation of equality.

They should then verify the solution by substituting the value for the unknown into the original equation to determine if both sides of the equation simplify to the same value. To verify that $x = -2$ is the solution to $-2x + 3 = 7$, for example, students would substitute -2 into the original equation for x and simplify:

Left Side	Right Side
$-2x + 3$ $-2(-2) + 3$ $4 + 3$ 7	7

Since both sides of the equation result in 7, the solution $x = -2$ is correct.

General Outcome: Represent Algebraic Expressions in Multiple Ways

Suggested Assessment Strategies

Interview

- Ask students to explain each step in the following solution:

  $16 + 5m = 6$

  Step 1: $16 - 16 + 5m = 6 - 16$

  Step 2: $5m = -10$

  Step 3: $m = -2$

  Ask them how they could verify the solution $m = -2$ is correct.

  (8PR2.3, 8PR2.4)

Performance

- Provide students with a set of cards. Half of the cards should have a linear equation written on them. The other half of the cards should have the solution. Ask students to match each equation with its solution. They should explain their process.

  (8PR2.3, 8PR2.4)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 6.1: Solving Equations Using Models

Lesson 6.2: Solving Equations Using Algebra

Lesson 6.3: Solving Equations Involving Fractions

ProGuide: pp. 4-12, 13-18, 19-23

Master 6.11

CD-ROM: Master 6.20, 6.21, 6.22

SB: pp. 318-326, 327-332, 333-337

PB: pp.138-141, 142-143, 144-147

Strand: Patterns and Relations

Specific Outcomes

Students will be expected to

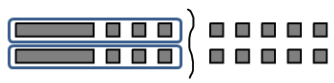
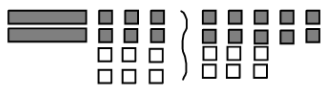
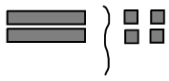
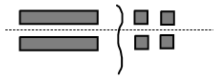
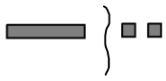
8PR2 Continued...

Achievement Indicator:

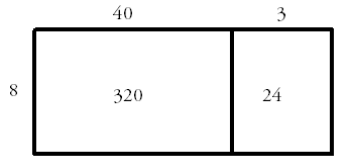
8PR2.5 *Apply the distributive property to solve a given linear equation of the form* $a(x+b)=c$.

Suggestions for Teaching and Learning

Students should use the distributive property to solve a given linear equation. As with other types of linear equations, modelling equations of the form $a(x + b) = c$ should precede solving symbolically. To model the equation $2(x + 3) = 10$, for example, students should recognize that they need 2 groups of 1 positive variable tile and 3 unit tiles.

Pictorial	Symbolic
	$2(x+3)=10$ $2x+6=10$
	$2x+6+(-6)=10+(-6)$
	$2x=4$
	$\frac{2x}{2}=\frac{4}{2}$
	$x=2$

Students worked with the distributive property as a strategy for multiplying a one digit number by a two digit number. They used the area model to multiply numbers such as, 8 × 43.



$8(40+3)$

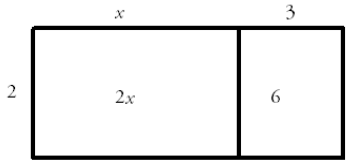
$320+24$

344

$320+24$

344

This strategy can also be used with algebraic expressions. The expression $2(x + 3)$ can be represented as:



$2(x + 3)$

$= 2(x) + 2(3)$

$= 2x + 6$

General Outcome: Represent Algebraic Expressions in Multiple Ways

Suggested Assessment Strategies

Paper and Pencil

- Ask students to solve $2(x-4)=-20$ by using a model. They should record their steps symbolically.

(8PR2.5)

- Ask students to solve the following equations and verify their solutions:
  (i) $3(m-2)=10$
  (ii) $-2(x+4)=-22$
  (iii) $-(p+4)=14$

(8PR2.5, 8PR2.3)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 6.4: The Distributive Property

Lesson 6.5: Solving Equations Involving the Distributive Property

ProGuide: pp. 24-29, 30-34

Master 6.14

CD-ROM: Master 6.23, 6.24

SB: pp. 338-343, 344-348

PB: pp.148-151

Strand: Patterns and Relations

Specific Outcomes

Students will be expected to

8PR2 Continued...

Achievement Indicators:

8PR2.5 *(Continued) Apply the distributive property to solve a given linear equation of the form* $a(x+b)=c$.

8PR2.3 *(Continued) Verify the solution to a given linear equation using a variety of methods, including concrete materials, diagrams and substitution.*

Suggestions for Teaching and Learning

After solving linear equations concretely and pictorially, students solve such equations symbolically. They should work with a variety of equations of the form $a(x + b) = c$:

- $4(x + 8) = 40$
- $-3(x + 4) = 12$
- $2(8 - h) = 10$
- $-6(-x + 4) = -36$

They should continue to verify their solutions algebraically.

When expanding an expression of the form $a(x + b)$ symbolically, a common error occurs when students only multiply the a by x and incorrectly record the result as $ax + b$. Referring back to the concrete and pictorial representations should reinforce that each term must be multiplied by a.

Students may need to be reminded that an expression such as $-(x + 3)$, for example, is equivalent to $-1(x + 3)$, where each term must be multiplied by -1.

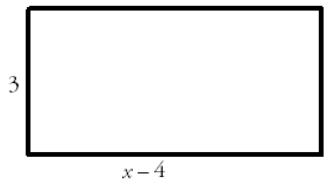
General Outcome: Represent Algebraic Expressions in Multiple Ways

Suggested Assessment Strategies

Paper and Pencil

- Ask students to respond to the following:

  (i) Write an expression that represents the area of the following rectangle:



  (ii) Determine the value of x if the area of the rectangle is 24 square units.

  (iii) Is it possible for $x = 2$? Explain.

(8PR2.5)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 6.4: The Distributive Property

Lesson 6.5: Solving Equations Involving the Distributive Property

ProGuide: pp. 24-29, 30-34

Master 6.14

CD-ROM: Master 6.23, 6.24

SB: pp. 338-343, 344-348

PB: pp.148-151

Strand: Patterns and Relations

Specific Outcomes

Students will be expected to

8PR2 Continued...

Achievement Indicator:

8PR2.6 *Identify and correct an error in a given incorrect solution of a linear equation.*

Suggestions for Teaching and Learning

Students should be provided with worked solutions of linear equations to verify. They should be able to identify and communicate the error that occurred and provide the correct solution. Encourage them to verify their solution once complete. The ability to analyze another student's work to identify and correct errors will strengthen student understanding of solving linear equations. Error analysis reinforces the importance of verifying solutions and recording steps, rather than only producing the final answer. Students should analyze solutions to questions such as:

Three friends are comparing solutions to the equation $4(s - 3) = 288$. They each had a diffferent solution. Which student has the correct answer? Identify and explain the error that occured in the other students' work.

Sam's Solution	Leah's Solution	Paul's Solution
$4(s-3)=288$	$4(s-3)=288$	$4(s-3)=288$
$4s-12=288$	$4s-12=288$	$4s-3=288$
$4s-12-12=288-12$	$4s-12+12=288+12$	$4s-3+3=288+3$
$\frac{4s}{4}=\frac{276}{4}$	$\frac{4s}{4}=\frac{300}{4}$	$\frac{4s}{4}=\frac{291}{4}$
$s=69$	$s=75$	$s=\frac{291}{4}$ **or** 72.75

Students should make the following conclusions:

- Leah's solution is correct.
- Sam made an error in Step 2. He subtracted 12 from both sides of the equation instead of adding 12 to both sides of the equation.
- Paul made an error in Step 1. He did not multiply 4 by -3 when applying the distributive property.

General Outcome: Represent Algebraic Expressions in Multiple Ways

Suggested Assessment Strategies

Paper and Pencil

- Ask students to verify each of the solutions provided below. Identify and correct any errors.

(i)

$x+4=3$

$x+4-4=3+4$

$x=7$

(ii)

$5+4x=13$

$5+4x-4x=13-4x$

$\frac{5}{9}=\frac{9x}{9}$

$\frac{5}{9}=x$

(iii)

$56=8(x+3)$

$56=8x+24$

$56-24=8x+24-24$

$\frac{32}{32}=\frac{8x}{32}$

$x=\frac{8}{32}$

$x=\frac{1}{4}$

(iv)

$-2(x-1)=-22$

$-2x-2=-22$

$-2x-2+2=-22+2$

$\frac{-2x}{-2}=\frac{-20}{-2}$

$x=10$

(v)

$7x-2=-16$

$7x-2+2=-16+2$

$7x=-14$

$x=-2$

(vi)

$\frac{s}{6}+3=11$

$6\left(\frac{s}{6}\right)+3=6(11)$

$s+3=66$

$s+3-3=66-3$

$s=63$

(8PR2.6)

Observation

- Ask students to solve each equation individually and then exchange their solutions with a partner. They should check each other's work, identifying errors (if any) and explain how to correct any errors that they notice.

(i) $5m=-10$

(ii) $2(x-3)=24$

(iii) $-9+2x=-13$

(8PR2.4, 8PR2.5, 8PR2.6)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 6.1: Solving Equations Using Models

Lesson 6.2: Solving Equations Using Algebra

Lesson 6.3: Solving Equations Involving Fractions

Lesson 6.5: Solving Equations Involving the Distributive Property

ProGuide: pp. 4-12, 13-18, 19-23, 30-34

Master 6.11

SB: pp. 325, 331, 337, 347, 348

PB: pp.143, 147, 149

Strand: Patterns and Relations

Specific Outcomes

Students will be expected to

8PR2 Continued...

Achievement Indicator:

8PR2.7 *Solve a given problem using a linear equation and record the process.*

Suggestions for Teaching and Learning

Students should be exposed to a variety of problems that can be solved using linear equations. They may find the the following four-step process useful when approaching these problems:

- understand the problem by identifying given information and the unknown (includes identifying a variable to represent the unknown)
- write a linear equation to represent the given problem
- solve the equation
- verify the solution is correct

Teachers could model this process with a problem such as:

Jordan went to the local fair. She spent $3.00 on each ride and $15 for food. She spent a total of $27.00. How many rides was Jordan on?

Understand the Problem: *I am told that the cost of each ride is $3.00. I know that Jordan spent an additional $15.00 on food and that, altogether, she spent $27.00. I need to find the number of rides. Let 'r' represent the number of rides that Jordan goes on.*

Write a Linear Equation: *Since each ride costs $3.00, I can multiply r by 3. This represents the amount of money she spent on rides. I can then add this to the cost of food ($15.00). This must equal $27.00, the total amount of money Jordan spent. An equation to represent this situation is*
$3r + 15 = 27$.

Solve the equation: *I can solve this equation to determine the number of rides that Jordan goes on:*

$3r + 15 = 27$
$3r + 15 - 15 = 27 - 15$
$3r = 12$
$\frac{3r}{3} = \frac{12}{3}$
$r = 4$
Jordan was on 4 rides.

Verify the Solution: *If each ride was $3.00 and she went on 4 rides, Jordan spent $12.00 on rides. Adding the $15.00 she spent on food, would equal $27.00.*
Using substitution, the solution can be verified algebraically.

LS	RS	
$3r + 15$	27	Since both the left hand side and right hand side of the equation simplify to 27, the solution is correct.
$3(4) + 15$		
$12 + 15$		
27		

General Outcome: Represent Algebraic Expressions in Multiple Ways

Suggested Assessment Strategies

Paper and Pencil

- Ask students to solve the following problems:
  - (i) A baker is packaging 61 cookies in identical boxes. She fills seven boxes and has 5 cookies left over. Determine how many cookies are in each box. (8PR2.2, 8PR2.4, 8PR2.7)
  - (ii) A taxicab company charges a basic rate of $3.75 plus $2.00 for every kilometre driven. Determine the distance travelled if the if the fare was $33.75. (8PR2.4, 8PR2.7)
  - (iii) Gabe would like to purchase a tablet that costs $250. Each app downloaded costs $2.00. If he has $278, how many apps can he download? Verify your solution. (8PR2.4, 8PR2.7)

Journal

- Point of Most Significance strategy can be used to help students reflect back on the key points about a lesson. Ask them to respond to the following prompt:

  Today we worked with a process for solving problems involving linear equations. Describe the most significant point that contributed to your learning. (8PR2.7)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 6.1: Solving Equations Using Models

Lesson 6.2: Solving Equations Using Algebra

Lesson 6.3: Solving Equations Involving Fractions

Lesson 6.5: Solving Equations Involving the Distributive Property

ProGuide: pp. 4-12, 13-18, 19-23, 30-34

Master 6.12, 6.13, 6.15

CD-ROM: Master 6.20, 6.21, 6.22, 6.24

SB: pp.318-326, 327-332, 333-337, 344-348

PB: pp.141, 143, 145, 146, 147, 151

Patterns and Relations

Specific Outcomes

Students will be expected to

8PR1 Continued...

