

# Mathematics

*Grade 9*



*Curriculum Guide*

*2014*

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## INTRODUCTION

### Background

*The curriculum guide communicates high expectations for students.*

The Mathematics curriculum guides for Newfoundland and Labrador have been derived from *The Common Curriculum Framework for K-9 Mathematics: Western and Northern Canadian Protocol*, January 2008. These guides incorporate the conceptual framework for Kindergarten to Grade 9 Mathematics and the general outcomes, specific outcomes and achievement indicators established in the common curriculum framework. They also include suggestions for teaching and learning, suggested assessment strategies, and an identification of the associated resource match between the curriculum and authorized, as well as recommended, resource materials.

This Grade 9 Mathematics course was originally implemented in 2010.

### Beliefs About Students and Mathematics

*Mathematical understanding is fostered when students build on their own experiences and prior knowledge.*

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in developing mathematical literacy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. Through the use of manipulatives and a variety of pedagogical approaches, teachers can address the diverse learning styles, cultural backgrounds and developmental stages of students, and enhance within them the formation of sound, transferable mathematical understandings. Students at all levels benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions provide essential links among concrete, pictorial and symbolic representations of mathematical concepts.

The learning environment should value and respect the diversity of students' experiences and ways of thinking, so that students feel comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. They must come to understand that it is acceptable to solve problems in a variety of ways and that a variety of solutions may be acceptable.

## Affective Domain

*To experience success, students must learn to set achievable goals and assess themselves as they work toward these goals.*

A positive attitude is an important aspect of the affective domain and has a profound impact on learning. Environments that create a sense of belonging, encourage risk taking and provide opportunities for success help develop and maintain positive attitudes and self-confidence within students. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must learn to set achievable goals and assess themselves as they work toward these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting, assessing and revising personal goals.

## Goals For Students

*Mathematics education must prepare students to use mathematics confidently to solve problems.*

The main goals of mathematics education are to prepare students to:

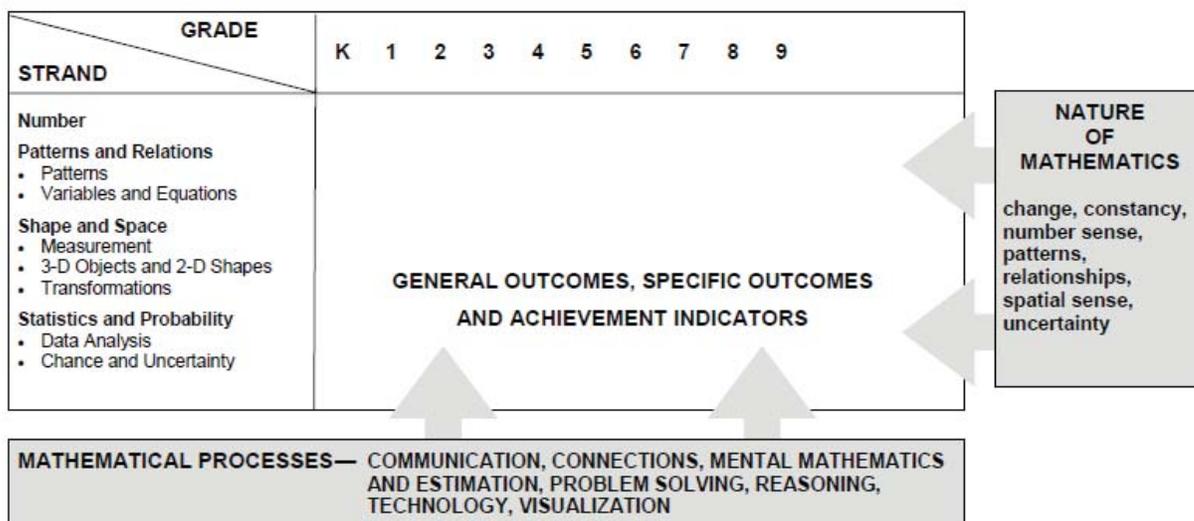
- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity.

# CONCEPTUAL FRAMEWORK FOR K-9 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.



## Mathematical Processes

- *Communication [C]*
- *Connections [CN]*
- *Mental Mathematics and Estimation [ME]*
- *Problem Solving [PS]*
- *Reasoning [R]*
- *Technology [T]*
- *Visualization [V]*

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and embrace lifelong learning in mathematics.

Students are expected to:

- communicate in order to learn and express their understanding
- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
- demonstrate fluency with mental mathematics and estimation
- develop and apply new mathematical knowledge through problem solving
- develop mathematical reasoning
- select and use technologies as tools for learning and for solving problems
- develop visualization skills to assist in processing information, making connections and solving problems.

This curriculum guide incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

**Communication [C]**

*Students must be able to communicate mathematical ideas in a variety of ways and contexts.*

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication helps students make connections among concrete, pictorial, symbolic, oral, written and mental representations of mathematical ideas.

**Connections [CN]**

*Through connections, students begin to view mathematics as useful and relevant.*

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. “Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding ... Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching” (Caine and Caine, 1991, p.5).

## Mental Mathematics and Estimation [ME]

*Mental mathematics and estimation are fundamental components of number sense.*

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

“Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math” (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics “... become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving” (Rubenstein, 2001, p. 442).

Mental mathematics “... provides the cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers” (Hope, 1988, p. v).

Estimation is used for determining approximate values or quantities or for determining the reasonableness of calculated values. It often uses benchmarks or referents. Students need to know when to estimate, how to estimate and what strategy to use.

Estimation assists individuals in making mathematical judgements and in developing useful, efficient strategies for dealing with situations in daily life.

## Problem Solving [PS]

*Learning through problem solving should be the focus of mathematics at all grade levels.*

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, “How would you know?” or “How could you ...?”, the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

A problem-solving activity requires students to determine a way to get from what is known to what is unknown. If students have already been given steps to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly seek and engage in a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk takers.

**Reasoning [R]**

*Mathematical reasoning helps students think logically and make sense of mathematics.*

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for students to develop their ability to reason. Students can explore and record results, analyze observations, make and test generalizations from patterns, and reach new conclusions by building upon what is already known or assumed to be true.

Reasoning skills allow students to use a logical process to analyze a problem, reach a conclusion and justify or defend that conclusion.

**Technology [T]**

*Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.*

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

Technology can be used to:

- explore and demonstrate mathematical relationships and patterns
- organize and display data
- extrapolate and interpolate
- assist with calculation procedures as part of solving problems
- decrease the time spent on computations when other mathematical learning is the focus
- reinforce the learning of basic facts
- develop personal procedures for mathematical operations
- create geometric patterns
- simulate situations
- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels.

## Visualization [V]

*Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.*

Visualization “involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world” (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and which estimation strategies to use (Shaw and Cliatt, 1989).

## Nature of Mathematics

- *Change*
- *Constancy*
- *Number Sense*
- *Patterns*
- *Relationships*
- *Spatial Sense*
- *Uncertainty*

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this curriculum guide. The components are change, constancy, number sense, patterns, relationships, spatial sense and uncertainty.

## Change

*Change is an integral part of mathematics and the learning of mathematics.*

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, ... can be described as:

- the number of a specific colour of beads in each row of a beaded design
- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain.

(Steen, 1990, p. 184).

## Constancy

*Constancy is described by the terms stability, conservation, equilibrium, steady state and symmetry.*

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS-Benchmarks, 1993, p.270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

- The ratio of the circumference of a teepee to its diameter is the same regardless of the length of the teepee poles.
- The sum of the interior angles of any triangle is  $180^\circ$ .
- The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

## Number Sense

*An intuition about number is the most important foundation of a numerate child.*

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p.146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Mastery of number facts is expected to be attained by students as they develop their number sense. This mastery allows for facility with more complex computations but should not be attained at the expense of an understanding of number.

Number sense develops when students connect numbers to their own real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. The evolving number sense typically comes as a by product of learning rather than through direct instruction. It can be developed by providing rich mathematical tasks that allow students to make connections to their own experiences and their previous learning.

## Patterns

*Mathematics is about recognizing, describing and working with numerical and non-numerical patterns.*

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands of mathematics.

Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with, and understanding of, their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps students develop algebraic thinking, which is foundational for working with more abstract mathematics.

## Relationships

*Mathematics is used to describe and explain relationships.*

Mathematics is one way to describe interconnectedness in a holistic worldview. Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves collecting and analyzing data and describing relationships visually, symbolically, orally or in written form.

## Spatial Sense

*Spatial sense offers a way to interpret and reflect on the physical environment.*

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes and to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of shapes and objects. Spatial sense allows students to make predictions about the results of changing these dimensions; e.g., doubling the length of the side of a square increases the area by a factor of four. Ultimately, spatial sense enables students to communicate about shapes and objects and to create their own representations.

**Uncertainty**

*Uncertainty is an inherent part of making predictions.*

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

**Essential Graduation Learnings**

Essential graduation learnings are statements describing the knowledge, skills and attitudes expected of all students who graduate from high school. Essential graduation learnings are cross-curricular in nature and comprise different areas of learning: *aesthetic expression, citizenship, communication, personal development, problem solving, technological competence and spiritual and moral development.*

**Aesthetic Expression**

*Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.*

**Citizenship**

*Graduates will be able to assess social, cultural, economic and environmental interdependence in a local and global context.*

**Communication**

*Graduates will be able to use the listening, viewing, speaking, reading and writing modes of language(s) and mathematical and scientific concepts and symbols to think, learn and communicate effectively.*

**Personal Development**

*Graduates will be able to continue to learn and to pursue an active, healthy lifestyle.*

**Problem Solving**

*Graduates will be able to use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts.*

**Technological Competence**

*Graduates will be able to use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.*

## Spiritual and Moral Development

*Graduates will be able to demonstrate an understanding and appreciation for the place of belief systems in shaping the development of moral values and ethical conduct.*

See *Foundations for the Atlantic Canada Mathematics Curriculum*, pages 4-6.

The mathematics curriculum is designed to make a significant contribution towards students' meeting each of the essential graduation learnings (EGLs), with the communication, problem solving and technological competence EGLs relating particularly well to the mathematical processes.

## Strands

- *Number*
- *Patterns and Relations*
- *Shape and Space*
- *Statistics and Probability*

The learning outcomes in the mathematics program are organized into four strands across the grades K–9. Some strands are further divided into substrands. There is one general outcome per substrand across the grades K–9.

The strands and substrands, including the general outcome for each, follow.

### Number

#### *Number*

- Develop number sense.

### Patterns and Relations

#### *Patterns*

- Use patterns to describe the world and to solve problems.

#### *Variables and Equations*

- Represent algebraic expressions in multiple ways.

### Shape and Space

#### *Measurement*

- Use direct and indirect measurement to solve problems.

#### *3-D Objects and 2-D Shapes*

- Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

#### *Transformations*

- Describe and analyze position and motion of objects and shapes.

### Statistics and Probability

#### *Data Analysis*

- Collect, display and analyze data to solve problems.

#### *Chance and Uncertainty*

- Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

## Outcomes and Achievement Indicators

### *General Outcomes*

The curriculum is stated in terms of general outcomes, specific outcomes and achievement indicators.

**General outcomes** are overarching statements about what students are expected to learn in each course.

### *Specific Outcomes*

**Specific outcomes** are statements that identify the specific skills, understanding and knowledge that students are required to attain by the end of a given course.

In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are not requirements that must be addressed to fully meet the learning outcome.

### *Achievement Indicators*

**Achievement indicators** are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome.

Specific curriculum outcomes represent the means by which students work toward accomplishing the general curriculum outcomes and ultimately, the essential graduation learnings.

## Summary

The conceptual framework for K-9 Mathematics (p.3) describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed. The components are not meant to stand alone. Activities that take place in the mathematics classroom should result from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between topics.

# ASSESSMENT AND EVALUATION

## Purposes of Assessment

What learning is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others about what is really valued.

*Assessment* techniques are used to gather information for evaluation. Information gathered through assessment helps teachers determine students' strengths and needs in their achievement of mathematics and guides future instructional approaches.

Teachers are encouraged to be flexible in assessing the learning success of all students and to seek diverse ways in which students might demonstrate what they know and are able to do.

*Evaluation* involves the weighing of the assessment information against a standard in order to make an evaluation or judgment about student achievement.

Assessment has three interrelated purposes:

- assessment *for* learning to guide and inform instruction;
- assessment *as* learning to involve students in self-assessment and setting goals for their own learning; and
- assessment *of* learning to make judgements about student performance in relation to curriculum outcomes.

## Assessment *for* Learning

Assessment *for* learning involves frequent, interactive assessments designed to make student understanding visible. This enables teachers to identify learning needs and adjust teaching accordingly. It is an ongoing process of teaching and learning.

Assessment *for* learning:

- requires the collection of data from a range of assessments as investigative tools to find out as much as possible about what students know
- provides descriptive, specific and instructive feedback to students and parents regarding the next stage of learning
- actively engages students in their own learning as they assess themselves and understand how to improve performance.

**Assessment *as* Learning**

Assessment *as* learning actively involves students' reflection on their learning and monitoring of their own progress. It focuses on the role of the student as the critical connector between assessment and learning, thereby developing and supporting metacognition in students.

Assessment *as* learning:

- supports students in critically analyzing their learning related to learning outcomes
- prompts students to consider how they can continue to improve their learning
- enables students to use information gathered to make adaptations to their learning processes and to develop new understandings.

**Assessment *of* Learning**

Assessment *of* learning involves strategies to confirm what students know, demonstrate whether or not they have met curriculum outcomes, or to certify proficiency and make decisions about students' future learning needs. Assessment *of* learning occurs at the end of a learning experience that contributes directly to reported results.

Traditionally, teachers relied on this type of assessment to make judgments about student performance by measuring learning after the fact and then reporting it to others. Used in conjunction with the other assessment processes previously outlined, however, assessment *of* learning is strengthened.

Assessment *of* learning:

- provides opportunities to report evidence to date of student achievement in relation to learning outcomes, to parents/guardians and other stakeholders
- confirms what students know and can do
- occurs at the end of a learning experience using a variety of tools.

Because the consequences of assessment *of* learning are often far-reaching, teachers have the responsibility of reporting student learning accurately and fairly, based on evidence obtained from a variety of contexts and applications.

## **Assessment Strategies**

Assessment techniques should match the style of learning and instruction employed. Several options are suggested in this curriculum guide from which teachers may choose, depending on the curriculum outcomes, the class and school/district policies.

### **Observation (formal or informal)**

This technique provides a way of gathering information fairly quickly while a lesson is in progress. When used formally, the student(s) would be aware of the observation and the criteria being assessed. Informally, it could be a frequent, but brief, check on a given criterion. Observation may offer information about the participation level of a student for a given task, use of a concrete model or application of a given process. The results may be recorded in the form of checklists, rating scales or brief written notes. It is important to plan in order that specific criteria are identified, suitable recording forms are ready, and all students are observed within a reasonable period of time.

### **Performance**

This curriculum encourages learning through active participation. Many of the curriculum outcomes promote skills and their applications. In order for students to appreciate the importance of skill development, it is important that assessment provide feedback on the various skills. These may be the correct manner in which to use a manipulative, the ability to interpret and follow instructions, or to research, organize and present information. Assessing performance is most often achieved through observing the process.

### **Paper and Pencil**

These techniques can be formative or summative. Whether as part of learning, or a final statement, students should know the expectations for the exercise and how it will be assessed. Written assignments and tests can be used to assess knowledge, understanding and application of concepts. They are less successful at assessing processes and attitudes. The purpose of the assessment should determine what form of paper and pencil exercise is used.

### **Journal**

Journals provide an opportunity for students to express thoughts and ideas in a reflective way. By recording feelings, perceptions of success, and responses to new concepts, a student may be helped to identify his or her most effective learning style. Knowing how to learn in an effective way is powerful information. Journal entries also give indicators of developing attitudes to mathematical concepts, processes and skills, and how these may be applied in the context of society. Self-assessment, through a journal, permits a student to consider strengths and weaknesses, attitudes, interests and new ideas. Developing patterns may help in career decisions and choices of further study.

**Interview**

This curriculum promotes understanding and applying mathematics concepts. Interviewing a student allows the teacher to confirm that learning has taken place beyond simple factual recall. Discussion allows a student to display an ability to use information and clarify understanding. Interviews may be a brief discussion between teacher and student or they may be more extensive. Such conferences allow students to be proactive in displaying understanding. It is helpful for students to know which criteria will be used to assess formal interviews. This assessment technique provides an opportunity to students whose verbal presentation skills are stronger than their written skills.

**Presentation**

The curriculum includes outcomes that require students to analyze and interpret information, to be able to work in teams, and to communicate information. These activities are best displayed and assessed through presentations. These can be given orally, in written/pictorial form, by project summary, or by using electronic systems such as video or computer software. Whatever the level of complexity, or format used, it is important to consider the curriculum outcomes as a guide to assessing the presentation. The outcomes indicate the process, concepts and context for which a presentation is made.

**Portfolio**

Portfolios offer another option for assessing student progress in meeting curriculum outcomes over a more extended period of time. This form of assessment allows the student to be central to the process. There are decisions about the portfolio, and its contents, which can be made by the student. What is placed in the portfolio, the criteria for selection, how the portfolio is used, how and where it is stored, and how it is evaluated are some of the questions to consider when planning to collect and display student work in this way. The portfolio should provide a long-term record of growth in learning and skills. This record of growth is important for individual reflection and self-assessment, but it is also important to share with others. For all students, it is exciting to review a portfolio and see the record of development over time.

## INSTRUCTIONAL FOCUS

### Planning for Instruction

Consider the following when planning for instruction:

- Integration of the mathematical processes within each topic is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency and must be integrated throughout the program.
- There should be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using manipulatives and be developed concretely, pictorially and symbolically.
- Students bring a diversity of learning styles and cultural backgrounds to the classroom. They will be at varying developmental stages.

### Teaching Sequence

The curriculum guide for Grade 9 Mathematics is organized by units. This is only a suggested teaching order for the course. There are a number of combinations of sequences that would be appropriate.

Each two page spread lists the topic, general outcome, and specific outcome.

### Instruction Time Per Unit

The suggested number of weeks of instruction per unit is listed in the guide at the beginning of each unit. The number of suggested weeks includes time for completing assessment activities, reviewing and evaluating. The timelines at the beginning of each unit are provided to assist in planning. The use of these timelines is not mandatory. However, it is mandatory that all outcomes are taught during the school year, so a long term plan is advised. Teaching of the outcomes is ongoing, and may be revisited as necessary.

### Resources

The authorized resource for Newfoundland and Labrador for students and teachers is *Math Makes Sense 9* (Pearson). Column four of the curriculum guide references *Math Makes Sense 9* for this reason. Teachers may use any other resource, or combination of resources, to meet the required specific outcomes.

## GENERAL AND SPECIFIC OUTCOMES

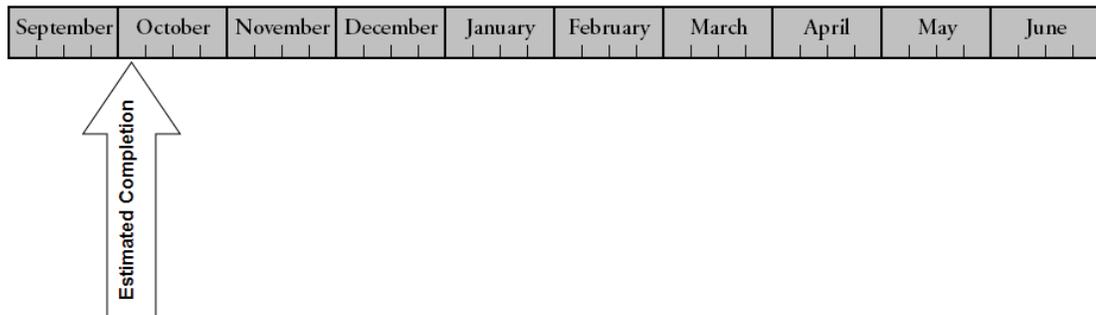
### GENERAL AND SPECIFIC OUTCOMES WITH ACHIEVEMENT INDICATORS (pages 19-162)

This section presents general and specific outcomes with corresponding achievement indicators and is organized by unit. The list of indicators contained in this section is not intended to be exhaustive but rather to provide teachers with examples of evidence of understanding that may be used to determine whether or not students have achieved a given specific outcome. Teachers may use any number of these indicators or choose to use other indicators as evidence that the desired learning has been achieved. Achievement indicators should also help teachers form a clear picture of the intent and scope of each specific outcome.

Grade 9 Mathematics is organized into nine units: *Square Roots and Surface Area, Powers and Exponent Laws, Rational Numbers, Linear Relations, Polynomials, Linear Equations and Inequalities, Similarity and Transformations, Circle Geometry, and Probability and Statistics.*

# Square Roots and Surface Area

Suggested Time: 3 Weeks



# Unit Overview

## Focus and Context

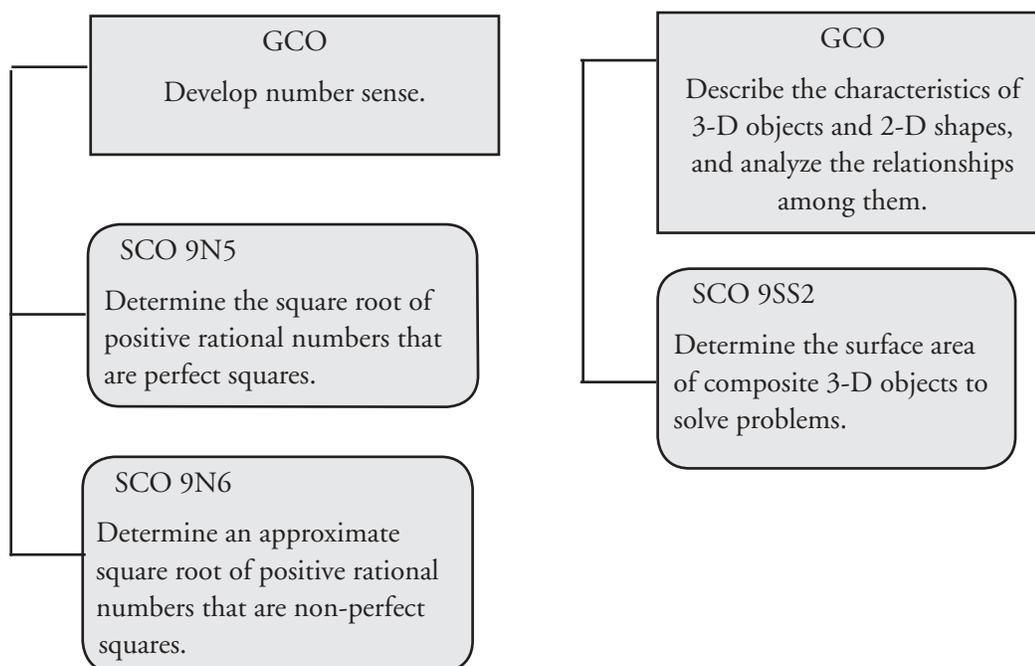
In this unit, students will build on their knowledge of squares and square roots by exploring these concepts as they pertain to positive rational numbers in the form of fractions and decimals. Calculators and the properties of square numbers will be used to determine the square roots.

Students will also use estimation, number lines, and benchmarks to determine an approximate square root of positive fractions and decimals that are non-perfect squares. Calculators will be utilized to give a closer approximation and to emphasize the difference between exact and approximate values of square root.

Students will determine the surface area of composite 3-D objects made from right cylinders, right rectangular prisms and right triangular prisms while taking into consideration the effects of overlap areas. The focus is on the development of formulae in a way that decreases a dependence on memorization and makes the process more beneficial for the student.

Developing a good understanding of squares and square roots and surface area of 3-D objects will aid in making a connection with many real-life situations. The exponential growth of bacteria, cell division, cost efficiency and minimal wastage of material, are examples of such processes.

## Outcomes Framework



## SCO Continuum

Mathematics 8	Mathematics 9	Mathematics Level I	
		1201	1202
<b>Number</b>			
<p>8N1. Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V]</p> <p>8N2. Determine the approximate square root of numbers that are not perfect squares (limited to whole numbers). [C, CN, ME, R, T]</p> <p>8N6. Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]</p>	<p>9N5. Determine the square root of positive rational numbers that are perfect squares. [C, CN, PS, R, T]</p> <p>9N6. Determine an approximate square root of positive rational numbers that are non-perfect squares. [C, CN, PS, R, T]</p>	<p>AN1. Demonstrate an understanding of factors of whole numbers by determining the:</p> <ul style="list-style-type: none"> <li>• prime factors</li> <li>• greatest common factor</li> <li>• least common factor</li> <li>• square root</li> <li>• cube root.</li> </ul> <p>[CN, ME, R]</p>	<p>not addressed</p>
<b>Shape and Space</b>			
<p>8SS1. Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, V, T]</p> <p>8SS3. Determine the surface area of:</p> <ul style="list-style-type: none"> <li>• right rectangular prisms</li> <li>• right triangular prisms</li> <li>• right cylinders to solve problems.</li> </ul> <p>[C, CN, PS, R, V]</p>	<p>9SS2. Determine the surface area of composite 3-D objects to solve problems. [C, CN, PS, R, V]</p>	<p>M3. Solve problems using SI and imperial units, that involve the surface area and volume of 3-D objects including:</p> <ul style="list-style-type: none"> <li>• right cones</li> <li>• right cylinders</li> <li>• right prisms</li> <li>• right pyramids</li> <li>• spheres.</li> </ul> <p>[CN, PS, R, V]</p>	<p>M4. Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimals and fractional measurements, and verify the solutions. [C, CN, ME, PS]</p>

### Mathematical Processes

[C] Communication  
 [CN] Connections  
 [ME] Mental Mathematics and Estimation

[PS] Problem Solving  
 [R] Reasoning  
 [T] Technology  
 [V] Visualization

**Strand: Number****Outcomes**

*Students will be expected to*

**9N5 Determine the square root of positive rational numbers that are perfect squares.**

[C, CN, PS, R, T]

**Achievement Indicator:**

9N5.1 *Determine whether or not a given rational number is a square number and explain the reasoning.*

**Elaborations—Strategies for Learning and Teaching**

In this unit, students will determine the square root of positive perfect squares, including fractions and decimals. In Grade 8, they were exposed to perfect squares and square roots, limited to whole numbers (8N1). This will now be extended to include all rational numbers.

Students have used materials and strategies, such as square shapes and grid paper, to determine whether or not a given whole number is a perfect square. Similarly, a fraction or a decimal is a perfect square if it can be represented as the area of a square.

To determine if a fraction is a perfect square students should determine if the numerator and denominator are perfect squares. They should notice that  $\frac{4}{81}$  is a perfect square, since both the numerator and denominator are perfect squares. If the numerator and denominator are not perfect squares, it may be possible for students to write an equivalent fraction so that the numerator and denominator are perfect squares. The rational number  $\frac{8}{50}$  is a perfect square, for example, since it is equivalent to both  $\frac{4}{25}$  and  $\frac{16}{100}$ .

Provide students with examples that include decimals. A decimal is a perfect square if its fractional equivalent is a perfect square. In Grade 7, students expressed both terminating and repeating decimals as fractions (7N4). To determine if a rational number such as 1.44 is a perfect square, ask students to examine its fractional equivalent,  $\frac{144}{100}$ . Similar to the example above, they should conclude that both 144 and 100 are perfect squares and consequently  $\frac{144}{100}$  is a perfect square as well.

Using the numerator and denominator may be students' preferred method for determining if a rational number is a perfect square. However, other methods do exist. Ask students if 144 is a perfect square. They should confirm that 12 is the square root of 144 making 144 a perfect square. Since  $12 \times 12 = 144$ , students should realize that  $1.2 \times 1.2 = 1.44$ , and conclude that 1.44 is also a perfect square.

Some students may try to generalize the numerator and denominator approach to include mixed numbers. Caution them that  $16\frac{4}{9}$  is not necessarily a perfect square simply because 16, 4 and 9 are all perfect squares. Converting  $16\frac{4}{9}$  to the improper fraction  $\frac{148}{9}$  shows that it is not a perfect square.

## General Outcome: Develop Number Sense.

### Suggested Assessment Strategies

#### *Journal*

- Ask students to respond to the following:  
Explain the term perfect square. Give an example of a whole number, a fraction, and a decimal that are perfect squares. Using diagrams, show why they are perfect squares.  
(9N5.1)

#### *Paper and Pencil*

- Ask students to identify which of the following are perfect squares: 0.49, 4.9, and 0.0049. They should explain their reasoning.  
(9N5.1)
- Mary creates stained glass mosaics for a hobby. She wants to make one with an area of  $3.24 \text{ cm}^2$ . Ask students to determine if she can make it in the shape of a perfect square.  
(9N5.1)
- Ask students to identify the perfect squares from the given list:  
 $\frac{8}{50}, \frac{39}{25}, 2.89, 4\frac{9}{25}, \frac{18}{32}$   
(9N5.1)

### Resources/Notes

#### Authorized Resource

#### *Math Makes Sense 9*

Prep Talk Video: Square Roots and Surface Area

#### Lesson 1.1: Square Roots of Perfect Squares

ProGuide: pp.4-11

CD-ROM: Master 1.16

Student Book (SB): pp. 6-13

Preparation and Practice Book (PB): pp. 2-9

#### Note:

*Math Makes Sense 9* does not address or provide examples for determining if a mixed number is a perfect square.

**Strand: Number****Outcomes**

*Students will be expected to*

9N5 Continued...

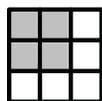
**Achievement Indicator:**

9N5.2 Determine the square root of a given positive rational number that is a perfect square.

**Elaborations—Strategies for Learning and Teaching**

In Grade 8, perfect square numbers were connected to the area of squares. When determining the square root of positive rational numbers, students should once again be encouraged to view the area as the perfect square, and either dimension of the square as the square root.

To determine  $\sqrt{\frac{4}{9}}$ , for example, ask students how they could calculate the side length of a square if the area is  $\frac{4}{9}$  square units. Ask students to create a 3 by 3 square, resulting in an area of 9 units<sup>2</sup>, and to shade 4 of those 9 regions.



They should observe that the side of the square has a length of 3 units and two of those units are shaded. The square with an area of  $\frac{4}{9}$  units<sup>2</sup> has a side length of  $\frac{2}{3}$  units. This should lead students to conclude that  $\sqrt{\frac{4}{9}} = \frac{2}{3}$ . Ask them to verify their answer by checking that  $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$ .

This method can also be used to determine the square root of a decimal. To determine the value of  $\sqrt{0.64}$ , for example, students must first convert 0.64 to the fraction  $\frac{64}{100}$ . Students should be thinking about a 100-square with 64 blocks shaded. Using the square as a visual, students should determine that  $\sqrt{0.64} = \frac{8}{10}$  or 0.8.