Achievement Indicators:

8PR1.1 *Create a table of values by substituting values for a variable in the equation of a given linear relation.*

8PR1.2 *Determine the missing value in an ordered pair for a given equation.*

Suggestions for Teaching and Learning

In Mathematics 7, students represented oral and written patterns using linear relations. They created and graphed the corresponding table of values and analyzed the graphs to draw conclusions and solve problems. Students will continue their study of linear relations in this unit by graphing and analyzing two-variable linear relations.

Teachers should provide students with various equations of linear relations and ask them to construct a table of values by substituting values for a variable in the equation. In the equation $y = 20 - 4x$, students could substitute 1, 2, 3, 4, 5, 6 for x to create the table of values shown:

x	1	2	3	4	5	6
y	16	12	8	4	0	-4

They should understand that the related pair of x and y values in the table is an ordered pair, (x, y), and that the input values correspond to x and the output values correspond to y. Through investigation, students should recognize that when the change in the x values is constant and the change in the y values is constant the relation is linear.

They should determine the missing value in an ordered pair for a given equation. To determine a missing y-coordinate in an ordered pair, the given x value is substituted into the linear relation. To determine a missing x-coordinate in an ordered pair, students should set up the corresponding linear equation and solve it. If asked to determine the missing y coordinate for the ordered pair (__, -10) for the relation $y = -3x + 5$, for example, students should substitute -10 into the equation for y and solve for x.
Alternatively, when students are provided with multiple ordered pairs, as in the example below, they could use patterning.

The equation of a linear relation is $y = -3x + 5$. Some of the ordered pairs for this relation are: (0, 5), (1, 2), (2, -1), (3, ___). Determine the missing value.

Students should recognize that there is a pattern in the y values. There is a constant decrease of 3. To determine the missing value, they would subtract 3 from -1 to obtain a value of -4.

General Outcome: Use Patterns to Describe the World and Solve Problems

Suggested Assessment Strategies

Paper and Pencil

- Ask students to use the equation $y = -3x + 4$ to complete the following:
  (i) Determine the missing values in the table.

x	-1	0	1	2	3	4
y						

  (ii) Determine the value of y for the ordered pair $(11, y)$.

  (iii) Determine the value of x for the ordered pair $(x, 13)$.

  (8PR1.1. 8PR1.2)

- Mary has started a new exercise program. The first day she does 9 sit ups, the second day she does 13, the third day 17, and the fourth day 21. This can be represented by $s = 4d + 5$, where s represents the number of sit ups and d represents the day. Ask students to complete the following:
  (i) Construct a table of values to represent this relationship for the first five days.
  (ii) Construct a graph of this linear relationship.
  (ii) How many sit ups will she do on the 5th day? 6th day? 10th day? 20th day? 50th day? What restrictions come into play as this pattern continues?

  (8PR1.2, 8PR1.3)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 6.6: Creating a Table of Values

ProGuide: pp.37-44

Master 6.16

CD-ROM: Master 6.25

SB: pp.351-358

PB: pp.152-154

Patterns and Relations

Specific Outcomes

Students will be expected to

8PR1 Continued...

Achievement Indicators:

8PR1.1 *(Continued) Create a table of values by substituting values for a variable in the equation of a given linear relation.*

8PR1.2 *(Continued) Determine the missing value in an ordered pair for a given equation.*

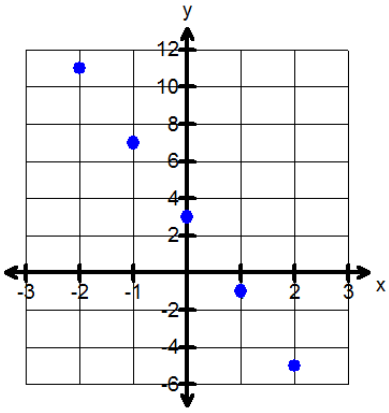
8PR1.3 *Construct a graph from the equation of a given linear relation (limited to discrete data).*

8PR1.4 *Describe the relationship between the variables in a given graph.*

Suggestions for Teaching and Learning

Constructing graphs from linear equations and their corresponding table of values allows students to visualize linear relationships. When the ordered pairs are graphed on a coordinate plane they fall along a straight line. Although many of the graphs for contextual problems are located in Quadrant 1, it is important that students be exposed to graphing ordered pairs that involve negative values. All work with graphing in this unit is limited to discrete data. As students graph linear relations, encourage them to describe the relationship between the variables in the graph. Teachers could begin by providing a linear relation, such as $y = -4x + 3$, and ask students to graph the relation for integer values of x from -2 to 2.

x	Calculations	y
-2	$y = -4(-2) + 3$	11
-1	$y = -4(-1) + 3$	7
0	$y = -4(0) + 3$	3
1	$y = -4(1) + 3$	-1
2	$y = -4(2) + 3$	-5



Students should recognize from the graph that as the x-coordinate increases by one, the y-coordinate decreases by 4. The points lie on a line that goes down to the right.

As a culminating activity for graphing, teachers could present a problem such as the following:

Zachary is planning a swimming party. It costs $30.00 to rent the pool. After swimmming everyone will have a snack. The snack costs $3.00 per person.

(i) Write an equation to represent this relation.
(ii) Use the equation to create a table of values.
(iii) Graph the relation.
(iv) Describe the relationship between the variables in the graph.
(v) If Zachary invited a total of 8 people, how much would his party cost? Write this information as an ordered pair.
(vi) If Zachary has a budget of $60, what is the maximum number of people he can invite to his party.

General Outcome: Use Patterns to Describe the World and Solve Problems

Suggested Assessment Strategies

Paper and Pencil

- A cell phone company charges a basic monthly rate of $20 and a $4 for each game downloaded. This can be represented by the equation $C = 4g + 20$, where C represents the Cost and g represents the number of games downloaded. Ask students to answer the following:
  (i) Determine the cost of using your cell phone by completing the table below.

Number of Games (g)	Cost (C)
0	
1	
2	
3	
4	
5	

  (ii) Create a graph using the data from the table of values.
  (iii) Matthew's cell phone bill for the first month was $100. Use the graph to find the number of games he downloaded. Use the equation to determine how many games Matthew downloaded during the first month.

(8PR1.1, 8PR1.2, 8PR1.3)

- The table of values represents a linear relation (discrete data).

x	1	2	3	4	5	6
y	5	10	15	20	25	30

  Ask students to complete the following:
  (i) Graph the ordered pairs in the table of values.
  (ii) What is the difference in consecutive y-values ? What is the difference in consecutive x-values ?
  (iii) Describe, in words, the relationship between the x-values and the y-values .
  (iv) Write an equation for y in terms of x?

(8PR1.3, 8PR1.4)

- James determined the mass of five pieces of copper, representing a linear relationship. The table shows his results. However, James made one error in finding the masses.

Volume (cm^3)	8	9	10	11	12
Mass (g)	88	99	110	121	144

  Ask students to complete the following:
  (i) Identify James' error and explain your thinking. What is the correct mass?
  (ii) Graph the ordered pairs from James' table of values.
  (iii) How could you use the graph to show which value is incorrect?

(8PR1.3, 8PR1.4)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 6.6: Creating a Table of Values

Lesson 6.7: Graphing Linear Relations

ProGuide: pp. 37-44, 45-51

Master 6.16, 6.17

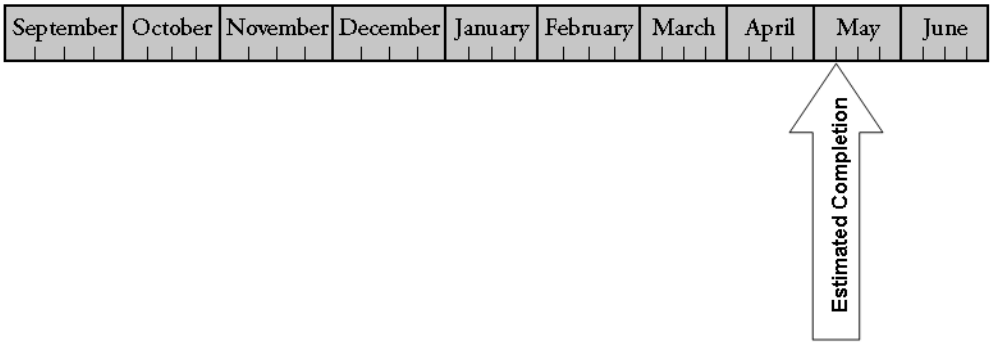
CD-ROM: Master 6.25, 6.26

SB: pp.351-358, 359 - 365

PB: pp.152-154, 155-157

Data Analysis and Probability

Suggested Time: 2 Weeks



Unit Overview

Focus and Context

In this unit, students will examine and create many types of graphs that are common to data management: circle graphs, bar graphs, pictographs, line graphs and double bar graphs. They will make informed decisions about which graph best represents a data set and also learn how to support that decision. In many cases one type of graph is better than another when representing a data set, and provides a visual image that is more informative and useful. Students will differentiate between graphs that are accurate and graphs that are misleading. They will also learn to recognize the false conclusions that misleading graphs try to represent.

They will study the basic principles of probability as it pertains to single events, independent events, and more than one independent event occurring simultaneously or in sequence. These principles will be applied to problem solving situations.

Statistics are used to measure a nation's economic performance and help governments make informed decisions about budgets, social programs, population issues and health care. It is a branch of mathematics that assists with tracking such things as inflation, the efficacy of medicines, and the performance of athletes, as well as predicting weather patterns.

Outcomes Framework

GCO

Collect, display and analyze data to solve problems.

SCO 8SP1

Critique ways in which data is presented.

GCO

Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

SCO 8SP2

Solve problems involving the probability of independent events.

GCO

Develop number sense.

SCO 8N5

Provide a context in which $\frac{a}{b}$ represents a:

- fraction
- rate
- ratio
- quotient
- probability

SCO Continuum

Grade 7	Grade 8	Grade 9
Statistics and Probability		
7SP1 Demonstrate an understanding of central tendency and range by: • determining the measures of central tendency (mean, median, mode) and range • determining the most appropriate measures of central tendency to report findings. [C, PS, R, T] 7SP2. Determine the effect on the mean, median and mode when an outlier is included in a data set. [C, CN, PS, R] 7SP3. Construct, label and interpret circle graphs to solve problems. [C, CN, PS, R, T, V] 7SP4. Express probabilities as ratios, fractions and percents. [C, CN, R, T, V] 7SP5 Identify the sample space (where the combined sample space has 36 or fewer elements) for a probability experiment involving two independent events. [C, ME, PS] 7SP6 Conduct a probability experiment to compare the theoretical probability (determined using a tree diagram, table or other graphic organizer) and experimental probability of two independent events. [C, PS, R, T]	8SP1 Critique ways in which data is presented. [C, R, T, V] 8SP2 Solve problems involving probability of independent events. [C, CN, PS, T]	9SP1 Describe the effect of: • bias • use of language • ethics • cost • time and timing • privacy • cultural sensitivity on the collection of data. [C, CN, R, T] 9SP2 Select and defend the choice of using either a population or a sample of a population to answer a question. [C, CN, PS, R] 9SP3 Develop and implement a project plan for the collection, display and analysis of data by: • formulating a question for investigation • choosing a data collection method that includes social considerations • selecting a population or a sample • collecting the data • displaying the collected data in an appropriate manner • drawing conclusions to answer the question. [C, PS, R, T, V] 9SP4Demonstrate an understanding of the role of probability in society. [C, CN, R, T]

Mathematical Processes

[C] Communication	**[PS] Problem Solving**
[CN] Connections	**[R] Reasoning**
[ME] Mental Mathematics and Estimation	**[T] Technology**
	[V] Visualization

Statistics and Probability (Data Analysis)

Specific Outcomes

Students will be expected to

8SP1 Critique ways in which data is presented.