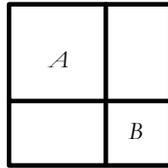
This pictorial method is an effective way to introduce square roots of rational numbers, and can be useful when finding square roots of relatively small numbers. It becomes inefficient, however, as the numbers get larger. Students should recognize a pattern emerging as they use the square models to determine square roots. The square root of a rational number, or a quotient, equals the quotient of the square roots of the numerator and denominator. That is,  $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ . To determine the value of  $\sqrt{9.61}$ , for example, students convert the decimal to a fraction resulting in  $\sqrt{\frac{961}{100}} = \frac{\sqrt{961}}{\sqrt{100}} = \frac{31}{10} = 3.1$ .

As students work through several examples of finding the square root of a positive rational number that is a perfect square, they should notice the value results in a terminating decimal.

In Grade 8, students used the Pythagorean theorem to find missing side measures of right triangles (8SS1). They should continue to use the Pythagorean theorem in problem solving situations.

**General Outcome: Develop Number Sense.****Suggested Assessment Strategies***Paper and Pencil*

- A square auditorium is divided into 4 sections. Sections A and B are also squares. Section A has an area of  $16 \text{ m}^2$  and Section B has an area of  $9 \text{ m}^2$ . Ask students to determine the combined area of the remaining space in the auditorium.



(9N5.2)

- A toy maker has a miniature picture frame that is 1.0 cm by 1.5 cm. She also has a square photo with an area of  $2.25 \text{ cm}^2$ . Ask students to answer the following questions:

- What are the dimensions of the photo?
- How much, if any, will she need to trim from the photo so that it will fit in the frame?

(9N5.2)

- Ask students to determine the value of each square root in simplest form:

$$(i) \sqrt{\frac{9}{36}} \quad (ii) \sqrt{\frac{25}{100}} \quad (iii) \sqrt{\frac{196}{49}}$$

(9N5.2)

- Jim lives in the downtown area of a city where the houses are very close together. He wants to paint a window sill on the second floor. The window sill is 3.5 m above the ground. The only ladder available is 5 m long. The space between the houses is only 2 m, and the window is on the side of the house. Ask students:

- if he places a ladder at the height of the window sill, how far away from the house will the base of the ladder need to be?
- if he places the ladder as far away from the house as the house next door will allow, how far up the side of the house will the ladder reach?
- to discuss whether the length of this ladder makes it suitable for painting the window.

(9N5.2)

**Resources/Notes****Authorized Resource***Math Makes Sense 9***Lesson 1.1: Square Roots of Perfect Squares**

ProGuide: pp.4-11

CD-ROM: Master 1.16

SB: pp. 6-13

PB: pp. 2-9

**Strand: Number****Outcomes**

*Students will be expected to*

9N5 Continued...

**Achievement Indicators:**

9N5.2 *Continued*

9N5.3 *Identify the error made in a given calculation of a square root, e.g., Is 3.2 the square root of 6.4?*

9N5.4 *Determine a positive rational number given the square root of that positive rational number.*

**Elaborations—Strategies for Learning and Teaching**

From their work with whole numbers, students are aware that square roots are less than the number. If a rational number is between 0 and 1, however, the square root will be greater than the number.

A class discussion on both positive and negative square roots may be warranted here. Until now, most students would have assumed that the square root of a number was positive. Students should realize that  $(-3)^2$  and  $(+3)^2$  both equal 9. Mathematicians use the radical sign  $\sqrt{\quad}$  to represent the principal, or positive, square root. When the question is written as  $\sqrt{9}$ , the answer is 3. Students should understand that in problem solving situations the answer is almost always the positive square root of a number because it is the only value that makes sense in most contexts. It is useful, however, for students to recognize that both the positive and negative values exist. Although this is not a focus of attention at this level, it will be important for solving equations in higher grade levels.

Engaging students in error analysis heightens awareness of common student errors. One of the perfect squares students frequently work with is 4. Students may incorrectly compute square roots by dividing the given number by 2 rather than finding its square root. The misconception probably arises from the fact that half of 4 and the square root of 4 both have the same value.

Students may also struggle with the correct placement of decimals when finding square roots of rational numbers. Reminding them that if a rational number is between 0 and 1, the square root will be greater than the number itself should help with this. Ask students to evaluate the following square roots and to observe the pattern of the place value.

$$\sqrt{8100} = 90$$

$$\sqrt{81} = 9$$

$$\sqrt{0.81} = 0.9$$

$$\sqrt{0.0081} = 0.09$$

In Grade 8, students determined the square of a given whole number (8N1). This will now be extended to include the squares of fractions and decimals. This provides an opportunity to revisit multiplying fractions (8N6), as well as multiplying decimals (7N2).

**General Outcome: Develop Number Sense.****Suggested Assessment Strategies***Interview*

- Ask students to verify the answers, correct any errors they discover, and explain where the mistakes were made.

(i)  $\sqrt{6.4} = 3.2$                       (iii)  $\sqrt{0.04} = 0.02$

(ii)  $\sqrt{4.9} = 0.7$                       (iv)  $\sqrt{1.44} = 12$

(9N5.3)

*Paper and Pencil*

- Katelyn calculated that the square root of 3.6 was 1.8. Nick determined that the answer was 0.6 and Renee estimated the answer to be 1.9. Ask students to decide who is correct, and explain the mistakes made in the two incorrect solutions.

(9N5.3)

- Ask students to find the squares of rational numbers such as:

(i)  $\frac{5}{17}$                       (ii) 1.21                      (iii) 0.5

(9N5.4)

- A square measures 5.7 cm on each side. Ask students to answer the following questions:

- What is the area of the square?
- What is the length of a diagonal of the square?
- What is the area of a circle that fits perfectly inside the square?

(9N5.4)

*Performance*

- For the activity *Commit and Toss* provide students with a selected response question, as shown below. Students write their answer, crumble their solutions into a ball, and toss the papers into a basket. Once all papers are in the basket, ask students to take one out. They then move to the corner of the room designated to match the selected response on the paper they have taken. In their respective corners, they should discuss the similarities and differences in the explanations provided and report back to the class.

What is the square root of 0.64?

(A) 0.032

(B) 0.08

(C) 0.32

(D) 0.8

Explain your reasoning.

(9N5.2, 9N5.3)

**Resources/Notes****Authorized Resource***Math Makes Sense 9***Lesson 1.1: Square Roots of Perfect Squares**

ProGuide: pp.4-11

Master 1.6

CD-ROM: Master 1.16

SB: pp. 6-13

PB: pp. 2-9

**Strand: Number**

**Outcomes**

*Students will be expected to*

**9N6 Determine an approximate square root of positive rational numbers that are non-perfect squares.**

[C, CN, PS, R, T]

**Achievement Indicators:**

9N6.1 *Estimate the square root of a given rational number that is not a perfect square using the roots of perfect squares as benchmarks.*

9N6.2 *Determine an approximate square root of a given rational number that is not a perfect square using technology, e.g. calculator, computer.*

9N6.3 *Explain why the square root of a given rational number as shown on a calculator may be an approximation.*

**Elaborations – Strategies for Learning and Teaching**

In Grade 8, students approximated square roots of numbers that were not perfect squares, limited to whole numbers (8N2).

Estimation is a useful skill that allows students to assess whether answers from a calculator are reasonable. Strategies used in Grade 8 to estimate square roots of non-perfect square whole numbers can be adapted to estimate square roots of non-perfect square rational numbers. Just as with whole numbers, students can use perfect squares as benchmarks to estimate a square root of fractions or decimals that are not perfect squares. Encourage the use of number lines to help students visualize where the numbers lie in relation to each other.

As students estimate the square root of  $\frac{14}{22}$ , for example, they should identify the perfect squares closest to 14 and 22 as 16 and 25 respectively. Students should notice  $\frac{14}{22} \div \frac{16}{25}$  and since  $\sqrt{\frac{16}{25}} = \frac{4}{5}$ , the square root of  $\frac{14}{22}$  is approximately  $\frac{4}{5}$ .

Students may have more difficulty determining the perfect square benchmarks when working with decimals. To estimate the square root of 1.30, the closest perfect square rational numbers on either side are 1.21 and 1.44. Students can then visualize where these numbers lie with respect to each other on a number line.



Since the square root of 1.21 is 1.1 and the square root of 1.44 is 1.2, they should conclude that the square root of 1.30 is between 1.1 and 1.2. Examining the location of 1.30 relative to the other two values should lead students to decide that its square root is closer to 1.1 than 1.2 and a reasonable estimate would be 1.14.

Once students are comfortable estimating without the aid of technology, a calculator can be used to approximate square roots. A calculator provides an efficient means of approximating square roots and it usually gives a closer approximation than an estimate does. As students use their calculator to determine their estimate, they should notice the square root of a non-perfect square results in a decimal that is non-repeating and non-terminating. This decimal value is approximate, not exact. This is a good place to discuss how accurate an approximation should be. Often more decimal places results in a better approximation of a square root. However, one or two decimal places is acceptable.

## General Outcome: Develop Number Sense.

### Suggested Assessment Strategies

#### *Paper and Pencil*

- Ask students to estimate the side length, to the nearest tenth, of a square with an area of  $31.5 \text{ cm}^2$ . Ask them to provide an explanation. (9N6.1)
  
- A metal worker needs to fabricate a square metal plate that has an area of  $17\frac{7}{8} \text{ cm}^2$ . Ask students to determine the approximate dimensions of the metal plate. (9N6.1)
  
- Ask students to complete the following activity:
  - (i) Use a calculator to determine the value of  $\sqrt{5}$ . Record all the digits displayed on the calculator.
  - (ii) Clear your calculator and enter the number recorded in step (i).
  - (iii) Square this number and record the result.
  - (iv) Compare the answer in step (iii) to the number 5. Are they the same? Identify any differences and explain why they exist. (9N6.2, 9N6.3)

#### *Journal*

- Ask students to respond to the following:  
How could you use estimation to determine that 0.7 and 0.007 are not reasonable values for  $\sqrt{4.9}$ ? (9N6.1)
  
- Ask students to explain why an answer displayed on a calculator may not be an exact answer. (9N6.3)

#### *Interview*

- A ladder leans against a building and exactly reaches the roof. The base of the ladder is 1.5 m away from the base of the building, and the length of the ladder is 5 m. Using the measurements given, Brigitte found the height of the building to be 5.2 m. Ask students if this answer is reasonable. Ask them to justify their decisions without formal calculations. (9N6.1)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 9*

#### Lesson 1.2: Square Roots of Non-Perfect Squares

ProGuide: pp. 12-18

Master 1.7

CD-ROM: Master 1.17

SB: pp. 14-20

PB: pp. 10-21

**Strand: Number**

**Outcomes**

*Students will be expected to*  
 9N6 Continued...

**Achievement Indicator:**

9N6.4 *Identify a number with a square root that is between two given numbers.*

**Elaborations – Strategies for Learning and Teaching**

In Grade 8, students were expected to identify a whole number whose square root lies between two given numbers (8N2). They could have been asked “*What whole number has a square root between 7 and 8?*” They should have determined that any whole number between 49 and 64 has a square root between 7 and 8. The realization that there is more than a single correct answer is important. Now the possibilities are extended to include rational numbers, and there are an infinite number of rationals between 49 and 64.

An example such as “*What number has a square root between 2.3 and 2.31?*” can emphasize the infinite number of possibilities. Ask students to complete the following table to help them visualize that any rational number between 5.29 and 5.3361 has a square root between 2.3 and 2.31.

Possible Square Roots	Possible Squares	Possible Numbers
<b>2.30</b>	$(2.30)^2$	5.29
2.302	$(2.302)^2$	5.299204
2.304	$(2.304)^2$	5.308416
2.306	$(2.306)^2$	5.317636
2.308	$(2.308)^2$	5.326864
<b>2.310</b>	$(2.310)^2$	5.3361

Guide students through the algebraic process of identifying a rational number whose square root lies between two given numbers.

$$2.3 < \sqrt{n} < 2.31$$

$$(2.3)^2 < n < (2.31)^2$$

$$5.29 < n < 5.3361$$

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**General Outcome: Develop Number Sense.**

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**Suggested Assessment Strategies***Paper and Pencil*

- Ask students to make a list of as many numbers with a square root between 18.1 and 18.2 as they can think of in two minutes. After the time is up, ask them to pass their list to another student. Then ask them, one at a time, to read an entry from the list in front of them. Everybody who has that number on their list will cross it off. At the end, the list with the most remaining entries is the winner. Students could be paired up for this activity.

(9N6.3, 9N6.4)

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**Resources/Notes****Authorized Resource***Math Makes Sense 9***Lesson 1.2: Square Roots of Non-Perfect Squares**

ProGuide: pp. 12-18

Master 1.7

CD-ROM: Master 1.17

SB: pp. 14-20

PB: pp. 10-21

**Strand: Shape and Space (3-D Objects and 2-D Shapes)**

**Outcomes**

*Students will be expected to*

**9SS2 Determine the surface area of composite 3-D objects to solve problems.**

[C, CN, PS, R, V]

**Achievement Indicators:**

9SS2.1 *Determine the overlap in a given concrete composite 3-D object, and explain its effect on determining the surface area (limited to right cylinders, right rectangular prisms and right triangular prisms).*

9SS2.2 *Determine the surface area of a given concrete composite 3-D object (limited to right cylinders, right rectangular prisms and right triangular prisms).*

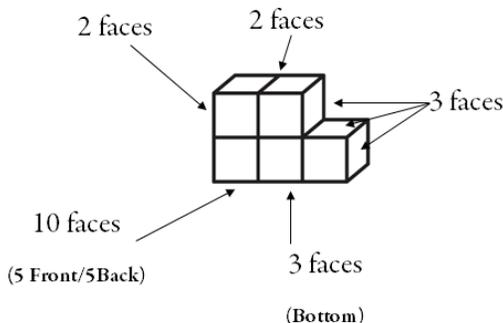
9SS2.3 *Solve a given problem involving surface area.*

**Elaborations – Strategies for Learning and Teaching**

In Grade 8, students determined the surface area of right rectangular prisms, right triangular prisms and right cylinders (8SS3). The focus was on gaining a conceptual understanding of surface area through the use of nets rather than on the use of formulae. This will now be extended to include composite 3-D objects.

Students should explore how the surface area changes when right cylinders, right rectangular prisms and right triangular prisms are arranged collectively, causing some of the faces to be covered as a result of contact with other objects. When objects are combined, there will be an area of overlap.

Consider the following composite object.



Many faces on these 5 cubes are no longer visible on the surface because of contact with an adjacent cube. The composite figure has 20 faces on the surface. When viewed separately, the 5 cubes have a total of 30 faces. The composite figure has 10 fewer faces because there are five areas of overlap and each removes two faces from the surface. As a result, this object has a surface area that is smaller than that of 5 individual cubes. Students should consider how such a shape is made from its component parts, determine the surface area of each part, and remove the area of overlapping surfaces. Alternatively, they could determine the area of each exposed surface and add to find the total area. A review of area formulae for rectangles, triangles and circles, as well as the formula for circumference, may be necessary.

There are many ways to decompose a composite object. The way in which the object is decomposed may affect its area of overlap, but not its surface area. When decomposing a composite object, encourage students to look for component parts such as triangular prisms, rectangular prisms and cylinders.

**General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.**

### Suggested Assessment Strategies

#### *Performance*

- Distribute sets of geometric solids to groups of students. Ask students to create composite shapes from the solids. Ask them to determine the surface area of the composite figure and explain how they found the surface area.

An extension to this may involve giving students the amount of wrapping paper equal to their calculated surface area. Ask students to wrap their object with the paper to see if their calculation was accurate.