[C, R, T, V]

Achievement Indicators:

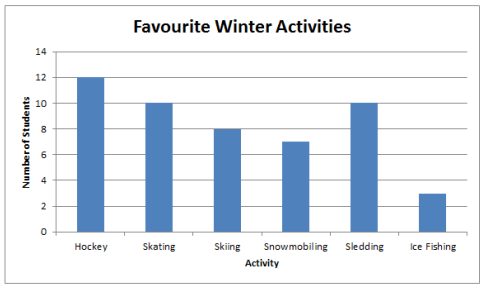
8SP1.1 *Compare the information that is provided for the same data set by a given set of graphs, including circle graphs, line graphs, bar graphs, double bar graphs and pictographs, to determine the strengths and limitations of each graph.*

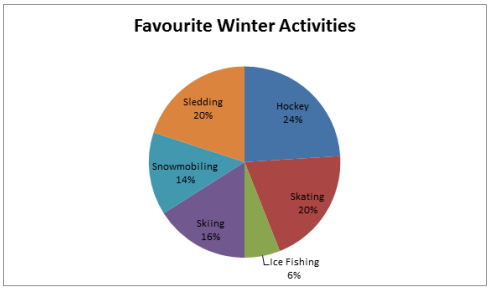
8SP1.2 *Identify the advantages and disadvantages of different graphs, including circle graphs, line graphs, bar graphs, double bar graphs and pictographs, in representing a specific given set of data.*

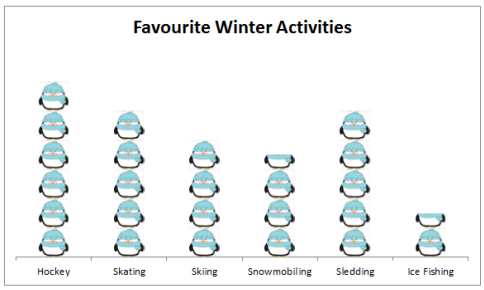
Suggestions for Teaching and Learning

In previous grades, students constructed and interpreted a variety of graphs. The focus of this outcome is on interpreting data and discussing the advantages and disadvantages of various graphical representations rather than constructing the graphs.

Provide students with a set of graphs, such as the following, that display the same set of data.







Key: = 2 students

Ask students questions such as the following:

- Which of the graphs do you find easiest to interpret? Why?
- Which graph would you use to identify the percentage of students whose favourite winter activity is skating? Why?
- Could you use the other graphs to determine the percentage of students whose favourite winter activity is skating? Explain.
- Which graph could you use to find the total number of students surveyed?
- How are the graphs similar? How are they different?
- What are the advantages and disadvantages of each graph?
- Which graph is most appropriate to display the data?

Through discussion, students should conclude that the suitability of a graph will depend on the type of information that is collected or given, and what you want the graph to display.

General Outcome: Collect, Display and Analyze Data to Solve Problems

Suggested Assessment Strategies

Presentation

- Ask students to conduct a survey with the class on favourite colour or sport, TV show, etc. Divide the class into groups and assign each group a different graph to represent the data collected on large chart paper. Each group should present their graph and discuss what they found easy about creating the graph and what challenges they encountered. Once all graphs have been presented, ask students to decide which type of graph best represents the data collected.

(8SP1.1, 8SP1.2)

Paper and Pencil

- A questionnaire can be used to determine students' familiarity with terminology. Ask students to select a response based on their level of familiarity with the mathematical term.

Example:

Circle Graph ☐ I have never heard of this. ☐ I have heard of this but I'm not sure what it means. ☐ I have some idea what it means. ☐ I clearly know what it means and can describe it.	**Discrete Data** ☐ I have never heard of this. ☐ I have heard of this but I'm not sure what it means. ☐ I have some idea what it means. ☐ I clearly know what it means and can describe it.
Continuous Data ☐ I have never heard of this. ☐ I have heard of this but I'm not sure what it means. ☐ I have some idea what it means. ☐ I clearly know what it means and can describe it.	**Pictograph** ☐ I have never heard of this. ☐ I have heard of this but I'm not sure what it means. ☐ I have some idea what it means. ☐ I clearly know what it means and can describe it.

For the third and fourth selected responses, consider leaving a blank space to have students describe their ideas about the term.

The questionnaire can be administered again as a post-assessment at the end of the unit.

(8SP1.1)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 7.1: Choosing an Appropriate Graph

ProGuide: pp. 4-12

CD-ROM: Master 7.17

Student Book (SB): pp. 382-390

Practice and Homework Book (PB): pp. 167-170

Statistics and Probability (Data Analysis)

Specific Outcomes

Students will be expected to

8SP1 Continued...

Achievement Indicators:

8SP1.1 *(Continued) Compare the information that is provided for the same data set by a given set of graphs, including circle graphs, line graphs, bar graphs, double bar graphs and pictographs, to determine the strengths and limitations of each graph.*

8SP1.2 *(Continued) Identify the advantages and disadvantages of different graphs, including circle graphs, line graphs, bar graphs, double bar graphs and pictographs, in representing a specific given set of data.*

Suggestions for Teaching and Learning

Students could brainstorm advantages and disadvantages for each type of graph. The table below is not intended to be exhaustive.

Type of Graph	Advantages	Disadvantages
Circle (Discrete)	• compares "part to whole" • data is displayed as a percent (shows proportion) • size of sector can be easily compared to other sectors	• does not show actual amount for each category • more difficult to construct (time and accuracy) • data must have obvious part-to-whole relationship • too many categories makes it look crowded
Line (Continuous)	• shows change over time • can see trends easily • can be used to interpolate and extrapolate • easy to construct	• limited to continuous data • may be difficult to read accurately depending on scale • may be difficult to compare categaories
Bar (Discrete)	• shows numbers of items in specific categories • easy to compare data • easy to construct	• may be difficult to read accurately depending on scale • not able to interpolate or extrapolate
Double Bar (Discrete)	• contains two sets of data that show numbers of items in categories • useful for comparing one set of data to another	• may be difficult to read accurately depending on scale • not able to interpolate or extrapolate
Pictograph (Discrete)	• visually appealing • useful for small data values • subject of graph is apparent from symbol • easy to compare	• limited to symbols that can be divided into smaller pieces • accuaracy when drawing may be challening

General Outcome: Collect, Display and Analyze Data to Solve Problems

Suggested Assessment Strategies

Journal

- Ask students to select two different types of graphs they have studied. They should provide an example of a set data that could be displayed using this type of graph and discuss the advantages and disvantages of each.

(8SP1.2)

Observation

- In groups, ask students to discuss the advantages and disadvantages of different types of graphs using "Talking Chips." Each student is given an equal number of game chips. They take turns giving an advantage or disadvantage about the different types of graphs. Each time a student contributes to the discussion they place a chip into the center of the group. The activity ends when each student is out of chips. The group should record the advantages and disadvantages in a foldable. This could be used as pre-assessment or post-assessment.

(8SP1.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 7.1: Choosing an Appropriate Graph

ProGuide: pp. 4-12

CD-ROM: Master 7.17

SB: pp. 382-390

PB: pp. 167-170

Statistics and Probability (Data Analysis)

Specific Outcomes

Students will be expected to

8SP1 Continued...

Achievement Indicator:

8SP1.3 *Justify the choice of a graphical representation for a given situation and its corresponding data set.*

Suggestions for Teaching and Learning

Students should be able to justify the choice of a graphical representation that has been used for a given situation. Teachers could provide examples of various graphs that have been used to display data. Consider the following:

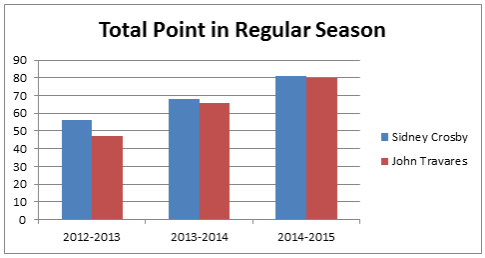
Sidney Crosby

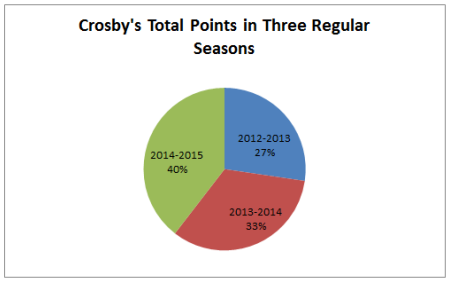
Season	Games Played	Goals	Assists	Total Points
2012-2013	36	15	41	56
2013-2014	80	36	54	68
2014-2015	74	27	54	81

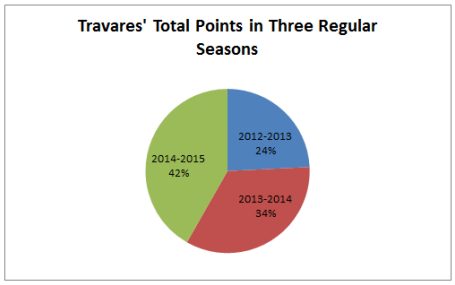
John Tavares

Season	Games Played	Goals	Assists	Total Points
2012-2013	48	28	19	47
2013-2014	59	24	42	66
2014-2015	79	35	45	80

Sandra used a double bar graph to display the data, while John chose a circle graphs:







Ask students questions such as:

- What conclusions can you make based on each graph?
- What trends do you notice in each graph?
- Which graphical representation do you think best represents the data? Justify your choice.

General Outcome: Collect, Display and Analyze Data to Solve Problems

Suggested Assessment Strategies

Observation

- In small groups, ask students to decide which graph they would use to represent the data in each of the following situations:
  (i) The cost of car insurance over the past 20 years
  (ii) Prices of different brands of athletic shoes
  (iii) The favourite after school activities of boys and girls
  (iv) The percentage of favourite ice cream flavors of grade 8 students.

  Students should be able to justify their choice. Encourage each group to share their results with the rest of the class.

  (8SP1.3)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 7.1: Choosing an Appropriate Graph

ProGuide: pp.4-12

Master 7.6a, 7.6b

CD-ROM: Master 7.17

SB: pp.382-390

PB: pp.167-170

Statistics and Probability (Data Analysis)

Specific Outcomes

Students will be expected to

8SP1 Continued...

Achievement Indicators:

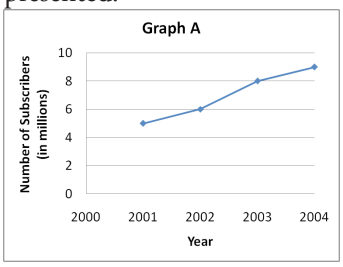
8SP1.4 *Explain how the format of a given graph, such as the size of the intervals, the width of bars and the visual representation, may lead to misinterpretation of the data.*

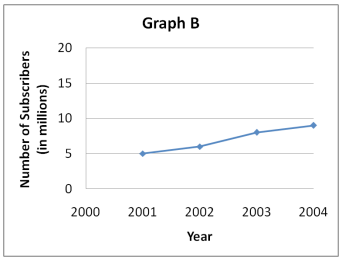
8SP1.5 *Explain how a given formatting choice could misrepresent the data.*

8SP1.6 *Identify conclusions that are inconsistent with a given data set or graph and explain the misinterpretation.*

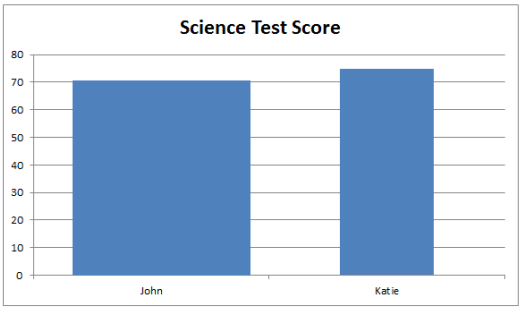
Suggestions for Teaching and Learning

Changes to the format of a given graph, such as the size of the intervals, the width of the bars and the visual representation, affect how the information is interpreted. As students examine various graphs they should consider how the format could lead to misinterpreting the data presented.