*Note: If sets of geometric solids are not available some advance planning might be necessary. Teachers could ask students to bring in boxes, cans, paper towel rolls, etc., from home for this activity.*

(9SS2.1, 9SS2.2, 9SS2.3)

#### *Paper and Pencil*

- Ask students to calculate the total surface area of a composite figure made up of 42 centimeter cubes with 12 overlaps.

(9SS2.2, 9SS2.3)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 9*

**Lesson 1.3: Surface Area of Objects Made from Right Rectangular Prisms**

**Lesson 1.4: Surface Area of Other Composite Objects**

ProGuide: pp. 23-30, 31-41

Master 1.8, 1.9

CD-ROM: Master 1.18, 1.19

SB: pp. 25-32, 33-43

PB: pp. 22-32, 33-43

Try It Virtual Manipulatives

*This program allows you to build models with virtual linking cubes.*

**Strand: Shape and Space (3-D Objects and 2-D Shapes)****Outcomes**

*Students will be expected to*

9SS2 Continued...

**Achievement Indicators:**

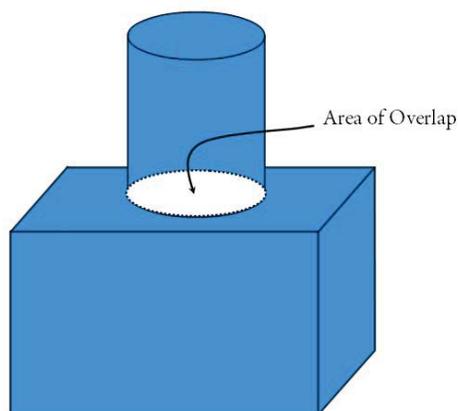
9SS2.1, 9SS2.2, 9SS2.3

*Continued*

**Elaborations—Strategies for Learning and Teaching**

In general, when determining the surface area of composite geometric figures, all sides, except overlap, are included. In contextual situations, however, sides other than the overlap may also need to be excluded.

The composite figure below is composed of a right cylinder and a right rectangular prism.



Ask students to identify the area of overlap. They should conclude the area of overlap is circular. Students can determine the surface area of the composite figure by calculating:

$$\text{Surface Area}_{\text{prism}} + \text{Surface Area}_{\text{cylinder}} - 2\text{Area}_{\text{circle}}$$

Ask students to now consider this object as the base for a patio post that sits on the ground. Ask them if they would paint the bottom of the rectangular prism. They should reason that when painting this object the surface touching the ground would not be painted and therefore would not be included in the surface area calculation.

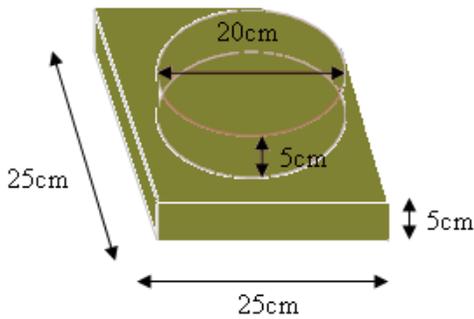
Discuss with students other examples where it is important to keep the context in mind. To determine how much paint is needed to paint a flat-bottom dresser, for example, the area of the bottom would be omitted because it would not be painted. Similarly, when icing a cake, the bottom of the cake would not be covered with icing.

**General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.**

**Suggested Assessment Strategies**

*Journal*

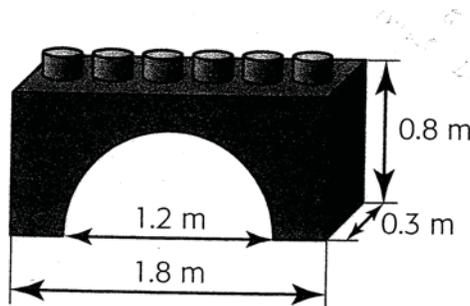
- Todd makes a two-layer cake. He puts strawberry jam between the layers instead of icing. He plans to cover the outside of the cake with icing. Ask students to describe how he can calculate the area that needs icing.



(9SS2.1, 9SS2.2, 9SS2.3)

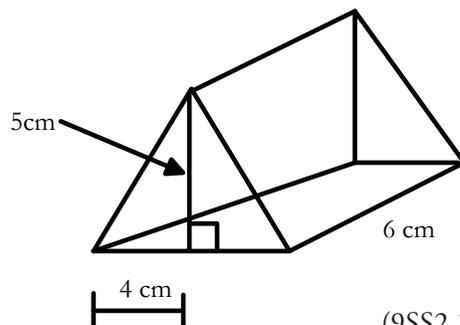
*Paper and Pencil*

- A set for a school theatre production uses giant linking cubes. Each cylindrical connector is 0.20 m in diameter and 0.15 m high. Ask students to calculate the total area that must be painted.



(9SS2.1, 9SS2.2, 9SS2.3)

- Ask students to calculate the surface area of the composite figure:



(9SS2.1, 9SS2.3, 9SS2.3)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 9*

**Lesson 1.3: Surface Area of Objects Made from Right Rectangular Prisms**

**Lesson 1.4: Surface Area of Other Composite Objects**

ProGuide: pp. 23-30, 31-41

Master 1.8, 1.9

CD-ROM: Master 1.18, 1.19

SB: pp. 25-32, 33-43

PB: pp. 22-32, 33-43

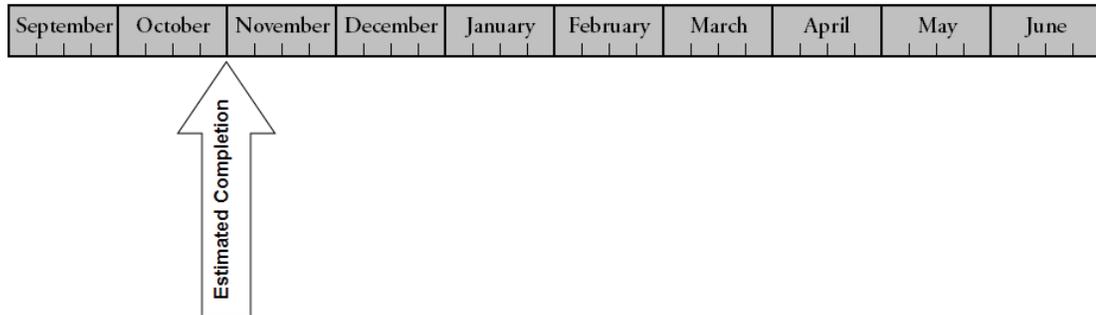
**Try It Virtual Manipulatives**

*This program allows you to build models with virtual linking cubes.*



# Powers and Exponent Laws

Suggested Time: 3 Weeks



# Unit Overview

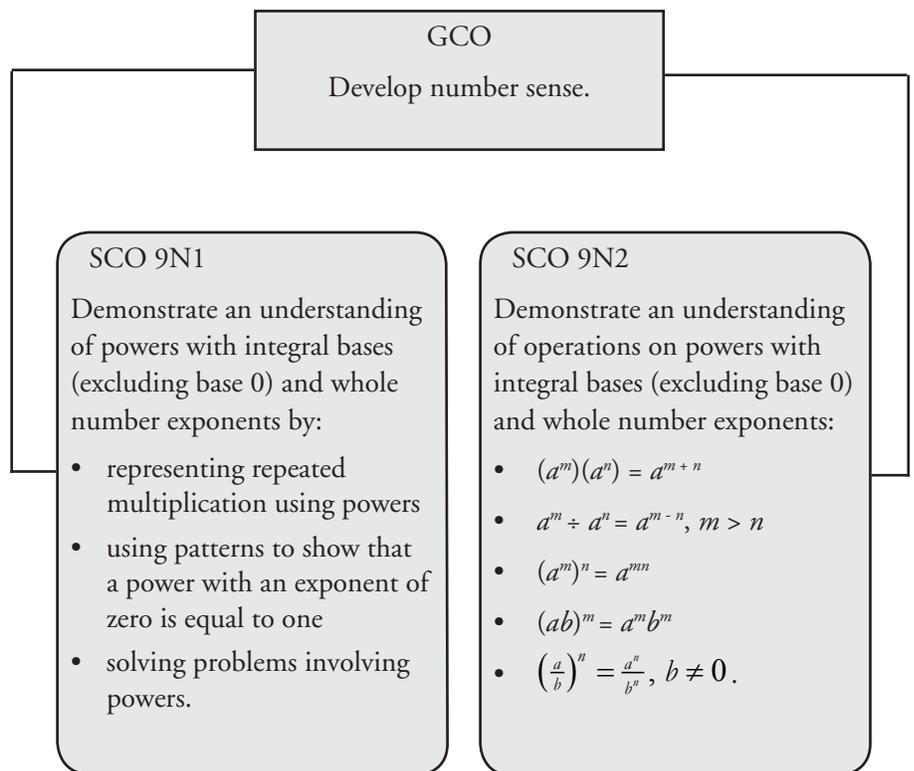
## Focus and Context

In this unit, students will demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by representing repeated multiplication using powers, using patterns to show that a power with an exponent of zero is equal to one, and solving problems involving powers. Students will be asked to evaluate exponential expressions to determine their value in standard form.

Evolving from these concepts, the laws of exponents will be developed. Students will be expected to discover the rules pertaining to the use of exponents as opposed to being given them to learn. They will also demonstrate an understanding of the order of operations on powers with integral bases (excluding base 0) and whole number exponents. Calculators can be used by students as they develop these skills, but an efficient use of the laws of exponents will be encouraged.

Exponents are an important part of our world. They can be used to make multiplication easier or to write large and small numbers in a more compact way. The strength of an earthquake, the pH of a solution, or the size of a number can all be related using exponents.

## Outcomes Framework



## SCO Continuum

Mathematics 8	Mathematics 9	Mathematics Level I	
		1201	1202
Number			
<p>8N1. Demonstrate an understanding of perfect squares and square roots, concretely, pictorially and symbolically (limited to whole numbers). [C, CN, R, V]</p> <p>8SS1. Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, V, T]</p>	<p>9N1. Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:</p> <ul style="list-style-type: none"> <li>representing repeated multiplication using powers</li> <li>using patterns to show that a power with an exponent of zero is equal to one</li> <li>solving problems involving powers.</li> </ul> <p>[C, CN, PS, R]</p> <p>9N2. Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents:</p> <ul style="list-style-type: none"> <li><math>(a^m)(a^n) = a^{m+n}</math></li> <li><math>a^m \div a^n = a^{m-n}, m &gt; n</math></li> <li><math>(a^m)^n = a^{mn}</math></li> <li><math>(ab)^m = a^m b^m</math></li> <li><math>\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0</math></li> </ul> <p>[C, CN, PS, R, T]</p>	<p>AN3. Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]</p>	not addressed

## Mathematical Processes

[C] Communication  
[CN] Connections  
[ME] Mental Mathematics and Estimation

[PS] Problem Solving  
[R] Reasoning  
[T] Technology  
[V] Visualization

**Strand: Number****Outcomes**

*Students will be expected to*

**9N1 Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:**

- representing repeated multiplication using powers
- using patterns to show that a power with an exponent of zero is equal to one
- solving problems involving powers.

[C, CN, PS, R]

**Achievement Indicators:**

9N1.1 *Demonstrate the difference between the exponent and the base by building models of a given power, such as  $2^3$  and  $3^2$ .*

9N1.2 *Explain, using repeated multiplication, the difference between two given powers in which the exponent and base are interchanged, e.g.,  $10^3$  and  $3^{10}$ .*

9N1.3 *Express a given power as a repeated multiplication.*

9N1.4 *Express a given repeated multiplication as a power.*

**Elaborations—Strategies for Learning and Teaching**

The use of exponents was introduced in Grade 8 as students explored perfect squares (8N1) and worked with the Pythagorean theorem (8SS1). This previous exposure was limited to square numbers. In this unit, powers with whole number exponents will be developed through repeated multiplication.

Students will be expected to identify the base, the exponent, and the power in an expression in exponential form. As they explore such expressions, students should become aware that the base and the exponent are not interchangeable. As such, switching them will not always result in the power having the same value. This can be demonstrated through the use of models.

From Grade 8, students are familiar with representing a power as a square region.

Ask students to draw a square with side lengths of 3 units.

They should conclude the area of this square is  $3^2$  or 9 units<sup>2</sup>.

An exponent of 2 results in a square number.

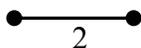


Ask students to draw a cube with side lengths of 2 units.

They should conclude the volume of this cube is  $2^3$  or 8 units<sup>3</sup>. An exponent of 3 results in a cube number.



Students should observe that  $3^2$  yields a two-dimensional image (length, width) and that  $2^3$  yields a three-dimensional image (length, width, and height). Ask students what type of image models the expression  $2^1$  and what measurements are involved. This should lead to a discussion of one-dimensional images which possess only length. The value  $2^1$  would produce:



Ultimately, students should recognize that powers with an exponent equal to 1 will have a value equal to the base (i.e.,  $a^1 = a$ ,  $3^1 = 3$ ). Ask students if the reverse is true. If students do not see an exponent on a number, it is implied that the exponent is 1 (i.e.,  $b = b^1$ ,  $8 = 8^1$ ).

Through the use of models and repeated multiplication, students should conclude that  $3^2 \neq 2^3$  and that exponents and bases are not interchangeable. Ask them if they can think of an example of powers where the exponent and base interchanged result in the same value (i.e.,  $2^4$  and  $4^2$ ).

Just as repeated addition can be represented as multiplication (e.g.,  $3 + 3 + 3 + 3 + 3 = 3 \times 5$ ), repeated multiplication can be represented by a power (e.g.,  $3 \times 3 \times 3 \times 3 \times 3 = 3^5$ ).

## General Outcome: Develop Number Sense.

### Suggested Assessment Strategies

#### *Performance*

- Ask students to create models for  $4^2$  and  $4^3$  using linking cubes. They should describe how these models are the same, and also how they are different. Ask students if they can model  $4^1$  using linking cubes and to explain their reasoning.

(9N1.1)

- Teachers could create a set of flash cards containing pairs that display powers in exponential form on one card and repeated multiplication on the other. One card is distributed to each student in the room randomly. Ask students to find their partner with the matching card and to explain why they are a match. The goal is to be both quick and correct. This is a simple game to reinforce the meaning of exponents.

(9N1.3, 9N1.4)

#### *Paper and Pencil*

- Ask students to represent the numbers below using as many different powers as possible. Ask them if some of them can only be represented in one way. If so, they should identify which ones.

- (i) 144
- (ii) 32
- (iii) 64
- (iv) 81
- (v) 125
- (vi) 70

(9N1.1)

### Resources/Notes

#### Authorized Resource

#### *Math Makes Sense 9*

Prep Talk Video: Powers and Exponent Laws

#### Lesson 2.1: What is a Power?

ProGuide: pp.4-9

Master 2.10a, 2.10b

CD-ROM: Master 2.17

Student Book (SB): pp. 52-57

Preparation and Practice Book (PB): pp. 54-58

**Strand: Number****Outcomes**

*Students will be expected to*

9N1 Continued...