The smaller scale used in Graph A suggests the increase in subscribers is greater than the increase shown in Graph B. However, both graphs actually show the same increase.



The wider bar could create the impression that John's score is much higher than Katie when, in fact, he scored lower than her.

Students should understand that sometimes the creator of a graph will intentionally misrepresent the data. This is usually done to emphasize or draw the reader's attention to an intended interpretation. The following are some ways in which graphs can be misrepresented:

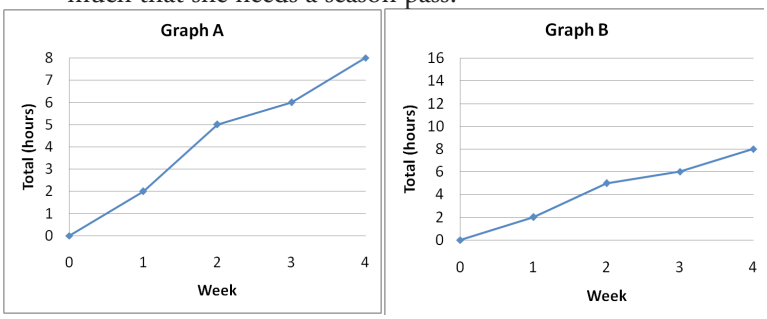
- Starting the scale at a number other than zero
- Using bars of different widths (area)
- Absence of a scale
- Larger symbols used for a particular category on a pictograph
- No key given for pictograph symbol
- Sections of a pie graph pulled away from the other sectors to emphasize it

General Outcome: Collect, Display and Analyze Data to Solve Problems

Suggested Assessment Strategies

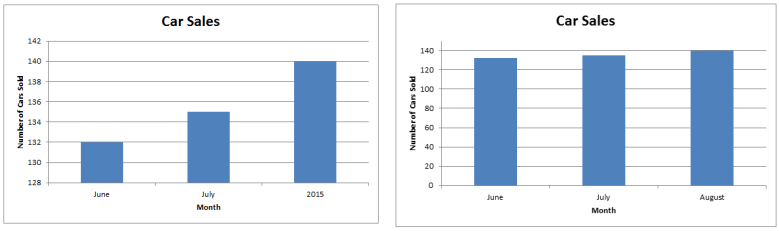
Journal

- Ask students to respond to the following:
  (i) The two graphs below show Sarah's hours of skiing during the month of February. Which graph would be best used to convince her parents that she has increased her skiing time so much that she needs a season pass?



(8SP1.4)

  (ii) You are going to ask your boss for a raise. Which of the following graphs would you use to convince your boss to grant the raise? Explain your choice.



(8SP1.4)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 7.2: Misrepresenting Data

ProGuide: pp.16-27

CD-ROM: Master 7.18

SB: pp. 394-402

PB: pp. 171-174

Statistics and Probability (Data Analysis)

Specific Outcomes

Students will be expected to

8SP1 Continued ...

Achievement Indicators:

8SP1.4 *(Continued) Explain how the format of a given graph, such as the size of the intervals, the width of bars and the visual representation, may lead to misinterpretation of the data.*

8SP1.5 *(Continued) Explain how a given formatting choice could misrepresent the data.*

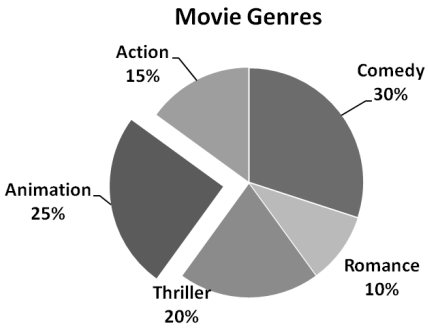
8SP1.6 *(Continiued) Identify conclusions that are inconsistent with a given data set or graph and explain the misinterpretation.*

Suggestions for Teaching and Learning

Provide students with a conclusion that is inconsistant with a given graph. They should study each graph and explain the misinterpretation. Consider the following examples:

- 

  Michelle says that company profits have tripled since January.

- 

  Animation is the most popular movie genre.

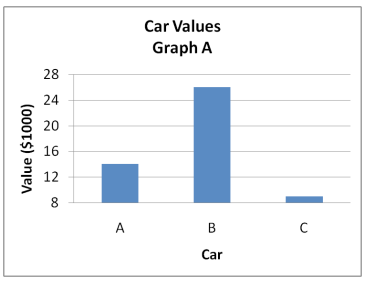
Students should also be able to explain how they could change the graph to accurately represent the data.

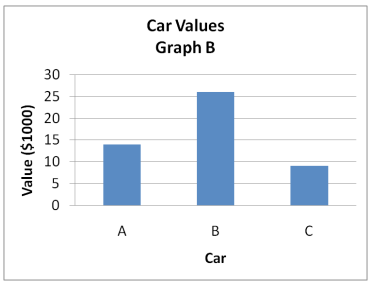
General Outcome: Collect, Display and Analyze Data to Solve Problems

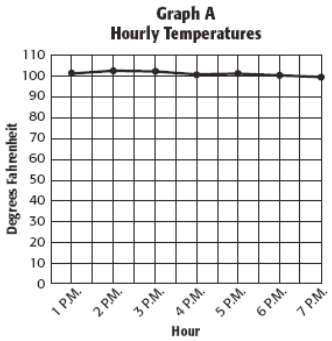
Suggested Assessment Strategies

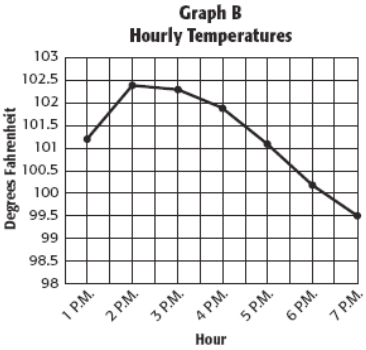
Paper and Pencil

- Provide students with a set of graphs and ask them to answer the following questions:
  - (i) How is the data in each graph misrepresented?
  - (ii) Why would the creator of each graph choose to portray the information this way?
  - (iii) Explain how the interpretation for Graph A could be different from the interpretation for Graph B.









(8SP1.5, 8SP1.6)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 7.2: Misrepresenting Data

ProGuide: pp.16-27

Master 7.7

CD-ROM: Master 7.18

SB: pp.394-402, 403-405

PB: pp.171-174

Statistics and Probability (Chance and Uncertainty)

Specific Outcomes

Students will be expected to

8SP2 Solve problems involving the probability of independent events.

[C, CN, PS, T]

8N5 Solve problems that involve rates, ratios and proportional reasoning.

[C, CN, PS, R]

Achievement Indicators:

8SP2.1 *Determine the probability of two given independent events and verify the probability using a different strategy.*

8N5.2 *Provide a context in which* $\frac{a}{b}$ *represents a:*

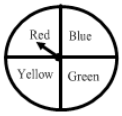
- *fraction*
- *rate*
- *ratio*
- *quotient*
- ***probability***

Suggestions for Teaching and Learning

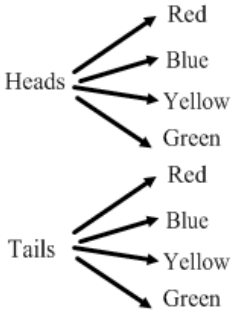
In Grade 7, students expressed probabilities as ratios, fractions and percents. They identified the sample space for a probability experiment involving two independent events and conducted experiments to compare the theoretical and experimental probabilities. In this unit, students will generalize and apply a rule for determining the probability of independent events.

To activate prior knowledge, ask students questions such as the following:

- How do you determine the theoretical probability of an event?
- What are independent events?
- Are rolling a die and flipping a coin independent or dependent events?
- What is the probability of flipping a heads on a dime and spinning red on the spinner shown below?



Students should organize the possible outcomes for this question by constructing a graphic organizer such as a table or a tree diagram:



	Red	Blue	Yellow	Green
Head	RH	BH	YH	GH
Tail	RT	BT	YT	GT

Based on their graphic organizer, students should conclude that $P(\text{heads and red}) = \frac{1}{8}$.

General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty

Suggested Assessment Strategies

Performance

- Ask students to work with a partner and give each group two dice. Player A will score a point if the sum of the two dice is even and player B will score a point if the sum is odd. Once students have finished their game ask them to use graphic organizers, such as a tree diagram or a table, to find the outcomes for the independent events.

  (i) Is the game fair? If not, how could you make the game fair?
  (ii) What is the probability of getting a sum of 2, 8, 11, etc.?

  Ask students to repeat the activity using the product of the dice to receive their points.

  (8SP2.1)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 7.3: Probability of Independent Events

ProGuide: pp. 29-35

Master 7.8a, 7.8b

CD-ROM: Master 7.19

SB: pp.407-413

PB: pp.175-178

Statistics and Probability (Chance and Uncertainty)

Specific Outcomes

Students will be expected to

8SP2 Continued...

8N5 Continued...

Achievement Indicators:

8SP2.2 *Generalize and apply a rule for determining the probability of independent events.*

8N5.2 *(Continued) Provide a context in which* $\frac{a}{b}$ *represents a:*

- *fraction*
- *rate*
- *ratio*
- *quotient*
- ***probability***

Suggestions for Teaching and Learning

Through various investigations, students should recognize that creating a graphic organizer can be very time consuming in determining the probability of independent events (especially in the case of more than two events). An example such as the following could help students generalize a rule for determining the probability of independent events.

What is the probability of flipping a heads on a dime and spinning red on the spinner shown below?

Students used a graphic organizer to determine P(heads and red)$=\frac{1}{8}$.

Ask students questions such as the following:

- What is P(heads)?
- What is P(red)?
- How does the probability of the individual events relate to the probability of the combined events?

Students should recognize that P(heads and red) is the product of the two individual probabilities: P(heads and red) = P(heads) × P(red).

This rule can be generalized for any events:

P(Event 1 and Event 2) = P(Event 1) × P(Event 2).

P(A and B) = P(A) × P(B)

This rule can be extended to three or more independent events.

General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty

Suggested Assessment Strategies

Journal

- Ask students to respond to the following:

  Justin was asked to determine *P*(G, 4, T), using the spinner shown below, a standard die and a dime.

  He decided to use a table to list the possible outcomes for this probability experiment. Is this a good idea? Why or why not? Explain a more efficient method of determining the probability.

  (8SP2.2)

- The probability of two independent events is $\frac{5}{12}$. Ask students to respond to the following:
  (i) What could the two events be?
  (ii) If one of the events is tossing heads, what could the other event be?

  (8SP2.2)

Presentation

- Ask students to develop a game using the probability of independent events.

  (8SP2.1)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 7.3: Probability of Independent Events

ProGuide: pp.29-35

Master 7.9

CD-ROM: Master 7.19

SB: pp.407-413

PB: pp.175-178

Statistics and Probability (Chance and Uncertainty)

Specific Outcomes

Students will be expected to

8SP2 Continued...

8N5 Continued...

Achievement Indicators:

8SP2.3 *Solve a given problem that involves determining the probability of independent events.*

8N5.2 *(Continued) Provide a context in which* $\frac{a}{b}$ *represents a:*

- *fraction*
- *rate*
- *ratio*
- *quotient*
- ***probability***

Suggestions for Teaching and Learning

Students should solve a variety of problems that involve determining the probability of independent events. Consider the following:

- Using a standard die and the three section spinner shown below, determine the probability of rolling a number less than 6 and spinning blue.

- The school cafeteria offers a lunch special. Sandra must choose one item from each category to complete her combo:

  Lunch: Pizza, Sub or Salad
  Drink: Milk, Juice or Bottled Water
  Dessert: Cookie or Muffin

- Wanda and her friends are planning on attending an outdoor movie on Saturday and a fireworks dispay on Sunday. The forecast is reporting a 30% chance of rain on Saturday and a 65% chance of rain on Sunday. What is the probability that it will rain on both days? What is the probability that it will not rain on either day?

- The chances of winning a prize in Tim Hortons© Roll Up the Rim to Win Contest is 1 out of 6. Anna buys a hot chocolate every day. Determine the probability of Anna winning a prize three days in a row. Determine the probability that Anna will not win a prize on either day.

General Outcome: Use Experimental or Theoretical Probabilities to Represent and Solve Problems Involving Uncertainty

Suggested Assessment Strategies

Paper and Pencil

- The cafeteria offers chicken burgers, pizza, or nachos as entrees and fruit slushies, water, or milk as beverages.

  Ask students to determine the probability that your friend chooses the following:

  (i) *P*(pizza and milk)?
  (ii) *P*(chicken burger and water)?
  (iii) *P*(nachos and not milk)?

  (8SP2.3, 8N5.2)

- In the game of Monopoly, you must roll doubles on the dice in order to get out of jail. Ask students to determine the probability of rolling doubles on your next turn.

  (8SP2.3, 8N5.2)

- Both you and your friend have a bag of fruit snacks. Each bag has 3 grape, 4 strawberry, 3 orange and 2 lemon. Ask students to determine the probability of you picking orange and your friend picking grape fruit snack.

  (8SP2.3, 8N5.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 7.3: Probability of Independent Events

Lesson 7.4: Solving Problems Involving Independent Events

ProGuide: pp. 29-35, 39-44

Master 7.9

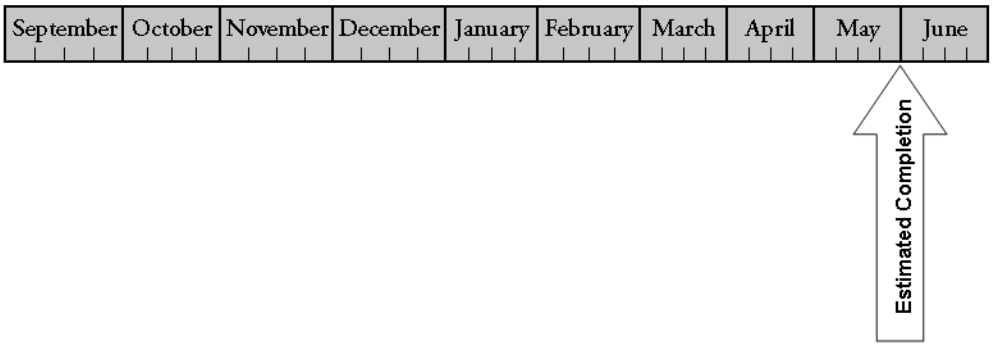
CD-ROM: Masters 7.19, 7.20

SB: pp. 407-413, 417-422

PB: pp. 175-181

Geometry

Suggested Time: 3 Weeks



Unit Overview

Focus and Context

In this unit, students will use models and drawings to create a variety of views (front, top and side) of three-dimensional objects. They will discover that more than one 2-D drawing can be created for each object depending on the viewpoint of the observer. Students will apply their knowledge of transformations to tessellations. They will create their own tessellations and make connections to tesellations that occur in their environment.

As students learn to describe the position of objects in structures and pictures, and transform and construct shapes, the development of spatial sense is supported. Developing a good understanding of representing 3-D objects and transformations will better prepare students for the higher order mathematics they will study in high school and beyond. Engineering, carpentry, surveying, interior decorating and architecture all require a strong knowledge of geometric concepts. Being able to visualize spatially and analyze changes in objects increase students' higher order thinking skills which are crucial in our technological and information based society.

Outcomes Framework

GCO
Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

SCO 8SS5

Draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms.

GCO
Describe and analyze position and motion of objects and shapes.

SCO 8SS6

Demonstrate an understanding of tessellation by:

- explaining the properties of shapes that make tessellating possible
- creating tessellations
- identifying tessellations in the environment.

SCO Continuum

Grade 7	Grade 8	Grade 9
Strand: Shape and Space (3-D Objects and 2-D Shapes)		
Specific Outcomes	**Specific Outcomes**	**Specific Outcomes**
7SS3 Perform geometric constructions, including: • perpendicular line segments • parallel line segments • perpendicular bisectors • angle bisectors. [CN, R, V]	8SS5 Draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms. [C, CN, R, T, V]	9SS2 Determine the surface area of composite 3-D objects to solve problems. [C, CN, PS, R, V] 9SS3 Demonstrate an understanding of similarity of polygons. [C, CN, PS, R, V]
Grade 7	**Grade 8**	**Grade 9**
Strand: Shape and Space (Transformations)		
7SS4 Identify and plot points in the four quadrants of a Cartesian plan, using integral ordered pairs. [C, CN, V] 7SS5 Perform and describe transformations (translations, rotations or reflections) of a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices). [C, CN, PS, T, V]	8SS6 Demonstrate an understanding of tessellation by: • explaining the properties of shapes that make tessellating possible • creating tessellations • identifying tessellations in the environment. [C. CN. PS. T. V]	9SS4 Draw and interpret scale diagrams of 2-D shapes. [CN, R, T, V] 9SS5 Demonstrate an understanding of line and rotation symmetry. [C, CN, PS, V]

Mathematical Processes

[C] Communication	**[PS] Problem Solving**
[CN] Connections	**[R] Reasoning**
[ME] Mental Mathematics and Estimation	**[T] Technology**
	[V] Visualization

Shape and Space (3-D Objects and 2-D Shapes)

Specific Outcomes

Students will be expected to

8SS5 Draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms.

[C, CN, R, T, V]

Achievement Indicators:

8SS5.1 *Draw and label the top, front and side views for a given 3-D object on isometric dot paper.*

8SS5.2 *Compare different views of a given 3-D object to the object.*

Suggestions for Teaching and Learning

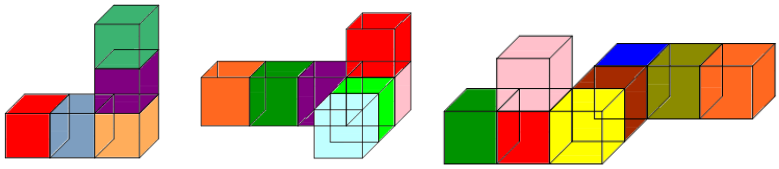
The study of 3-D drawings is new to Grade 8 students. However, some students may be familiar with these types of images in video games and 3-D movies.

In this unit, they will draw and compare the views of given 3-D objects and will interpret views in order to build the 3-D objects. Modelling these objects using linking cubes will strengthen student understanding of the top, front and side views as it will allow them to physically manipulate the objects.

Teachers could use everyday objects, such as a tissue box, to introduce the vocabulary for top view, front view, left view and right view.

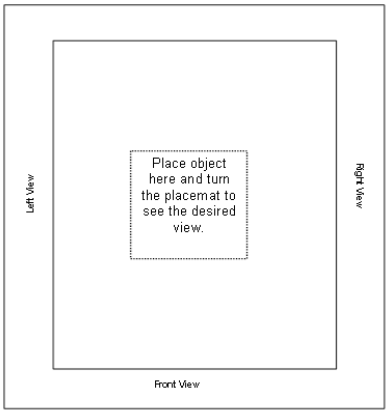
Top View: Front view: Left view: Right view:

Using linking cubes, students couldbuild various models, such as:



Using the models and square dot paper, they should identify and draw the top, front and side views of each model.

Some students may benefit from using a placemat, such as the one shown below, when working with manipulatives and other given objects. They should place the model or object on the mat. To see each view, students simply turn the mat or look at the object from above. They can then draw the views on square dot paper as they see them.



General Outcome: Describe the Characteristics of 3-D Objects and 2-D Shapes, and Analyze the Relationships Among Them

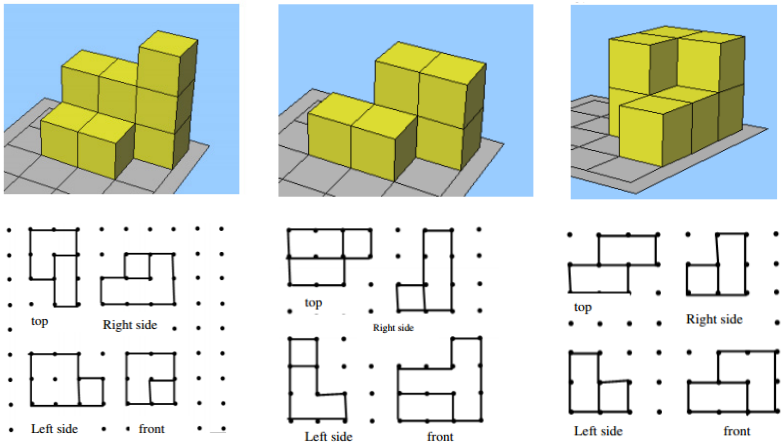
Suggested Assessment Strategies

Performance

- Provide students with interlocking cubes and ask them to build a shape, using a specified number of cubes.
  - (i) Ask students to sketch the top, front and side views on square dot paper.
  - (ii) Ask students to exchange their models and views with another student to verify they are correct.
  - (iii) Display the models and views prepared by students and ask students to match the view with the correct model.

(8SS5.1, 8SS5.2)

- Provide students with a deck of cards. Half of the cards should show a 3-D object and the other half should illustrate the corresponding front, top and side views (as shown below). Students should match each 3-D object with its views.



(8SS5.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 8.1: Sketching Views of Objects

ProGuide: pp. 4-10

Master 8.6a, 8.6b

CD-ROM: Master 8.26

Student Book (SB): pp. 434-439

Practice and Homework Book (PB): pp. 191-194

Shape and Space (3-D Objects and 2-D Shapes)

Specific Outcomes

Students will be expected to

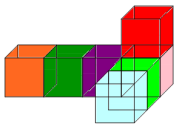
8SS5 Continued

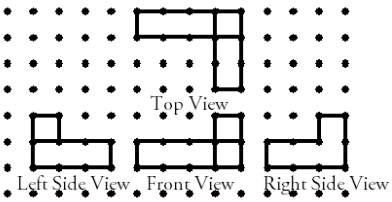
Achievement Indicators:

8SS5.1 *(Continued) Draw and label the top, front and side views for a given 3-D object on isometric dot paper.*

8SS5.2 *(Continued) Compare different views of a given 3-D object to the object.*

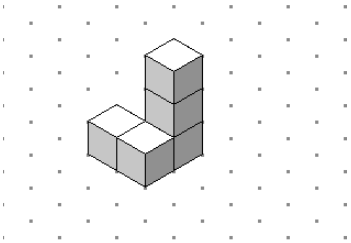
Suggestions for Teaching and Learning

In the case of  , students would sketch the following:

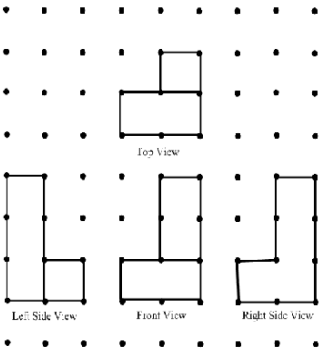


When drawing views, internal line segments are drawn only where the depth or thickness of the object changes.

Once students are able to identify and sketch these views from a physical model, teachers should present a 3-D image drawn on isometric paper. It is useful to have the faces of these drawings shaded to create the three dimensional look. Students should sketch the views of objects such as:



Encourage them to begin by identifying the top, front and side views on the given diagram. This should help them sketch the views:



Teachers should emphasize the relationship between each of the views and the actual object.