**Achievement Indicators:**

9N1.5 *Explain the role of parentheses in powers by evaluating a given set of powers, e.g.,  $(-2)^4$ ,  $(-2^4)$  and  $-2^4$ .*

9N1.6 *Demonstrate, using patterns, that  $a^0$  is equal to 1 for a given value of  $a$  ( $a \neq 0$ ).*

9N1.7 *Evaluate powers with integral bases (excluding base 0) and whole number exponents.*

**Elaborations—Strategies for Learning and Teaching**

Students have used parentheses to evaluate expressions involving the order of operations. They should have an understanding that the placement of parentheses in a mathematical statement sometimes changes the value of the expression (e.g.,  $2 + 3 \times 4 \neq (2 + 3) \times 4$ ). Similarly, the placement of parentheses in an exponential expression can affect the value of the expression. Ask students to simplify the expression  $-3^2$  and then  $(-3)^2$  and explain why their answers are different. Discussion about negative bases and negative powers, as well as the purpose of parentheses, will be necessary. Parentheses are used when a power has a negative base to show that the negative sign is part of the base. For example,  $-3^2$  is not equal to  $(-3)^2$ .

One way to demonstrate that  $a^0$  is equal to 1 is through the use of patterns. Ask students to simplify the following powers to verify a power with an exponent of zero is equal to 1 (excluding base 0).

$$\begin{array}{ll} 2^5 = 32 & 3^5 = 243 \\ 2^4 = 16 & 3^4 = 81 \\ 2^3 = 8 & 3^3 = 27 \\ 2^2 = 4 & 3^2 = 9 \\ 2^1 = 2 & 3^1 = 3 \\ 2^0 = 1 & 3^0 = 1 \end{array}$$

This can be revisited later in the unit when students are introduced to the laws of exponents (9N2). At this point, it is sufficient that students understand that  $a^0=1$  through the use of patterning.

Students should evaluate exponential expressions to determine their values in standard form. Such expressions may contain a single term or multiple terms. To continue to develop number sense, the use of mental math should be encouraged whenever possible.

The order of operations must be applied to evaluate expressions. Although students have used the order of operations in previous grades, this will be their first exposure to applying the order of operations to expressions containing powers. They must be made aware that powers are evaluated after any operations in brackets and before multiplying and dividing. Following the order of operations ensures consistency when evaluating expressions. Calculators should not replace an understanding of the order of operations.

**General Outcome: Develop Number Sense.****Suggested Assessment Strategies***Journal*

- Ask students to respond to the following:
  - Explain why  $-6^2 \neq (-6)^2$  but  $-6^3 = (-6)^3$ .
  - Explain why  $(-3)^2 > 0$  but  $(-3)^3 < 0$ .

(9N1.5)

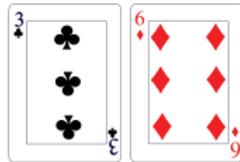
*Paper and Pencil*

- Ask students to evaluate powers such as the following:
  - $3 + 2^0$
  - $3^0 + 2^0$
  - $(3 + 2)^0$
  - $-3^0 + 2$
  - $-3^0 + (-2)^0$
  - $-(3 + 2)^0$

(9N1.6)

*Performance*

- From several decks of cards remove all the cards that are numbered between 1 and 5. These will be the game cards. Pair students off and give each pair a set of game cards. Place the set of cards face down in front of them and have them turn over two cards at once.



The first student to correctly determine which power is largest,  $3^6$  or  $6^3$ , will win that round. The game will continue in this manner until the set of cards has been exhausted. This is intended to be a non-calculator game.

(9N1.7)

**Resources/Notes****Authorized Resource***Math Makes Sense 9***Lesson 2.1: What is a Power?**

ProGuide: pp.4-9

Master 2.10a, 2.10b

CD-ROM: Master 2.17

SB: pp. 52-57

PB: pp. 54-58

**Lesson 2.2: Powers of Ten and the Zero Exponent**

ProGuide: pp.10-14

Master 2.11

CD-ROM: Master 2.18

SB: pp. 58-62

PB: pp. 61-63

**Lesson 2.3: Order of Operations with Powers**

ProGuide: pp.15-20

Master 2.6, 2.7, 2.7a, 2.8, 2.12

CD-ROM: Master 2.19

SB: pp. 63-68

PB: pp. 67-74

**Strand: Number****Outcomes**

*Students will be expected to*

**9N2 Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents:**

- $(a^m)(a^n) = a^{m+n}$
- $a^m \div a^n = a^{m-n}$ ,  $m > n$
- $(a^m)^n = a^{mn}$
- $(ab)^m = a^m b^m$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ ,  $b \neq 0$ .

[C, CN, PS, R, T]

**Achievement Indicator:**

9N2.1 *Explain, using examples, the exponent laws of powers with integral bases (excluding base 0) and whole number exponents:*

- $(a^m)(a^n) = a^{m+n}$
- $a^m \div a^n = a^{m-n}$ ,  $m > n$
- $(a^m)^n = a^{mn}$
- $(ab)^m = a^m b^m$
- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ ,  $b \neq 0$ .

**Elaborations—Strategies for Learning and Teaching**

Operations with exponents can be efficiently performed with the use of the laws of exponents. These laws will be developed with powers that have integral bases and whole number exponents. In Mathematics 1201, this will be extended to include powers with fractional exponents and negative exponents, as well as powers with rational and variable bases (AN3).

Focus should be placed on the development of an understanding of the laws of exponents. By expanding powers and completing the calculation, students should be able to predict the exponent laws. The following exponent laws should be developed:

- $(a^m)(a^n) = a^{m+n}$

Ask students to simplify the expression  $2^2 \times 2^3$  to a single power by expressing the two powers separately and then multiplying the results:  $2^2 \times 2^3 = (2 \cdot 2) \times (2 \cdot 2 \cdot 2)$ , or  $2^5$ . Students can repeat the process for other powers so that they may predict the rule.

- $a^m \div a^n = a^{m-n}$ ,  $m > n$ :  $5^6 \div 5^2$   
 $\frac{5^6}{5^2}$   
 $\frac{5 \times 5 \times 5 \times 5 \times 5 \times 5}{5 \times 5}$   
 $\frac{5}{5} \times \frac{5}{5} \times \frac{5}{1} \times \frac{5}{1} \times \frac{5}{1} \times \frac{5}{1}$   
 $1 \times 1 \times 5 \times 5 \times 5 \times 5$   
 $5^4$

For expressions like this, ensure that  $m > n$ , as students are only expected to work with whole number exponents here. They are already familiar with expressions where  $m = n$  resulting in a power with an exponent of zero. Integral exponents will be introduced in Mathematics 1201 (AN3).

- $(a^m)^n = a^{mn}$   
 $(2^4)^2 = 2^4 \cdot 2^4$

Students may then apply the rule  $a^m \cdot a^n = a^{m+n}$ , or simply write  $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$ . After repeating this with other examples, students should be able to predict a rule that applies when an exponent is raised to an exponent.

**General Outcome: Develop Number Sense.****Suggested Assessment Strategies***Performance*

- Ask students to create a foldable with an entry for each of the exponent laws accompanied by examples.

<b>Product of Powers</b> $(a^m)(a^n) = a^{m+n}$
<b>Quotient of Powers</b> $\frac{a^m}{a^n} = a^{m-n}$
<b>Power of a Power</b> $(a^m)^n = a^{mn}$
<b>Power of a Product</b> $(ab)^m = a^m \times b^m$
<b>Power of a Quotient</b> $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

(9N2.1)

**Resources/Notes****Authorized Resource***Math Makes Sense 9*

Lesson 2.4: Exponent Laws I

Lesson 2.5: Exponent Laws II

ProGuide: pp.25-30, 31-37

Master 2.9, 2.9a, 2.13, 2.14

CD-ROM: Master 2.20, 2.21

SB: pp. 73-78, 79-85

PB: pp. 76-81, 82-87

**Strand: Number****Outcomes***Students will be expected to*

9N2 Continued...

**Achievement Indicators:**9N2.1 *Continued***Elaborations – Strategies for Learning and Teaching**

- $(ab)^m = a^m b^m$   
 $(2 \times 3)^3$   
 $(2 \times 3)(2 \times 3)(2 \times 3)$   
 $(2 \times 2 \times 2)(3 \times 3 \times 3)$   
 $2^3 \times 3^3$

- $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$

$$\left(\frac{6}{2}\right)^3$$

$$\left(\frac{6}{2}\right)\left(\frac{6}{2}\right)\left(\frac{6}{2}\right)$$

$$\frac{(6)(6)(6)}{(2)(2)(2)}$$

$$\frac{6^3}{2^3}$$

Whenever possible, instruction should be designed so that students discover rules/relationships and verify discoveries.

9N2.2 *Evaluate a given expression by applying the exponent laws.*

Students can explore different solutions to problems to develop an appreciation for the efficiency that the exponent laws provide. For example, alternate solutions used to evaluate  $(2^3 \times 2^2)^2$  are shown here.

$\begin{aligned} &(2^3 \times 2^2)^2 \\ &= (2^{3+2})^2 \\ &= (2^5)^2 \\ &= 2^{10} \\ &= 1024 \end{aligned}$	$\begin{aligned} &(2^3 \times 2^2)^2 \\ &= (2^3)^2 \times (2^2)^2 \\ &= 2^6 \times 2^4 \\ &= 2^{6+4} \\ &= 2^{10} \\ &= 1024 \end{aligned}$	$\begin{aligned} &(2^3 \times 2^2)(2^3 \times 2^2) \\ &= (2^5)(2^5) \\ &= 2^{10} \\ &= 1024 \end{aligned}$
---	---	--

Students should be encouraged to use the laws as efficiently as possible. It is important that examples such as those shown above be done by applying the exponent laws instead of through the use of a calculator. This provides an important foundation for multiplying and dividing a polynomial by a monomial (9PR7).

**General Outcome: Develop Number Sense.****Suggested Assessment Strategies***Paper and Pencil*

- Ask students to answer the following:
  - Determine which operations (+, −, ×, ÷) should be placed in this expression in order to make the equation true.
 
$$5^2 \text{ \_\_\_\_\_ } 16 \text{ \_\_\_\_\_ } 2^2 \text{ \_\_\_\_\_ } 6 = 83$$

(9N2.2)
  - Determine the value of  $2^{x+3}$  if  $2^x = 16$ .
 

(9N2.2)

*Performance*

- This activity could be used as an exit card. Provide students with an index card where they are asked to simplify expressions as a single power. The student should not place their name on the card since the index cards could be used for error analysis during the next class. A sample list is shown:
  - $(-4)^2(-4)^2$
  - $3^6 \times 3^8$
  - $\frac{4^{10}}{4^9}$
  - $(-3)^6 \div (-3)^4$
  - $((-9)^2)^5$
  - $(3^2 \times 3^8) \div (3^3)^2$
  - $(2^3 \times 2^2)^3 - (3^2 \times 3)^2$
  - $2^5 \times 2 - 2^3 \times 2^2$

(9N2.2)
- Ask students to play the Exponent Bingo game. Give each student a blank Bingo card and ask them to number the card in any fashion from 1 to 24. Allow for a free space. Ask students to simplify various expressions, such as  $(2^4 \times 2^3) \div 2^6$ , find the value on their Bingo cards and cross it off. Traditional Bingo alignments win - horizontal, vertical and diagonal. Options such as four corners may also be used.
 

(9N2.2)

**Resources/Notes****Authorized Resource***Math Makes Sense 9***Lesson 2.4: Exponent Laws I****Lesson 2.5: Exponent Laws II**

ProGuide: pp.25-30, 31-37

Master 2.9, 2.9a, 2.13, 2.14

CD-ROM: Master 2.20, 2.21

SB: pp. 73-78, 79-85

PB: pp. 76-81, 82-87

**Strand: Number****Outcomes**

*Students will be expected to*

9N2 Continued...

**Achievement Indicators:**

9N2.3 Determine the sum of two given powers, e.g.,  $5^2 + 5^3$ , and record the process.

9N2.4 Determine the difference of two given powers, e.g.,  $4^3 - 4^2$ , and record the process.

9N2.5 Identify the error(s) in a given simplification of an expression involving powers.

**Elaborations – Strategies for Learning and Teaching**

Students should be reminded that the order of operations must always be applied when evaluating a mathematical expression. The terms with exponents, therefore, are evaluated before the powers can be added or subtracted.

Some common student errors with exponential calculations are highlighted here.

$$5^2 - 3$$

$$10 - 3$$

$$7$$

Students often apply the order of operations incorrectly to expressions involving powers.

$$7 + 2 \times 4^2 - 4$$

$$9 \times 4^2 - 4$$

$$36^2 - 4$$

$$1296 - 4$$

$$1292$$

Students should be able to identify and correct errors such as those present in the following evaluation:

$$\frac{(1+3)^2}{(3+5 \times 6^0) \div 2^2}$$

$$\frac{1^2+3^2}{(3+5 \times 1) \div 4}$$

$$\frac{1+9}{8 \div 4}$$

$$\frac{10}{2}$$

$$5$$

**General Outcome: Develop Number Sense.****Suggested Assessment Strategies***Journal*

- Ask students to respond to the following:  
You are at home when your friend, who was absent today, calls and asks you to explain how to do tonight's homework problem that he copied from the teacher's webpage. Explain how to evaluate  $5^2 - 2^3$ .  
(9N2.4)

*Paper and Pencil*

- Brainstorm with students to identify all mistakes that students are likely to make when evaluating the expression below. Ask students to explain to the group why these are mistakes, and then determine the correct solution.

$$32 \div (-2^3) + 5(4)^2$$

(9N2.5)

- Mary evaluated the following expression and the solution is shown:

$$\begin{aligned} (3 + 5)^2 \times 3 + 4 \\ = 8^2 \times 7 \\ = 64 \times 7 \\ = 448 \end{aligned}$$

Ask students to circle and explain the mistake, then evaluate the expression correctly showing all steps.

(9N2.5)

*Performance*

- Redistribute the index cards from the Performance activity on page 47. Students could work in pairs. They should identify and correct any errors that exist. At the end of the activity, each group should present to the rest of the class the common errors that were made.  
(9N2.2, 9N2.5)

**Resources/Notes****Authorized Resource***Math Makes Sense 9*

Lesson 2.4: Exponent Laws I

Lesson 2.5: Exponent Laws II

ProGuide: pp.25-30, 31-37

Master 2.9, 2.9a, 2.13, 2.14

CD-ROM: Master 2.20, 2.21

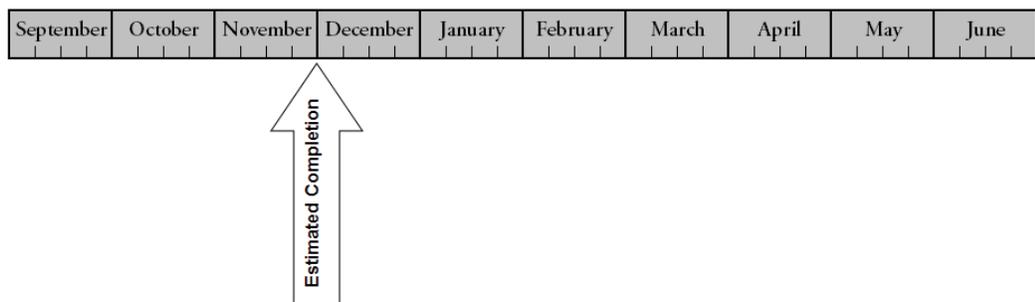
SB: pp. 73-78, 79-85

PB: pp. 76-81, 82-87



# Rational Numbers

Suggested Time: 4 Weeks

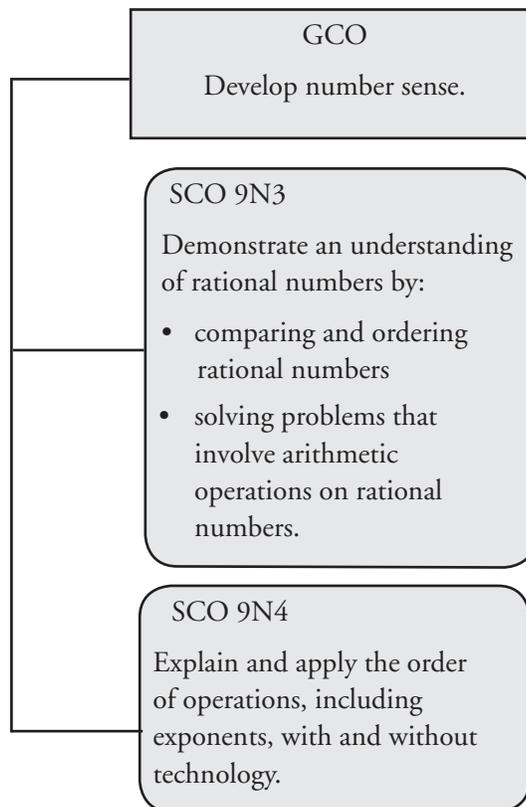


# Unit Overview

## Focus and Context

In this unit, students will compare and order rational numbers using common denominators, number lines, and place value. In earlier grades, students used concrete materials and pictorial representations to illustrate addition, subtraction, multiplication and division of positive fractions and decimals. This will now be extended to include negative rational numbers. Proficiency with paper and pencil algorithms for each operation is important since these algorithms provide fundamental building blocks for algebraic manipulations and add to general computational efficiency which is necessary for later grades. Students will also solve problems that involve arithmetic operations on rational numbers. They will explain and apply the order of operations, including exponents, with and without technology.

## Outcomes Framework



## SCO Continuum

Mathematics 8	Mathematics 9	Mathematics Level I	
		1201	1202
Number			
<p>8N6. Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially and symbolically. [C, CN, ME, PS]</p> <p>8N7. Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically. [C, CN, PS, R, V]</p>	<p>9N3. Demonstrate an understanding of rational numbers by:</p> <ul style="list-style-type: none"> <li>• comparing and ordering rational numbers</li> <li>• solving problems that involve arithmetic operations on rational numbers.</li> </ul> <p>[C, CN, PS, R, T, V]</p> <p>9N4. Explain and apply the order of operations, including exponents, with and without technology. [PS, T]</p>	<p>AN2. Demonstrate an understanding of irrational numbers by:</p> <ul style="list-style-type: none"> <li>• representing, identifying and simplifying irrational numbers</li> <li>• ordering irrational numbers.</li> </ul> <p>[CN, ME, R, V]</p> <p>AN3. Demonstrate an understanding of powers with integral and rational exponents. [C, CN, PS, R]</p>	not addressed

## Mathematical Processes

[C] Communication  
[CN] Connections  
[ME] Mental Mathematics and Estimation

[PS] Problem Solving  
[R] Reasoning  
[T] Technology  
[V] Visualization

**Strand: Number****Outcomes**

*Students will be expected to*

**9N3 Demonstrate an understanding of rational numbers by:**

- comparing and ordering rational numbers
- solving problems that involve arithmetic operations on rational numbers.

[C, CN, PS, R, T, V]

**Achievement Indicator:**

9N3.1 *Order a given set of rational numbers, in fraction and decimal form, by placing them on a number line,*

*e.g.,  $\frac{3}{5}$ ,  $-0.666\dots$ ,  $0.5$ ,  $-\frac{5}{8}$ .*

**Elaborations—Strategies for Learning and Teaching**

The focus of this unit is the set of rational numbers,  $\mathbb{Q}$ . Students need to have an understanding that a rational number is any number which can be written in the form  $\frac{p}{q}$ , where  $p$  and  $q$  are both integers and  $q \neq 0$ . This means that the set of rational numbers includes all fractions and all terminating or repeating decimals. Non-terminating and non-repeating decimals belong to the set of irrational numbers. Rational numbers make up the largest subset of real numbers. Teachers could use a graphic organizer, such as a Venn diagram, to help students visualize the various subsets of the real number system.

In Grade 6, students were introduced to the ordering of integers (6N7). In Grade 7, students compared and ordered positive fractions, positive decimals and whole numbers (7N7). Now they will compare and order rational numbers, including negative fractions and negative decimals. They have been exposed to a variety of strategies for comparing fractions and decimals. Such strategies can be revisited here in the context of comparing rational numbers.

All rational numbers being compared can be converted to fractions or decimals. When working with decimals, students can compare them using place value. When working with fractions, students can use strategies such as the following:

- Use common denominators. If both fractions have the same denominator, the larger numerator represents the larger fraction (e.g.,  $-\frac{3}{8} > -\frac{5}{8}$ ). If denominators differ, students can write equivalent fractions with like denominators and then compare the numerators.
- Use common numerators. If both fractions have the same positive numerator, the fraction with the smallest denominator is larger (e.g.,  $\frac{2}{7} > \frac{2}{9}$ ). If both fractions have the same negative numerator, the fraction with the largest denominator is larger (e.g.,  $-\frac{2}{9} > -\frac{2}{7}$ ).
- Place the fractions on a number line with benchmarks.

In Grade 7, students were expected to demonstrate an understanding of the relationship between positive terminating decimals and positive fractions and between positive repeating decimals and positive fractions (7N4).

## General Outcome: Develop Number Sense.

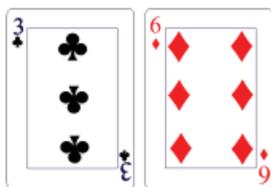
### Suggested Assessment Strategies

#### *Performance*

- Prepare a set of cards with a variety of rational numbers. Give each student one card. Place them in teams. Students must compare their cards and arrange themselves so that they are in order (ascending or descending). The team who correctly orders the cards first is the winner. A variation of this activity is the Clothesline Game. Students are required to 'pin' the cards on a clothesline that has been prepared in advance. The clothesline would be marked off at regular intervals, representing integers on a number line.

(9N3.1)

- From several decks of cards remove all the cards that are numbered between 1 and 9. These will be the game cards. Pair students off and give each pair a set of game cards. Students should place the set of cards face down and turn over two cards at once.



The black cards represent positives and the red cards represent negatives.

Students will form two fractions using the indicated cards. In the case above the fractions will be  $-\frac{3}{6}$  and  $-\frac{6}{3}$ .

The first student to determine which fraction is closest to zero wins that round. The game continues until all cards have been played.

This activity can be modified to work with decimals.

(9N3.1)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 9*

Prep Talk Video: Rational Numbers

#### Lesson 3.1: What is a Rational Number?

ProGuide: pp.4-13

Master 3.7

CD-ROM: Master 3.18

Student Book (SB): pp. 94-103

Preparation and Practice Book (PB): pp. 93-101

**Strand: Number****Outcomes**

*Students will be expected to*

9N3 Continued...

**Achievement Indicator:**

9N3.2 *Identify a rational number that is between two given rational numbers.*

**Elaborations—Strategies for Learning and Teaching**

Students already have experience with identifying numbers that fall between two given numbers from Grade 7, using positive fractions and decimals (7N7). This will now be extended to the set of rational numbers. Ask students to find a fraction, for example, that is between  $-\frac{2}{3}$  and  $-\frac{3}{4}$ .

Students with a facility for working with fractions and decimals may be able to determine an answer using mental math. Most will have to rewrite each fraction using a common denominator. It is important for them to realize that it may not necessarily be the lowest common denominator that will lead directly to a solution. If asked, for example, to identify a rational number between  $\frac{1}{2}$  and  $\frac{2}{3}$ , some will first write equivalent fractions  $\frac{3}{6}$  and  $\frac{4}{6}$ . It is still difficult to identify a rational number between these two fractions. Using a denominator of 12 makes it easier to identify  $\frac{7}{12}$  as a rational number between the two given numbers.

Students are also expected to identify a number that is between two rational numbers in decimal form, such as  $-0.7$  and  $-0.71$ . Adding trailing zeros to each of these numbers will give many possible solutions. The decimal numbers can be written as  $-0.700$  and  $-0.710$ . By comparing the last two digits of each, several solutions can be identified.  $-0.701$ ,  $-0.702$ ,  $-0.703$ ,  $-0.704$ ,  $-0.705$ ,  $-0.706$ ,  $-0.707$ ,  $-0.708$ , and  $-0.709$  are all numbers between  $-0.7$  and  $-0.71$ .

In a case when students are given a negative and positive rational number to start, they should quickly realize that zero is the most expedient solution.

Students have a choice of strategies when they are asked to identify a rational number between a fraction and a decimal. They could convert the fraction to a decimal, or vice versa, and then use the appropriate method.

**General Outcome: Develop Number Sense.****Suggested Assessment Strategies***Journal*

- Ask students to discuss how to identify all integers that are between  $\frac{11}{5}$  and  $-\frac{15}{4}$ , and then identify them. (9N3.2)

*Performance*

- Provide each student with an index card. Two of the cards will have a rational number already written on them and will serve as the ends of a number line. All other cards will be blank. The students must write a rational number that is between the two endpoints on a card. Then they must place their card in the correct position on the number line. (9N3.2)

*Paper and Pencil*

- Ask students to determine three rational numbers between two given decimal numbers, such as 0.6 and 0.61. They should choose one number and explain why it is between the two given numbers. (9N3.2)

**Resources/Notes****Authorized Resource***Math Makes Sense 9***Lesson 3.1: What is a Rational Number?**

ProGuide: pp.4-13

CD-ROM: Master 3.18

SB: pp. 94-103

PB: pp. 93-101

**Strand: Number****Outcomes**

*Students will be expected to*

9N3 Continued...

**Achievement Indicator:**

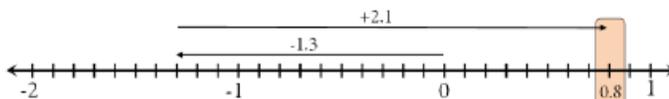
9N3.3 *Solve a given problem involving operations on rational numbers in fraction form and decimal form.*

**Elaborations—Strategies for Learning and Teaching**

In Grade 7, students worked with the four operations with positive decimals (7N2). They also worked with addition and subtraction of integers (7N6) and positive fractions (7N5). In Grade 8, they multiplied and divided integers (8N7) and positive fractions (8N6). Operations with negative rational numbers in fractional and decimal form are new at Grade 9.

Significant work has taken place using concrete materials and pictorial representations for fractions and integers. Most students should be able to work reasonably well with all four operations at the symbolic level by Grade 9. Some students, however, may require a brief review of operations with fractions and integers using concrete and pictorial models.

In Grade 7, students used number lines when adding and subtracting integers. This can also be an effective model for adding and subtracting rational numbers. The number line shown below, for example, can be used to determine that  $-1.3 + 2.1$  is 0.8.



It is important that students become proficient with standard algorithms. These algorithms provide fundamental building blocks for algebraic manipulations and add to general computational efficiency. Students are required to demonstrate mathematical proficiency with the algorithms. They should not become dependent on calculators to perform operations on rational numbers.

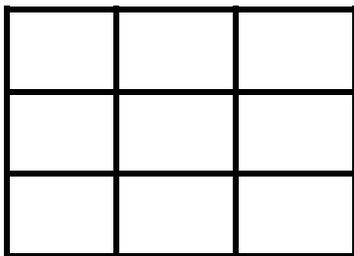
- Operations on rational numbers in the form of decimals combines the rules for operations on positive decimals with the rules for adding, subtracting, multiplying and dividing integers.
- Operations on rational numbers in the form of fractions combines the rules for adding, subtracting, multiplying and dividing positive fractions with the rules for operations on integers.

## General Outcome: Develop Number Sense.

### Suggested Assessment Strategies

#### *Paper and Pencil*

- Ask students to answer the following:  
Some of David's stocks are reported on the New York Stock Exchange using fractions, while the rest are reported on the Toronto Stock Exchange using decimals. How much did he lose if one holding of 150 shares reported a net change of  $-4\frac{1}{2}$  and a second holding of 6000 shares reported a net change of  $-0.25$ ?  
(9N3.3)
- In a magic square, the sum of each row, column and diagonal is the same. Ask students to:
  - create a magic square which uses a mixture of positive and negative rational numbers written in fractional form
  - create a magic square which uses a mixture of positive and negative rational numbers written in decimal form



(9N3.3)

#### *Performance*

- Pose the following to students:  
A children's wading pool has a small leak. During an afternoon, one-eighth of the water leaks out of the pool. What could each expression below describe about this situation?
  - $0.75 \times \left(-\frac{1}{8}\right)$
  - $-\frac{1}{8} \div 0.25$

Using *Think-Pair-Share*, give individual students time to think about the question. Students then pair up with a partner to discuss their ideas. After pairs discuss, students share their ideas in a small-group or whole-class discussion.  
(9N3.3)
- Students could work in small groups to put together a jigsaw puzzle where the expressions on the adjacent sides of the puzzle piece have to be equivalent. This activity provides students with the opportunity to practice adding, subtracting, multiplying and dividing rational numbers, make mathematical arguments about whether or not pieces fit together, and check and revise their work.  
(9N3.3)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 9*

Lesson 3.2: Adding Rational Numbers

Lesson 3.3: Subtracting Rational Numbers

Lesson 3.4: Multiplying Rational Numbers

Lesson 3.5: Dividing Rational Numbers

ProGuide: pp. 16-23, 24-30, 33-39, 40-46

Master 3.8, 3.9

CD-ROM: Master 3.19, 3.20, 3.21, 3.22

See It Videos and Animations:

Adding Rational Numbers

Subtracting Rational Numbers

SB: pp. 106-113, 115-120, 123-129, 130-136

PB: pp. 102-108, 109-116, 117-124, 125-129

#### Suggested Resource

<https://www.k12pl.nl.ca/curr/7-9/math/grade9/links/unit3.html>

- A Tarsia formulator can be used to customise jigsaws, domino activities and a variety of rectangular card sort activities.

**Strand: Number****Outcomes**

*Students will be expected to*

**9N4 Explain and apply the order of operations, including exponents, with and without technology.**

[PS, T]

**Achievement Indicators:**

9N4.1 *Solve a given problem by applying the order of operations without the use of technology.*

9N4.2 *Solve a given problem by applying the order of operations with the use of technology.*

9N4.3 *Identify the error in applying the order of operations in a given incorrect solution.*

**Elaborations – Strategies for Learning and Teaching**

Problems involving the use of multiple operations with rational numbers provide teachers with a good opportunity to observe whether students understand the four basic operations on rational numbers instead of applying them in rote fashion. It is important that students have a solid foundation in operations with rational numbers, since it is fundamental to the study of algebra.

Students used the order of operations, excluding exponents, with whole numbers, in Grade 6 (6N9). This was extended to include decimals in Grade 7 (7N2) and integers and fractions in Grade 8 (8N6, 8N7). Work in the previous unit, Powers and Exponent Laws, (9N1, 9N2) enables them to incorporate exponents into calculations involving multiple operations. Students will now complete calculations with rational numbers based on the following order:

- Perform the operations in brackets first
- Find the value of expressions involving exponents
- Multiply or divide in the order they appear from left to right
- Add or subtract in the order they appear from left to right

Students should be encouraged to perform operations without calculators as much as possible. However, some questions lend themselves to calculator use more than others. When evaluating with decimals, it is appropriate to use a calculator for more than 2-digit multipliers or more than 1-digit divisors. When verifying solutions, calculators can also be a useful tool. There are different types of scientific and graphing calculators available. Allow time for students to become familiar with inputting data into their calculators so that the order of operations will be applied correctly.