General Outcome: Describe the Characteristics of 3-D Objects and 2-D Shapes, and Analyze the Relationships Among Them

Suggested Assessment Strategies

Performance

- Teachers could create a set of puzzles, illustrating a 3-D shape and its corresponding top, front, and side views. Give each student a puzzle piece and ask them to complete their puzzle by finding the other students who have the missing views or 3-D object. (8SS5.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 8.1: Sketching Views of Objects

ProGuide: pp. 4-10

Master 8.6a, 8.6b

CD-ROM: Master 8.26

SB: pp. 434-439

PB: pp. 191-194

Shape and Space (3-D Objects and 2-D Shapes)

Specific Outcomes

Students will be expected to

8SS5 Continued...

Achievement Indicators:

8SS5.3 *Predict the top, front and side views that will result from a described rotation (limited to multiples of 90 degrees) and verify predictions.*

8SS5.4 *Draw and label the top, front and side views that result from a described rotation (limited to multiples of 90 degrees).*

8SS5.5 *Build a 3-D block object, given the top, front and side views, with or without the use of technology.*

8SS5.6 *Sketch and label the top, front and side views of a 3-D object in the environment with or without the use of technology.*

Suggestions for Teaching and Learning

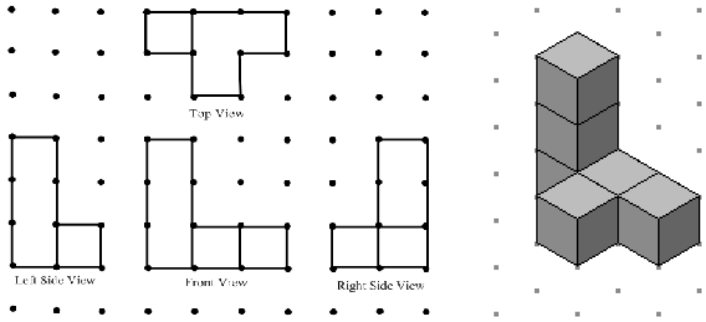
All rotations are limited to multiples of 90 degrees. Objects may be rotated along horizontal or vertical axes only.

Provide students with a linking cube 3-D object. They should sketch the top, front and side views for the given object. Ask students questions such as the following:

- How would the top, front and side views change if you rotated the object 90° clockwise? 90° counter clockwise?
- How would the top, front and side views change if you rotated the object 180° clockwise? 180° counter clockwise?
- How would the top, front and side views change if you rotated the object 270° counter clockwise? 270° counter clockwise?

Students should state and check their predictions by performing the indicated rotation. The use of the placemat with interlocking cubes could be useful to students as they determine the changes in views. They should draw and label the top, front and side views.

If given the top, front and side views, students should build the corresponding 3-D block object. They should carefully analyze each view to determine how many blocks are visible and where the changes in depth occur. Remind students that an internal line indicates a change in depth. Consider the following example:



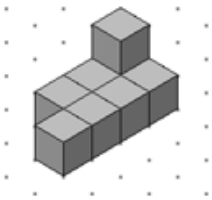
Students should use linking cubes to create the object. Interactive websites and software programs, such as those listed on the Professional Learning site should be used to assist students with their constructions. As a culminating activity, ask students to identify a 3-D object in their environment. They should sketch the top, front and side views of the object using square dot paper or techology. Students should predict how the views would change after a 270° rotation over a vertical axis, for example, and then sketch these views.

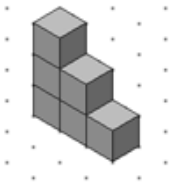
General Outcome: Describe the Characteristics of 3-D Objects and 2-D Shapes, and Analyze the Relationships Among Them

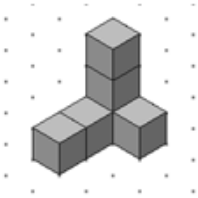
Suggested Assessment Strategies

Paper and Pencil

- Ask students to use the isometric drawings below to draw the top, front, left side and right side views on square dot paper.







  Repeat the above exercise for a rotation of 90 degrees clockwise.
  (8SS5.4)

- Ask students to sketch and label the top, front, right side and left side views of their school.
  (8SS5.6)

Journal

- Ask students to respond to the following:

  A classmate insists that you need all six views of an object to create a model. Is this correct? Explain.
  (8SS5.5)

Presentation

- Ask students choose an object of interest to them and draw the views of this object. Final products could be displayed in the classroom.
  (8SS5.6)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 8.2: Drawing Views of Rotated Objects

Lesson 8.3: Building Objects from their Views

ProGuide: pp. 11-16, 17-24

Master 8.7, 8.8a, 8.8b

CD-ROM: Master 8.27, 8.28

SB: pp. 441-446, 447-454

PB: pp. 195-198, 199-201

Suggested Resource

Resource Link: http://www.k12pl.nl.ca/curr/k-6/math/grade-8/links/unit8.html

- Link to interactive websites and software programs

Shape and Space (Transformations)

Specific Outcomes

Students will be expected to

8SS6 Demonstrate an understanding of tessellation by:

- **explaining the properties of shapes that make tessellating possible**
- **creating tessellations**
- **identifying tessellations in the environment.**

[C, CN, PS, T, V]

Achievement Indicator:

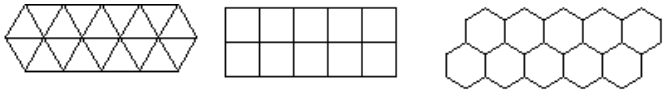
8SS6.1 *Identify, in a given set of regular polygons, those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices.*

Suggestions for Teaching and Learning

Students have been studying transformations since Grade 5, where they performed a single transformation on a 2-D shape. In Grade 6, they extended this knowledge to perform a combination of transformations on a 2-D shape and performed single transformations on a 2-D shape in the first quadrant of the Cartesian plane. In Grade 7, students performed and described transformations on a 2-D shape in all four quadrants of a Cartesian plane (limited to integral number vertices). In this unit, they will use this knowledge to study tessellations. A tessellation is a design created when congruent copies of a shape cover a plane without any gaps or overlaps. Another word for a tessellation is a tiling.

In Mathematics 6, students explored regular and irregular polygons. They learned that the the sides of a regular polygon are of the same length and that the angles of a regular polygon are of the same measure. They sorted polygons as regular or irregular, identified and described regular and irregular polygons in their enviroment. Teachers could activate prior knowlege by asking students to describe and sort regular and irregular polygons.

Provide small groups of students with a set of regular polygons. Ask them to investigate which of the polygons could cover the top of their desk with no gaps or overlaps. Through exploration, students should observe that only triangles, squares and hexagons can cover the top of their desk without gaps or overlaps:



Teachers should introduce the terms plane and tessellations in connection with this activity. A regular tessellation is a tessellation made up of congruent regular polygons.

Students should further explore tessellations in terms of angle measurement. Ask questions such as the following:

- Why do you think that some polygons tessellate and others do not?
- What do you notice about the sum of the angles at a point where the vertices meet?

They should conclude this sum is 360°.

General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

Journal

- Ask students to draw two different regular polygons that tessellate in a plane. They should explain why they know their figure will tessellate, including diagrams to support their answer.

  (8SS6.1)

- Ask students to respond to the following:

  Sarah is tiling her bedroom floor. Could Sarah choose ceramic tiles in the shape of a regular octagon? Explain your thinking.

  (8SS6.1)

Performance

- Provide students with a set of pattern blocks and ask them to complete the following:
  - (i) Determine which shapes will tessellate and trace the tessellation. Colour the traced blocks.
  - (ii) Using your sketches of the polygons which tessellate, find the sum of the angles at any given point on each sketch. What do you notice? Do you think this will always be the case? Why?

  (8SS6.1)

- Using the *Cut Out* template provided, ask students to cut out three copies of each polygon using cardstock. They should determine whether each polygon tessellates and record their results. Ask them to determine the sum of the angles at any given point where the vertices meet and state what they notice.

  (8SS6.1)

- Provide students with a set of pattern blocks. Ask them to determine whether the following combinations of pattern blocks can be used to create a tesselation:

  Combination 1: triangles and squares

  Combination 2: hexagons and squares

  Combination 3: hexagons and triangles

  Combination 4: hexagons, squares and triangles

  Combination 5: your choice

  They should trace their blocks as they explore. Using their sketches of the combinations that created tessellations, students should determine the sum of the angles at any given point on each sketch. They should state their observations.

  (8SS6.1)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 8.5: Constructing Tessellations

ProGuide: pp.32-40

CD-ROM: Master 8.30

SB: pp. 462-470

PB: pp. 205-207

Suggested Resource

Resource Link: http://www.k12pl.nl.ca/curr/k-6/math/grade-8/links/unit8.html

- Template for *Cut Out* activity

Shape and Space (Transformations)

Specific Outcomes

Students will be expected to

8SS6 Continued...

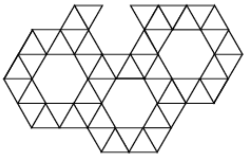
Achievement Indicator:

8SS6.1 *(Continued) Identify, in a given set of regular polygons, those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices.*

Suggestions for Teaching and Learning

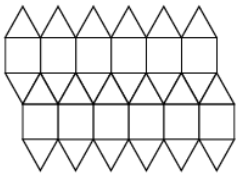
Students should explore the possibilities for creating a tessellation using a combination of regular polygons. Some of the following combinations might be suggested:

- Four triangles and a hexagon at each vertex point



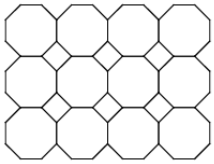
$$(60° + 60° + 60° + 60°) + 120° = 360°$$

- Three triangles and two squares at each vertex point



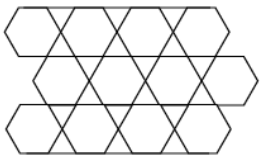
$$(60° + 60° + 60°) + (90° + 90°) = 360°$$

- Square and two octagons at each vertex point



$$90° + 135° + 135° = 360°$$

- Triangle, hexagon, triangle, hexagon at each vertex point



$$60° + 120° + 60° + 120° = 360°$$

Alternatively, students could use various technologies to explore which regular polygons and combination of regular polygons will tessellate. Students should explore the sum of the angle measures at a point where the vertices meet to confirm that in order for a polygon to tessellate, the sum of the angle measures where vertices meet must be $360°$. Students should be given the measures of each interior angles of regular polygons:

Polygon	Interior angle measure
Triangle	60°
Square	90°
Pentagon	108°
Hexagon	120°
Octagon	135°
Decagon	144°
Dodecagon	150°

They should be able to refer to this information when completing in class activities, assessments and evaluations.

General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

Performance

- Using the *Cut Out* template provided, ask students to cut out three copies of each polygon using cardstock. They should determine which of the following combinations of polygons can be used to create a tessellation:

  Combination 1: triangles and squares

  Combination 2: pentagons and triangles

  Combination 3: hexagons and squares

  Combination 4: hexagons and triangles

  Combination 5: pentagons and squares

  Combination 6: octagons and squares

  Combination 7: triangles and octagons

  Combination 8: hexagons, squares and triangles

  Combination 9: decagons and triangles

  Combination 10: dodecagons and triangles

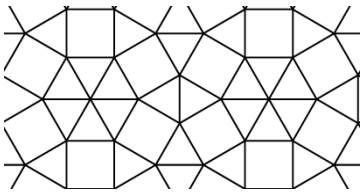
  Combination 11: your choice

  They should sketch the result for each combination. Using the sketches of those combinations that created tessellations, ask students to determine the sum of the angles at any given point on each sketch. They shoud state their observations.

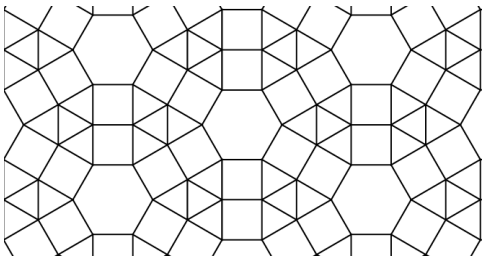
  (8SS6.1)

- Ask students to examine the tessellations below to identify which polygons were used. They should use angle measures to verify the tessellation:

  (i)



  (ii)



  (8SS6.1)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 8.5: Constructing Tessellations

ProGuide: pp. 32-40

CD-ROM: Master 8.30

SB: pp. 462-470

PB: pp. 205-207

Suggested Resource

Resource Link: http://www.k12pl.nl.ca/curr/k-6/math/grade-8/links/unit8.html

- Template for *Cut Out* activity

Shape and Space (Transformations)

Specific Outcomes

Students will be expected to

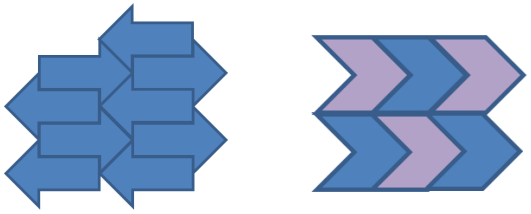
8SS6 Continued...