## General Outcome: Develop Number Sense.

### Suggested Assessment Strategies

#### Journal

- Ask students to explain why it is essential that the rules for order of operations for rational numbers be the same as the order of operations for integers. (9N4.1)
- Ask students how they could predict whether the sum, difference, product or quotient of two rational numbers will be greatest. (9N4.1)

#### Paper and Pencil

- Ask students to simplify and arrange from least to greatest:

$$(i) \quad -\frac{3}{4} - \left(-\frac{3}{4} + -\frac{4}{5}\right) \qquad (ii) \quad -\frac{3}{5} - \left(-\frac{3}{4}\right) - \left(-\frac{9}{10}\right)$$

$$(iii) \quad 6 \div \left(-\frac{1}{5}\right) - \left(-\frac{1}{2}\right) \qquad (iv) \quad \frac{3}{5} - \left(-\frac{3}{5} - -\frac{2}{3}\right)$$

(9N4.1)

- Ask students to add brackets where required to make this a true statement:  
 $3.5 + 4 \div 0.75 + (-8.1) = 1.9$  (9N4.1)

#### Performance

- Create a game where students are given a strip containing 4 rational numbers and ask students to use  $+$ ,  $-$ ,  $\times$ ,  $\div$ ,  $()$ ,  $\sqrt{\quad}$ ,  $x^2$  to create equations. The individual operations could be placed on pieces of paper to allow students to move them around the numbers. Teachers may have students create the equation with the largest solution or solution closet to zero, or any other variation.

Given:

-1.86	-2	5.3	9
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Students Create:

$(-1.86 + 2)^2 \times -5.3 - \sqrt{9}$
--

A variation of this activity is for a student to choose 4 numbers, create an expression and ask their partner to simplify the expression.

(9N4.1, 9N4.2)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 9*

#### Lesson 3.6: Order of Operations with Rational Numbers

ProGuide: pp. 47-52

CD-ROM: Master 3.23

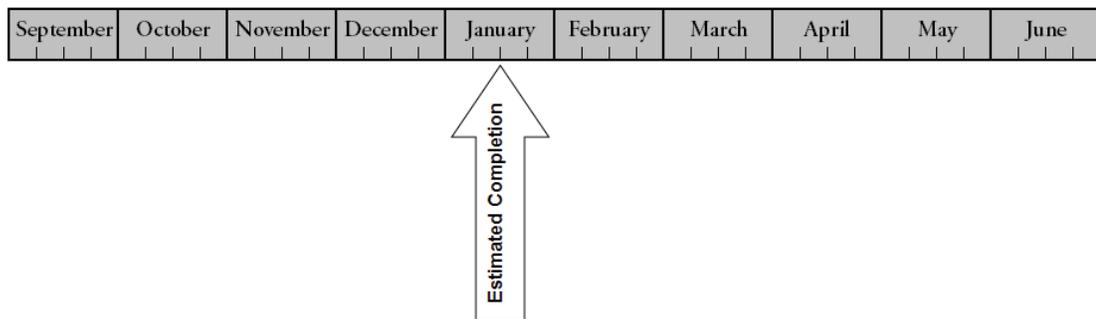
SB: pp. 137-142

PB: pp. 130-134



# Linear Relations

Suggested Time: 4 Weeks



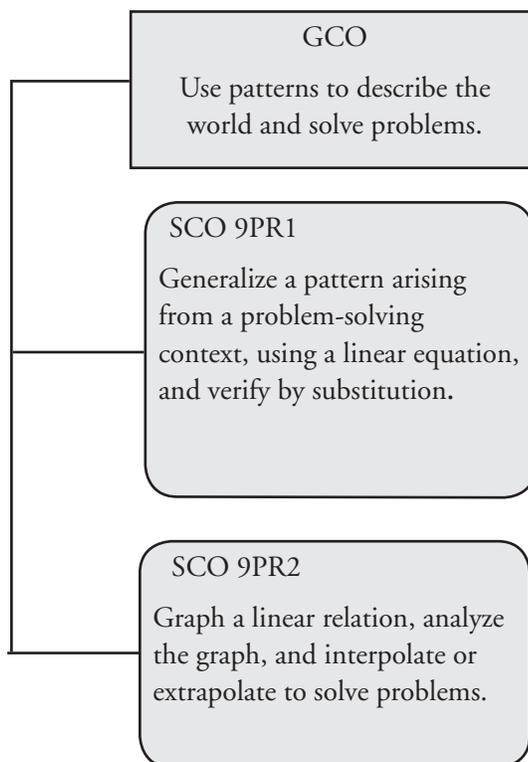
# Unit Overview

## Focus and Context

In this unit, students will work with patterns presented in tables, graphs, charts, pictures, and problem situations. They will describe in words and use expressions or equations to represent linear relations. Through investigation, students will identify the dependent and independent variables, any constants in the situation, and will discover that a linear relation requires constant change in the dependent and independent variables. Once equations have been created, students will use them to find the missing values of the independent and dependent variables. Graphing and substitution methods will be used to predict values in problem-solving situations.

Using real-world contexts enhances students' interest, and ultimately their understanding of mathematical concepts. An activity as simple as stringing beads could be used to engage students in determining equations and graphs of linear relations.

## Outcomes Framework



## SCO Continuum

Mathematics 8	Mathematics 9	Mathematics Level I	
		1201	1202
Patterns and Relations			
<p>8PR1. Graph and analyze two variable linear relations. [C, ME, PS, R, T,V]</p>	<p>9PR1. Generalize a pattern arising from a problem solving context, using a linear equation, and verify by substitution. [C, CN, PS, R, V]</p> <p>9PR2. Graph a linear relation, analyze the graph, and interpolate or extrapolate to solve problems. [C, CN, PS, R, T, V]</p>	<p>RF4. Describe and represent linear relations, using:</p> <ul style="list-style-type: none"> <li>• words</li> <li>• ordered pairs</li> <li>• tables of values</li> <li>• graphs</li> <li>• equations.</li> </ul> <p>[C, CN, R, V]</p> <p>RF5. Determine the characteristics of the graphs of linear relations, including the:</p> <ul style="list-style-type: none"> <li>• intercepts</li> <li>• slope</li> <li>• domain</li> <li>• range.</li> </ul> <p>[CN, PS, R, V]</p> <p>RF6. Relate linear relations expressed in:</p> <ul style="list-style-type: none"> <li>• slope–intercept form <math>(y = mx + b)</math></li> <li>• general form <math>(Ax + By + C = 0)</math></li> <li>• slope–point form <math>(y - y_1 = m(x - x_1))</math></li> </ul> <p>to their graphs. [CN, R, T, V]</p> <p>RF7. Determine the equation of a linear relation, given:</p> <ul style="list-style-type: none"> <li>• a graph</li> <li>• a point and the slope</li> <li>• two points</li> <li>• a point and the equation of a parallel or perpendicular line</li> </ul> <p>to solve problems. [CN, PS, R, V]</p>	not addressed

Mathematical Processes

[C] Communication  
[CN] Connections  
[ME] Mental Mathematics and Estimation

[PS] Problem Solving  
[R] Reasoning  
[T] Technology  
[V] Visualization

**Strand: Patterns and Relations (Patterns)****Outcomes**

*Students will be expected to*

**9PR1 Generalize a pattern arising from a problem-solving context, using a linear equation, and verify by substitution.**

[C, CN, PS, R, V]

**Achievement Indicators:**

9PR1.1 *Write an expression representing a given pictorial, oral or written pattern.*

9PR1.2 *Write a linear equation to represent a given context.*

9PR1.3 *Write a linear equation representing the pattern in a given table of values and verify the equation by substituting values from the table.*

**Elaborations—Strategies for Learning and Teaching**

In Grade 7, students used algebraic expressions to describe patterns, and constructed graphs from the corresponding table of values (7PR2, 7PR4). In Grade 8, students examined the various ways a relation can be expressed, including ordered pairs, table of values and graphs. They also used patterns to find missing values in a linear relation (8PR1). In previous grades, the algebraic expressions were often given to students. In Grade 7, students did formulate linear relations to represent the relationship in a given oral or written pattern. However, in Grade 8 most of the work was done with linear relations provided. In Grade 9, there is a focus on writing an expression or equation given the pictorial, oral or written form of the relation.

Students sometimes have difficulty distinguishing between an expression and an equation. Although they worked with both in Grade 7 (7PR4), teachers should reinforce the difference.

Students are expected to move interchangeably among the various representations that describe relations. They should be able to describe in words and use expressions and equations to represent patterns from tables, graphs, charts, pictures, and problem situations. Information presented in a variety of formats should be used to derive mathematical expressions and equations and to predict unknown values.

When a relation is represented using pictorial or written form, students should use patterns to derive the expression or equation. Students should examine the situation to determine what stays constant, what changes and how it relates to the expression or equation. Once the equation has been created, students are expected to use it to find missing values of the independent and dependent variable.

Patterns expressed in a table of values should also be represented with a linear equation. This can be achieved by examining the constant changes in the columns of the table. At this point, examples should be limited to increases of 1 in the independent variable. Later in the unit, when students have had experience dealing with graphing linear relations, increments other than 1 can be used for the independent variable. Through investigation, students should realize that a linear relation occurs when there is a constant change in the independent and dependent variables.

Students should make a connection between the constant change in the dependent variable and the equation. Students will use this connection to substitute values from the table to determine the equation. They can then verify this equation using substitution.

**General Outcome: Use patterns to describe the world and solve problems.**

**Suggested Assessment Strategies**

*Interview*

- Ask students to discuss the benefits of describing a relation using:
  - (i) a pictorial model
  - (ii) an algebraic representation
 Ask them which representation they prefer and why.  
 (9PR1.1, 9PR1.2)

*Paper and Pencil*

- Ask students to answer the following:  
 Jake is checking over his math assignment. He phones you to verify the equation for the following table of values:

$x$	$y$
3	8
4	10
5	12
6	14

He thinks the equation is  $y = 3x - 1$ , since the point (3, 8) satisfies this equation. Is he correct? Justify your answer.  
 (9PR1.3)

- On a questionnaire such as the following, students select a response based on their level of familiarity with the mathematical term.

<p><b>Expression</b></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I clearly know what it means and can describe it.</p>	<p><b>Equation</b></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I clearly know what it means and can describe it.</p>
<p><b>Table of Values</b></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I clearly know what it means and can describe it.</p>	<p><b>Linear Relation</b></p> <p><input type="checkbox"/> I have never heard of this.</p> <p><input type="checkbox"/> I have heard of this but I'm not sure what it means.</p> <p><input type="checkbox"/> I have some idea what it means.</p> <p><input type="checkbox"/> I clearly know what it means and can describe it.</p>

A blank space could be left for the third and fourth selected responses, to have students describe their ideas about the term. The questionnaire can be administered again as a post-assessment at the end of the unit.

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 9*

Prep Talk Video: Linear Relations

**Lesson 4.1: Writing Equations to Describe Patterns**

**Lesson 4.2: Linear Relations**

ProGuide: pp. 6-14, 15-25

Master 4.6

CD-ROM: Master 4.20, 4.21

Student Book (SB): pp.154-162, 164-173

Preparation and Practice Book (PB): pp. 142-148, 149-156

**Strand: Patterns and Relations (Patterns)****Outcomes**

*Students will be expected to*

9PR1 Continued...

**Achievement Indicators:**

9PR1.3 *Continued*

9PR1.4 *Solve, using a linear equation, a given problem that involves pictorial, oral and written linear patterns.*

9PR1.5 *Describe a context for a given linear equation.*

**Elaborations—Strategies for Learning and Teaching**

Asking questions such as the following can help guide students through the process of representing a pattern in a table of values with a linear equation.

$x$	$y$
0	3
1	5
2	7
3	9

- What pattern exists in the  $x$ -values?
- What pattern exists in the  $y$ -values?
- Can you predict the relationship between the  $x$ -values and the  $y$ -values?
- What equation represents this relationship?
- Substitute values from the table to check the equation.
- Does your equation need to be adjusted? If so, how can you determine what needs to be changed?

Students should realize that as the independent variable increases by 1, the dependent variable increases by 2. This suggests that the dependent variable could be 2 times the independent variable (i.e.,  $y = 2x$ ).

Substituting values from the table indicates that this equation results in  $y$ -values that are 3 less than those in the table. From this, students should conclude that the correct equation is  $y = 2x + 3$ .

Students should also work with decreasing patterns that can be represented with linear equations.

As students analyze pictures, tables and equations, they should recognize that each representational form is a viable way to solve a problem.

This understanding gives them a choice of representations to use and can lessen their reliance on procedural manipulation of the symbolic representation. Alternate representations can strengthen students' awareness of symbolic expressions and equations. For students to have this choice and this knowledge, they must have had experience with each type of representation.

In addition, students should connect their mathematical learning to contextual situations. Ask students to create a context to describe a given linear relation, such as  $C = 3a + 1$ .

**General Outcome: Use patterns to describe the world and solve problems.**

**Suggested Assessment Strategies**

**Resources/Notes**

*Paper and Pencil*

**Authorized Resource**

- Ask students to describe a situation which can be represented by each equation.

(i)  $y = 3x + 6$

(ii)  $12k = 60$

(iii)  $S = 5p - 2$

(9PR1.5)

*Math Makes Sense 9*

**Lesson 4.1: Writing Equations to Describe Patterns**

**Lesson 4.2: Linear Relations**

ProGuide: pp. 6-14, 15-25

Master 4.6

CD-ROM: Master 4.20, 4.21

SB: pp.154-162, 164-173

PB: pp. 142-148, 149-156

- Sean pays a one time fee of \$6.00 to download songs plus \$0.25 for each song. Ask students to answer the following:

(i) Write an equation to represent this situation.

(ii) How much would it cost to download 16 songs?

(iii) How many songs can be downloaded for \$13.00?

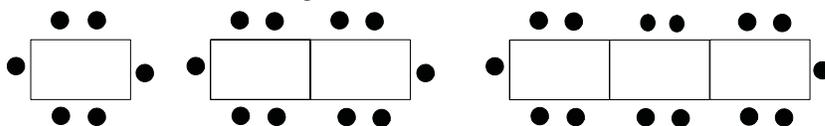
(9PR1.2, 9PR1.4)

- Ask students to answer the following:

(i) Sherry is on the graduation committee and she uses a column in the school newsletter to communicate to the senior class. Her recent column contained the equation  $C = 10d + 30$ . Which graduation activity could the equation represent? Justify your response.

(9PR1.5)

(ii) Sheila was having a party and could arrange the table and chairs as shown in the diagram below.



(a) Write and solve an equation to determine how many people would be seated at 15 tables.

(b) How many tables would be required to seat 200 people?

(9PR1.2, 9PR1.4)

- Matthew created the following number pattern:

6, 14, 22, 30.....

Ask students to answer the following:

(i) Determine the next three terms.

(ii) Develop an expression that can be used to determine the value of each term in the number pattern.

(iii) Use the expression to find the 20<sup>th</sup> term in this pattern.

(iv) Which term has a value of 94?

(9PR1.1, 9PR1.4)

**Strand: Patterns and Relations (Patterns)**

**Outcomes**

*Students will be expected to*

**9PR2 Graph a linear relation, analyze the graph, and interpolate or extrapolate to solve problems.**

[C, CN, PS, R, T, V]

**Achievement Indicators:**

9PR2.1 *Describe the pattern found in a given graph.*

9PR2.2 *Graph a given linear relation, including horizontal and vertical lines.*

**Elaborations—Strategies for Learning and Teaching**

In previous grades, students described the relationship between variables of a given graph. They also constructed and analyzed graphs of linear equations, with a focus on discrete data. In Grade 9, students will be expected to work with both discrete and continuous data. Exploration should quickly reveal that all linear relations lie in a straight line.