Achievement Indicators:

8SS6.2 *Identify, in a given set of irregular polygons, those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices.*

Suggestions for Teaching and Learning

Students should explore a set of irregular polygons to determine those shapes and combinations of shapes that will tesselate. Working with concrete models will allow them to observe how the shapes fit together, leave gaps or overlap each other. Through investigation they should discover that all triangles and all quadrilaterals tessellate. Some irregular convex and concave pentagons and hexagons will tessellate. They should once again make the connection that at any point where vertices meet the sum of the angle measures is $360°$.



8SS6.3 *Identify and describe tessellations in the enviornment.*

Teachers could brainstorm session with students to identify and describe tessellations in the enviroment. They may suggest:

- Floor tiles
- Quilting
- Fencing patterns
- Wall paper patterns
- Bricklaying patterns
- Company logos

Any such examples should be referenced as much as possible during discussions throughout the unit.

General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

Performance

- Using the template provided, ask students to complete the following:
  (i) Trace this isosceles triangle onto construction paper. Use the triangle to create a tessellation. Remember that the sum of the angles around any given point must be 360°.

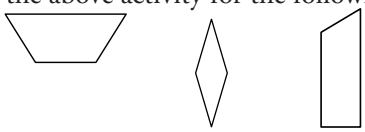
  (ii) Trace this scalene triangle onto construction paper. Use the triangle to create a tessellation. Remember that the sum of the angles around any given point must be 360°.

  (iii) Trace a triangle of your choice onto construction paper. Use the triangle to create a tessellation. Remember that the sum of the angles around any given point must be 360°.

  What do you notice about your tessellations?

(8SS6.2)

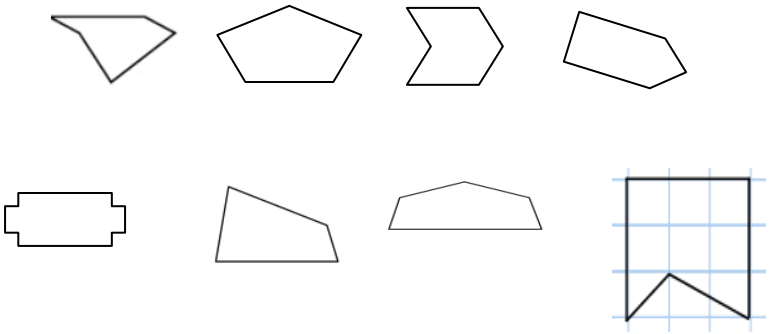
- Repeat the above activity for the following quadrilaterals.



(8SS6.2)

Paper and Pencil

- Ask students to identify which of the following irregular polygons can be used to create a tessellation:



(8SS6.2)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 8.5: Constructing Tessellations

ProGuide: pp. 32-40

CD-ROM: Master 8.30

SB: pp. 462-470

PB: pp. 205-207

Suggested Resource

Resource Link: http://www.k12pl.nl.ca/curr/k-6/math/grade-8/links/unit8.html

- Template for *Performance* task

Shape and Space (Transformations)

Specific Outcomes

Students will be expected to

8SS6 Continued

Achievement Indicators:

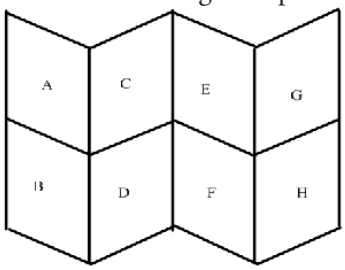
8SS6.4 *Identify a translation, reflection or rotation in a given tessellation.*

8SS6.5 *Identify a combination of transformations in a given tessellation.*

8SS6.3 *(Continued) Identify and describe tessellations in the environment.*

Suggestions for Teaching and Learning

Provide students with a variety of tessellations and ask them to identify which transformation(s) were used in the tessellation. Manipulatives, such as Miras™ and tracing paper, may help students identify these transformations. Consider the following example:



In identifying the transformations that were used to create this tessellation, students could make statements such as the following:

- Shape A can be transformed into shape E through a translation to the right.
- Shape A can be transformed into shape C by reflection in the side that is shared between them.
- Shape A can be transformed into shape B by rotating 180° around the midpoint of their shared side (or through a translation down).

Students should also be able to examine the given tessellation and identify combinations of transformations such as:

- Shape A can be transformed into shape F if it is translated to the right to position E and then rotated 180 degrees around the midpoint of the shared side between E and F.
- Shape A can be transformed into shape D by being rotated 180 degrees around the midpoint of the side shared between A and B and then reflected in the side shared between B and D.

It is important to note that when describing the transformations in a given tessellation multiple solutions are possible. Encourage students to share their solutions as they explore various tessellations, including those from their environment.

General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

Journal

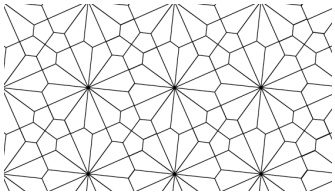
- John missed the class on tessellating patterns using transformations. Ask students to explain to him why some tessellating patterns made using translations can also be made using reflections?

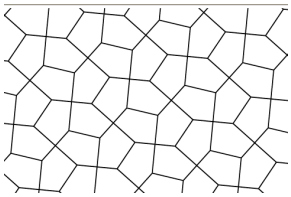
(8SS6.4)

Paper and Pencil

- Ask students to complete the following:

  The tessellations below were created using irregular polygons. Using different colours, shade in one of each type of polygon used in the tessellation. Describe how each tessellation was created. Which transformations were used?





(8SS6.4, 8SS6.5)

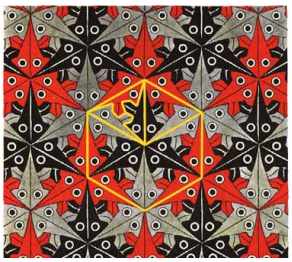
Performance

- Ask students to use the Internet to identify an example of a tessellation in their environments. They should print their tessellation and describe it in terms of the shapes used and the transformations used to create the tessellation. Students should share their example with the class.

(8SS6.3, 8SS6.4, 8SS6.5)

Observation

- Ask students to use the Escher tessellation shown to answer the questions which follow:



(i) Which regular polygon is used to create the tessellation?
(ii) Describe the transformations that are used to tessellate the polygon.
(iii) Which transformations are used to tessellate the shape within the polygon?

(8SS6.3, 8SS6.4, 8SS6.5)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 8.4: Identifying Transformations

Lesson 8.6: Identifying Transformations in Tessellations

ProGuide: pp. 26-31, 41-48

CD-ROM: Master 8.29, 8.31

SB: pp. 456-461, 471-478

PB: pp. 202-204, 208-210

Shape and Space (Transformations)

Specific Outcomes

Students will be expected to

8SS6 Continued...

Achievement Indicators:

8SS6.6 *Create a tessellation using one or more 2-D shapes, and describe the tessellation in terms of transformations and conservation of area.*

8SS6.7 *Create a new tessellating shape (polygon or non-polygon) by transforming a portion of a given tessellating polygon and describe the resulting tessellation in terms of transformations and conservation of area.*

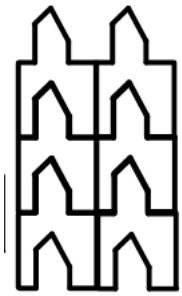
Suggestions for Teaching and Learning

One method of creating tessellations involves starting with a 2-D shape students know will tessellate, such as a quadrilatral.

Students should identify a shape to be cut out of one side:

They should then translate the cut out piece to the opposite side of the figure. This ensures the conservation of area within the figure:

Students can replicate the shape and use their knowledge of transformations to create their tessellation:



Students may make more complex designs by using a combination of 2-D shapes or by cutting shapes out of additional sides in the original polygon. Technology, such as Paint, Microsoft Word or Mircrosoft Excel could be used to help students create their tessellations. Encourage them to share their designs with the class and discuss the transformations used. Teachers could also consider inviting a community artist to participate in a class quilting or art project involving tessellations.

General Outcome: Describe and Analyze Position and Motion of Objects and Shapes

Suggested Assessment Strategies

Performance

- Ask students to create a tessellation by completing the following:
  (i) Using a piece of card stock, cut out a square. Determine the area of your square.
  (ii) Starting at the edge of the square, cut a small irregular shape out of the square. What has happened to the area of the square?
  (iii) Use a small piece of tape to attach the cut-out piece to another edge of the square. What is the area of your new shape?
  (iv) Use your new shape to create a tessellation.

  Repeat the above steps using a hexagon or a triangle.
  Students should share their tessellations with the class.

  (8SS6.7)

- Ask students to design a new pattern for flooring for their bedroom using a tessellation. Describe the tessellation used in terms of the shapes used and the transformations used to create the tessellation. They should present their designs to the class.

  (8SS6.6, 8SS6.7)

Journal

- Ask students to respond to the following:

  When creating a tile for an Escher-style tessellation, a piece of the original polygon is cut out and placed on another side of the original polygon. How do you know the area of the original polygon is maintained?

  (8SS6.6)

Resources/Notes

Authorized Resource

Math Makes Sense 8

Lesson 8.4: Identifying Transformations

Lesson 8.5: Constructing Tessellations

Lesson 8.6: Identifying Transformations in Tessellations

ProGuide: pp. 26-31, 32-40, 41-48

CD-ROM: Master 8.29, 8.30, 8.31

SB: pp. 456-461, 462-470, 471-478

PB: pp. 202-204, 205-207, 208-210

Appendix:

Outcomes with Achievement Indicators Organized by Topic

(With Curriculum Guide Page References)

Strand: Number	**General Outcome:** Develop number sense.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
8N1 Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V]	8N1.1 Represent a perfect square as a square region using materials, such as grid paper or square shapes.	pp. 22-25
	8N1.2 Determine whether or not a given number is a perfect square using materials and strategies, such as square shapes, grid paper or prime factorization, and explain the reasoning.	pp. 22-25
	8N1.3 Determine the factors of a given perfect square, and explain why one of the factors is a square root and the others are not.	pp. 26-29
	8N1.4 Determine the square root of a given perfect square and record it symbolically.	pp. 26-29
	8N1.5 Determine the square of a given number.	pp. 26-29
8N2 Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [C, CN, ME, R, T]	8N2.1 Estimate, or approximate, a square root of a given number that is not a perfect square using the roots of perfect squares as benchmarks.	p. 30
	8N2.2 Identify a whole number whose square root lies between two given numbers.	p. 30
	8N2.3 Approximate a square root of a given number that is not a perfect square using technology, e.g., calculator, computer.	p. 32
	8N2.4 Explain why a square root of a number shown on a calculator may be an approximation.	p. 32
8N3 Demonstrate an understanding of percents greater than or equal to 0% [CN, PS, R, V]	8N3.1 Provide a context where a percent may be more than 100% or between 0% and 1%.	p. 124
	8N3.2 Represent a given fractional percent using grid paper.	pp. 126-129
	8N3.3 Represent a given percent greater than 100 using grid paper.	pp. 126-129
	8N3.4 Determine the percent represented by a given shaded region on a grid and record it in decimal, fractional and percent form.	pp. 126-131
	8N3.5 Express a given percent in decimal or fractional form.	p. 130
	8N3.6 Express a given decimal in percent or fractional form.	p. 130
	8N3.7 Express a given fraction in decimal or percent form.	p. 130
	8N3.8 Solve a given problem involving percents.	pp. 132-135
	8N3.9 Solve a given problem involving combined percents.	p. 136
	8N3.10 Solve a given problem that involves finding the percent of a percent.	p. 138

Strand: Number	**General Outcome:** Develop number sense.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
8N4 Demonstrate an understanding of ratio and rate. [C, CN, V]	8N4.1 Express a two-term ratio from a given context in the forms 3:5 or 3 to 5.	p. 140
	8N4.2 Express a three-term ratio from a given context in the forms 4:7:3 or 4 to 7 to 3.	p. 140
	8N4.3 Express a part-to-part ratio as a part-to whole fraction.	p. 142
	8N4.4 Express a given ratio as a percent.	p. 142
	8N4.5 Identify and describe ratios from real-life examples, and record them symbolically.	p. 144
	8N4.6 Express a given rate using words or symbols, e.g., 20 L per 100 km or 20 L/100 km.	p. 146
	8N4.7 Identify and describe rates from real-life examples, and record them symbolically.	p. 146
	8N4.8 Explain why a rate cannot be represented as a percent.	p. 146
8N5 Solve problems that involve rates, ratios and proportional reasoning. [C, CN, PS, R]	8N5.1 Explain the meaning of $\frac{a}{b}$ within a given context.	pp. 142, 146
	8N5.2 Provide a context in which $\frac{a}{b}$ represents a: • fraction • rate • ratio • quotient • probability.	pp. 142, 146, 186-191
	8N5.3 Solve a given problem involving ratio.	p. 144
	8N5.4 Solve a given problem involving rates.	p. 148

Strand: Number	**General Outcome:** Develop number sense.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
8N6 Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]	8N6.1 Model multiplication of a positive fraction by a whole number concretely or pictorially and record the process.	p. 70
	8N6.2 Model multiplication of a positive fraction by a positive fraction concretely or pictorially using an area model and record the process.	p. 72
	8N6.3 Provide a context that requires the multiplication of two given positive fractions.	p. 72
	8N6.4 Estimate the product of two given positive proper fractions to determine if the product will be closer to 0, $\frac{1}{2}$, or 1.	p. 74
	8N6.5 Generalize and aply rules for multiplying positive proper fractions, including mixed numbers.	pp. 76-79
	8N6.6 Model division of a whole number and a positive proper fraction, concretely or pictorially and record the process.	pp. 80-85
	8N6.7 Model division of a positive proper fraction by a positive proper fraction pictorially and record the process.	p. 86
	8N6.8 Estimate the quotient of two given positive fractions and compare the estimate to whole number benchmarks.	p. 88
	8N6.9 Generalize and apply rules for dividing positive proper fractions.	pp. 90-93
	8N6.10 Model, generalize and apply rules for dividing fractions with mixed numbers.	p. 94
	8N6.11 Provide a context that requires the dividing of two given positive fractions.	p. 96
	8N6.12 Identify the operation required to solve a given problem involving positive fractions.	p. 96
	8N6.13 Solve a given problem involving positive fractions taking into consideration order of operations (limited to problems with positive solutions).	p. 98

Strand: Number	**General Outcome:** Develop number sense.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
8N7 Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically. [C, CN, PS, R, V]	8N7.1 Model the process of multiplying two integers using concrete materials or pictorial representations and record the process.	pp. 46-51
	8N7.2 Generalize and apply a rule for determining the sign of the product of integers.	p. 52
	8N7.3 Provide a context that requires multiplying two integers.	p. 54
	8N7.4 Solve a given problem involving the multiplication of integers.	p. 54
	8N7.5 Model the process of dividing an integer by an integer using concrete materials or pictorial representations and record the process.	p. 56
	8N7.6 Generalize and apply a rule for determining the sign of the quotient of integers.	p. 58
	8N7.7 Provide a context that requires dividing two integers.	p. 60
	8N7.8 Solve a given problem involving the division of integers (2-digit by 1-digit) without the use of technology.	p. 60
	8N7.9 Solve a given problem involving the division of integers (2-digit by 2-digit) with the use of technology.	p. 60
	8N7.10 Identify the operation required to solve a given problem involving integers.	p. 62
	8N7.11 Solve a given problem involving integers taking into consideration order of operations.	p. 64

Strand: Patterns and Relations (Patterns)	**General Outcome:** Use patterns to describe the world and solve problems.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
8PR1 Graph and analyze two-variable linear relations. [C, ME, PS, R, T, V]	8PR1.1 Create a table of values by substituting values for a variable in the equation of a given linear relation.	pp. 168-171
	8PR1.2 Determine the missing value in an ordered pair for a given equation.	pp. 168-171
	8PR1.3 Construct a graph from the equation of a given linear relation (limited to discrete data).	p. 170
	8PR1.4 Describe the relationship between the variables in a given graph.	p. 170

Strand: Patterns and Relations (Variables and Equations)	**General Outcome:** Use patterns to describe the world and solve problems.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
8PR2 Model and solve problems using linear equations of the form: • $ax = b$ • $\frac{x}{a} = b,\ a \neq 0$ • $ax + b = c$ • $\frac{x}{a} + b = c,\ a \neq 0$ • $a(x + b) = c$ where *a*, *b*, and *c* are integers.	8PR2.1 Model a given problem with a linear equation and solve the equation using concrete models.	pp. 156-159
	8PR2.2 Draw a visual representation of the steps used to solve a given linear equation and record each step symbolically.	pp. 156-159
	8PR2.3 Verify the solution to a given linear equation using a variety of methods, including concrete materials, diagrams and substitution.	pp. 158, 162
	8PR2.4 Solve a given linear equation symbolically.	p. 158
	8PR2.5 Apply the distributive property to solve a given linear equation of the form $a(x + b) = c$.	pp. 160-163
	8PR2.6 Identify and correct an error in a given incorrect solution of a linear equation.	p. 164
	8PR2.7 Solve a given problem using a linear equation and record the process.	p. 166

Strand: Shape and Space (Measurement)	**General Outcome:** Use direct or indirect measurement to solve problems.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
8SS1 Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V]	8SS1.1 Model and explain the Pythagorean theorem concretely, pictorially or using technology.	pp. 34-37
	8SS1.2 Determine the measure of the third side of a right triangle, given the measures of the other two sides, to solve a given problem.	pp. 36-39
	8SS1.3 Explain, using examples, that the Pythagorean theorem applies only to right triangles.	p. 38
	8SS1.4 Determine whether or not a given triangle is a right triangle by applying the Pythagorean theorem.	p. 38
	8SS1.5 Solve a given problem that involves Pythagorean triples, e.g. 3, 4, 5 or 5, 12, 13.	p. 40
8SS2 Draw and construct nets for 3-D objects.	8SS2.1 Match a given net to the 3-D object it represents.	p. 104
	8SS2.2 Draw nets for a given right circular cylinder, right rectangular prism, and verify by constructing the 3-D objects from the nets.	p. 104
	8SS2.3 Predict 3-D objects that can be created from a given net and verify the prediction.	p. 106
	8SS2.4 Construct a 3-D object from a given net.	p. 106
8SS3 Determine the surface area of: • right rectangular prisms • right triangular prisms • right cylinders to solve problems.	8SS3.1 Identify all the faces of a given prism, including right rectangular and right triangular prisms.	p. 108
	8SS3.2 Explain, using examples, the relationship between the area of a 2-D shape and the surface area of a given 3-D object.	p. 108
	8SS3.3 Describe and apply strategies for determining the surface area of a given right rectangular or right triangular prism.	p. 110
	8SS3.4 Solve a given problem involving surface area.	pp. 110-113
	8SS3.5 Describe and apply strategies for determining the surface area of a given right cylinder.	p. 112

Strand: Shape and Space (Measurement)	**General Outcome:** Use direct or indirect measurement to solve problems	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
8SS4 Develop and apply formulas for determining the volume of right prisms and right cylinders.	8SS4.1 Determine the volume of a given right prism, given the area of the base.	p. 114
	8SS4.2 Explain the connection between the area of the base of a given right 3-D objects and the formula for the volume of the object.	pp. 114-117
	8SS4.3 Generalize and apply a rule for determining the volume of right cylinders.	p. 116
	8SS4.4 Demonstrate that the position of a given 3-D object does not affect its volume.	p. 116
	8SS4.5 Apply a formula to solve a given problem involving the volume of a right cylinder or a right prism.	p. 118

Strand: Shape and Space (3-D Obects and 2-D Shapes)	**General Outcome:** Describe the characteristics of 3-D and 2-D shapes, and analyze the relationships among them.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
8SS5 Draw and interpret top, front and side views of 3-D objects composed of right rectangular prisms. [C, CN, R, T, V]	8SS5.1 Draw and label the top, front and side views for a given 3-D object on isometric dot paper.	pp. 196-199
	8SS5.2 Compare different views of a given 3-D object to the object.	pp. 196-199
	8SS5.3 Predict the top, front and side views that will result from a described rotation (limited to multiples of 90 degrees) and verify predictions.	p. 200
	8SS5.4 Draw and label the top, front and side views that result from a described rotation (limited to multiples of 90 degrees).	p. 200
	8SS5.5 Build a 3-D block object, given the top, front and side views, with or without the use of technology.	p. 200
	8SS5.6 Sketch and label the top, front and side views of a 3-D object in the environment with or without the use of technology.	p. 200

Strand: Shape and Space (Transformations)	**General Outcome:** Describe and analyze position and motion of objects and shapes.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
8SS6 Demonstrate an understanding of tessellation by: • explaining the properties of shapes that make tessellating possible • creating tessellations • identifying tessellations in the environment. [C. CN. PS. T. V]	8SS6.1 Identify, in a given set of regular polygons, those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices.	pp. 202-205
	8SS6.2 Identify, in a given set of irregular polygons, those shapes and combinations of shapes that will tessellate, and use angle measurements to justify choices.	p. 206
	8SS6.3 Identify and describe tessellations in the enviornment.	pp. 206-209
	8SS6.4 Identify a translation, reflection or rotation in a given tessellation.	p. 208
	8SS6.5 Identify a combination of transformations in a given tessellation.	p. 208
	8SS6.6 Create a tessellation using one or more 2-D shapes, and describe the tessellation in terms of transformations and conservation of area.	p. 210
	8SS6.7 Create a new tessellating shape (polygon or non-polygon) by transforming a portion of a given tessellating polygon and describe the resulting tessellation in terms of transformations and conservation of area.	p. 210

Strand: Statistics and Probability (Data Analysis)	**General Outcome:** Collect, display and analyze data to solve problems.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
8SP1 Critique ways in which data is presented. [C, R, T, V]	8SP1.1 Compare the information that is provided for the same data set by a given set of graphs, including circle graphs, line graphs, bar graphs, double bar graphs and pictographs, to determine the strengths and limitations of each graph.	pp. 176-179
	8SP1.2 Identify the advantages and disadvantages of different graphs, including circle graphs, line graphs, bar graphs, double bar graphs and pictographs, in representing a specific given set of data.	pp. 176-179
	8SP1.3 Justify the choice of a graphical representation for a given situation and its corresponding data set.	p. 180
	8SP1.4 Explain how the format of a given graph, such as the size of the intervals, the width of bars and the visual representation, may lead to misinterpretation of the data.	pp. 182-185
	8SP1.5 Explain how a given formatting choice could misrepresent the data.	pp. 182-185
	8SP1.6 Identify conclusions that are inconsistent with a given data set or graph and explain the misinterpretation.	pp. 182-185

Strand: Shape and Space (Chance and Uncertainty)	**General Outcome:** Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
8SP2 Solve problems involving probability of independent events. [C, CN, PS, T]	8SP2.1 Determine the probability of two given independent events and verify the probability using a different strategy.	p. 186
	8SP2.2 Generalize and apply a rule for determining the probability of independent events.	p. 188
	8SP2.3 Solve a given problem that involves determining the probability of independent events.	p. 190

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Western and Northern Canadian Protocol for Collaboration in Basic Education (Kindergarten to Grade 12). *The Common Curriculum Framework for K-9 Mathematics: Western and Northern Canadian Protocol* – May 2006 and *The Common Curriculum Framework for Grades 10-12* – January 2008. Reproduced (and/or adapted) by permission. All rights reserved.

September 2015
ISBN: 978-1-55146-561-6