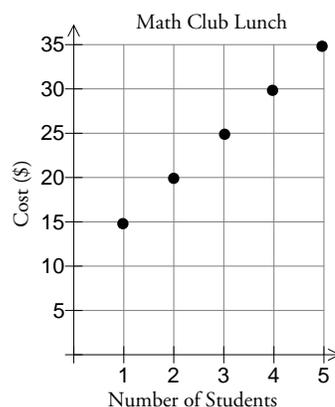
Students will be working with the concept of slope. The term ‘slope’, however, is not introduced until Grade 10. Comparison of the table and the graph should enable students to recognize that the constant change in the independent variable represents the horizontal change in the graph. Similarly, the constant change in the dependent variable represents the vertical change in the graph.

Students have experience graphing linear relations from Grade 8. They will now create a table of values and use ordered pairs to graph linear relations. The ‘slope  $y$ -intercept method’ is not introduced until Grade 10.

When graphing linear relations, students will be expected to distinguish between discrete and continuous data. Discrete data is data which can be counted so it does not contain fractions. When graphing data points that represent discrete data, points are not connected or are connected with a dashed line. A dashed line is used when the discrete data have values between the plotted points that are valid. If there are no valid values between the plotted points, then no line is drawn. Continuous data has an infinite number of values between data points. It makes sense to have fractions. When graphing points that represent continuous data, points are connected with a solid line.

Contextual situations such as the following should make this idea more concrete for students:

- Case 1: No line drawn



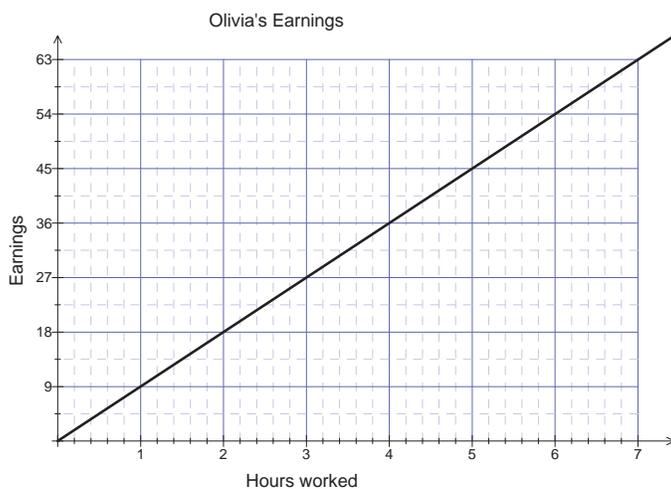
This graph has discrete data because it is not possible to have a fraction of a person. Since there are no valid data points between the plotted points, the points are not connected.

**General Outcome: Use patterns to describe the world and solve problems.****Suggested Assessment Strategies***Paper and Pencil*

- Ask students to graph a variety of linear relations using a table of values. Ask them to note any patterns in the table of values and to graph the relation.

(9PR2.1)

- Olivia works part time at a grocery store. Ask students to use the graph below to describe the pattern and explain what it represents.



(9PR2.1)

*Journal*

- Ask students to respond to the following:
  - You are given a linear equation. Describe the process you would follow to represent the equation on a graph. Use an example to support your answer.

(9PR2.2)

- Use examples and diagrams to help explain how horizontal and vertical lines and their equations are similar and how they are different.

(9PR2.2)

**Resources/Notes****Authorized Resource***Math Makes Sense 9***Lesson 4.2: Linear Relations****Lesson 4.3: Another Form of the Equation for a Linear Relation**

ProGuide: pp. 16-25, 26-32

Master 4.7, 4.8

CD-ROM: Master 4.21, 4.22

SB: pp. 164-173, 174-181

PB: pp. 149-156, 157-165

**Try It Virtual Manipulatives**

*This program allows you to create a table of values and graph a linear relation.*

**Strand: Patterns and Relations (Patterns)**

**Outcomes**

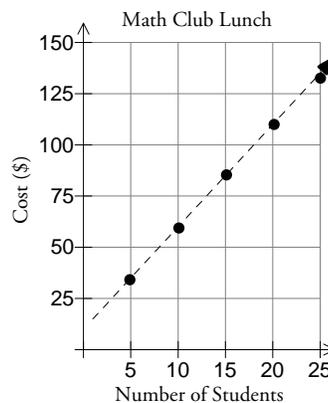
*Students will be expected to*  
 9PR2 Continued...

**Achievement Indicators:**

9PR2.1, 9PR2.2 *Continued*

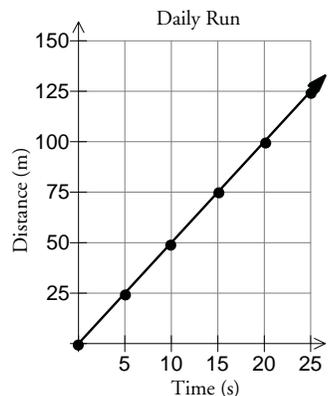
**Elaborations—Strategies for Learning and Teaching**

- Case 2: Dashed line is drawn



This graph also contains discrete data. Since there are some valid data points between the plotted points, the points are connected with a dashed line. Lunch for three students would cost \$25, for example, so the point (3, 25) is a valid point.

- Case 3: Solid line is drawn



On this graph the data is continuous because it makes sense to have fractional time. The points, therefore, are connected with a solid line.

Ask students to think about other situations involving discrete and continuous data. Examples for discrete data may include situations involving number of people, number of DVDs, number of pizza toppings, number of concert tickets, etc. Examples of continuous data may include situations involving temperatures that occur over time, height or weight over age, distance over time, etc. The decision about whether or not to join points on a graph is necessary only in contextual situations. If students are graphing a linear relation from a given equation without context, points are connected.

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**General Outcome: Use patterns to describe the world and solve problems.**


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**Suggested Assessment Strategies**
*Paper and Pencil*

- Ask students to create a table of values and a graph for the following linear equations:
  - (i)  $x = 4$
  - (ii)  $4x + y = 5$
  - (iii)  $y = 1$

(9PR2.2)
  
- Ask students to answer the following:  
 A common price for downloading apps is \$0.99.
  - (i) Write an equation that relates the total cost of buying apps,  $C$ , to the number of downloads,  $d$ .
  - (ii) Graph the equation. Should you connect the points on the graph? Explain.
  - (iii) What is the total cost for 100 downloads?
  - (iv) You have saved \$24.75. How many apps can you download?

(9PR1.2, 9PR1.4, 9PR2.2)

**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 9*
**Lesson 4.2: Linear Relations**
**Lesson 4.3: Another Form of the Equation for a Linear Relation**

ProGuide: pp. 16-25, 26-32

Master 4.7, 4.8

CD-ROM: Master 4.21, 4.22

SB: pp. 164-173, 174-181

PB: pp. 149-156, 157-165

**Try It Virtual Manipulatives**

*This program allows you to create a table of values and graph a linear relation.*

**Strand: Patterns and Relations (Patterns)**

**Outcomes**

*Students will be expected to*  
 9PR2 Continued...

**Achievement Indicators:**

9PR2.1, 9PR2.2 *Continued*

9PR2.3 *Match given equations of linear relations with their corresponding graphs.*

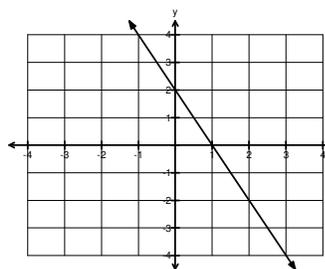
**Elaborations – Strategies for Learning and Teaching**

Students should graph horizontal, vertical and oblique lines. Oblique, or slanted, lines are neither perpendicular nor parallel to the  $x$  or  $y$ -axis. This is a new term for students, but they have previous experience with graphing oblique lines using the table of values method. The equations of horizontal and vertical lines contain only one of the variables. As a result,  $x$  or  $y$  is always constant. This results in a line that will be either perpendicular to the  $x$ -axis ( $x = a$ ) or perpendicular to the  $y$ -axis ( $y = a$ ).

When graphing oblique lines, students can substitute values of  $x$  into the equation, and use prior knowledge to solve the equation for  $y$ . At this point, they are not expected to rearrange equations when creating tables of values. They will work with more complex equations later in this course in the Linear Equations and Inequalities unit. For now, students can avoid having to solve equations with rational numbers by selecting convenient numbers to substitute for  $x$ .

Students should recognize that the graph, table of values and ordered pairs show a relationship between two variables. To match graphs with their corresponding equations, selected ordered pairs from the graph can be tested to see if they satisfy the given equation. Students should be encouraged to select at least two points to verify, as they can incorrectly match graphs when just one point satisfies the equation. For example, if students choose the point  $(0, 2)$  when matching the graph below with the correct equation, they may make an incorrect match. In this case, the ordered pair  $(0, 2)$  satisfies both equations. Testing a second ordered pair will ensure a correct match.

*Which equation matches the graph below?  $x + y = 2$  or  $2x + y = 2$  ?*



Other methods to match the equation with the graph, such as comparing the graph's slope and  $y$ -intercept to the equation, will **not** be explored until Grade 10.

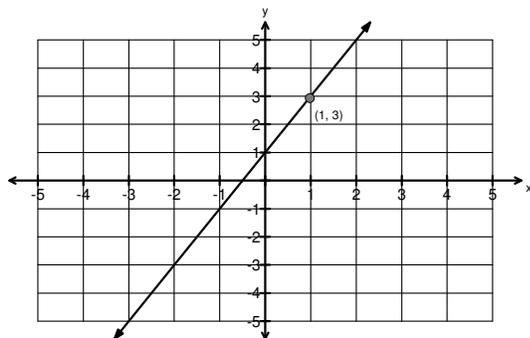
**General Outcome: Use patterns to describe the world and solve problems.**

**Suggested Assessment Strategies**

*Paper and Pencil*

- Ask students to answer the following:

June stated that the equation for the graph below is  $x + y = 4$ , since the point (1,3) satisfies the equation. Is she correct? Justify your answer.



(9PR2.3)

*Performance*

- Students can work in pairs to complete the following puzzle investigating the characteristics and graphs of various linear functions. They should work with 20 puzzle pieces (4 complete puzzles consisting of a function and four related characteristics) to correctly match the characteristics with each function. A sample is shown below.

As the $x$ -value increases by 1, the $y$ -value decreases by 2	The point (10, -19) satisfies the equation												
$2x + y = 1$													
<table style="margin-left: auto; margin-right: auto;"> <thead> <tr> <th style="border-right: 1px solid black; padding: 2px 5px;"><math>x</math></th> <th style="padding: 2px 5px;"><math>y</math></th> </tr> </thead> <tbody> <tr><td style="border-right: 1px solid black; padding: 2px 5px;">-2</td><td style="padding: 2px 5px;">5</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 5px;">-1</td><td style="padding: 2px 5px;">3</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 5px;">0</td><td style="padding: 2px 5px;">1</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 5px;">1</td><td style="padding: 2px 5px;">-1</td></tr> <tr><td style="border-right: 1px solid black; padding: 2px 5px;">2</td><td style="padding: 2px 5px;">-3</td></tr> </tbody> </table>	$x$	$y$	-2	5	-1	3	0	1	1	-1	2	-3	
$x$	$y$												
-2	5												
-1	3												
0	1												
1	-1												
2	-3												

(9PR2.1, 9PR2.2, 9PR2.3)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 9*

Lesson 4.2: Linear Relations

Lesson 4.3: Another Form of the Equation for a Linear Relation

ProGuide: pp. 16-25, 26-32

Master 4.7, 4.8

CD-ROM: Master 4.21, 4.22

SB: pp. 164-173, 174-181

PB: pp. 149-156, 157-165

Try It Virtual Manipulatives

*This program allows you to create a table of values and graph a linear relation.*

**Lesson 4.4: Matching Equations and Graphs**

ProGuide: pp. 35-42

Master 4.9, 4.9a

CD-ROM: Master 4.23

SB: pp.183-190

PB: pp. 166-169

**Suggested Resource**

<https://www.k12pl.nl.ca/curr/7-9/math/grade9/links/unit4.html>

- Quia - Graphs and Linear Equations is an activity that asks students to match linear equations and graphs.

**Strand: Patterns and Relations (Patterns)****Outcomes**

*Students will be expected to*

9PR2 Continued...

**Achievement Indicators:**

9PR2.4 *Interpolate the approximate value of one variable on a given graph given the value of the other variable.*

9PR2.5 *Extrapolate the approximate value of one variable from a given graph given the value of the other variable.*

9PR2.6 *Solve a given problem by graphing a linear relation and analyzing the graph.*

**Elaborations—Strategies for Learning and Teaching**

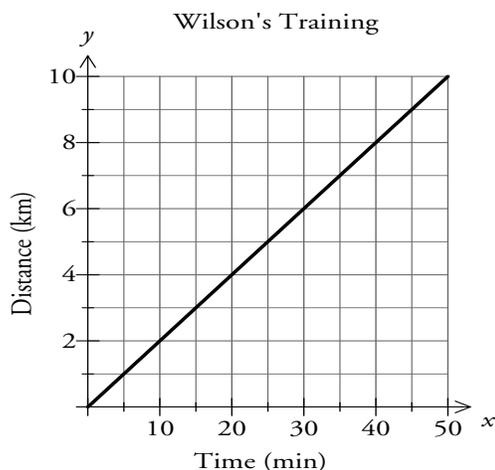
In previous grades, students made predictions for unknown quantities by “eyeballing” from the graph (7PR2) and using the equation (8PR1). The terms extrapolation and interpolation were not formally introduced. Students are now expected to make predictions by extending their graph. The focus here will be on interpreting the data and making predictions for unknown values. Interpolation is the prediction of a value between two known values. It is important for students to realize that when graphs display discrete data, interpolation is inappropriate because there are no data points between the known data points. Extrapolation is the prediction of a value which goes beyond the data that is given. Generally, students are less comfortable with extrapolation than with interpolation. There is opportunity here for students to work with real-life applications. By extending the graph, assumptions are being made that the pattern will continue. Students need to be aware that this is not always applicable in contextual situations. As students make inferences from a graph, it is important that they justify their interpolations and extrapolations.

**General Outcome: Use patterns to describe the world and solve problems.**

**Suggested Assessment Strategies**

*Paper and Pencil*

- Ask students to complete the following:  
Wilson is training for a 10 km race. The graph shows his times and distances at 10 minute intervals.



- (i) Determine how long it takes Wilson to run 3 km.
- (ii) Determine how far he can run in 45 minutes.
- (iii) Determine how fast he is running.
- (iv) Can you use this graph to predict how far he will run in 200 minutes? Why or why not?

(9PR2.4, 9PR2.5, 9PR2.6)

*Performance*

- Students can work in pairs to explore the relationship between the height of the top of a metre stick to the distance between the bottom of the stick and the wall. To begin, they stand a metre stick upright against a wall and record the measurements. They should then move the bottom end 10 cm away from the wall and measure the height of the top of the metre stick. This process continues until the metre stick is lying on the floor. Ask students to:

- (i) Record data in a table of values:

Distance between bottom of stick and wall (cm)	0	10	20	30	40	50	60	70	80	90	100
Height of top of stick (cm)											

- (ii) Create a graph.
- (iii) Analyze the graph and describe any relationships that exist.
- (iv) Write an equation for the relationship.
- (v) Use interpolation or extrapolation to make predictions for the given data.

(9PR1.2, 9PR2.1, 9PR2.2, 9PR2.4, 9PR2.5)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 9*

**Lesson 4.5: Using Graphs to Estimate Values**

ProGuide: pp. 43-51

CD-ROM: Master 4.24

SB: pp. 191-199

PB: pp. 170-172

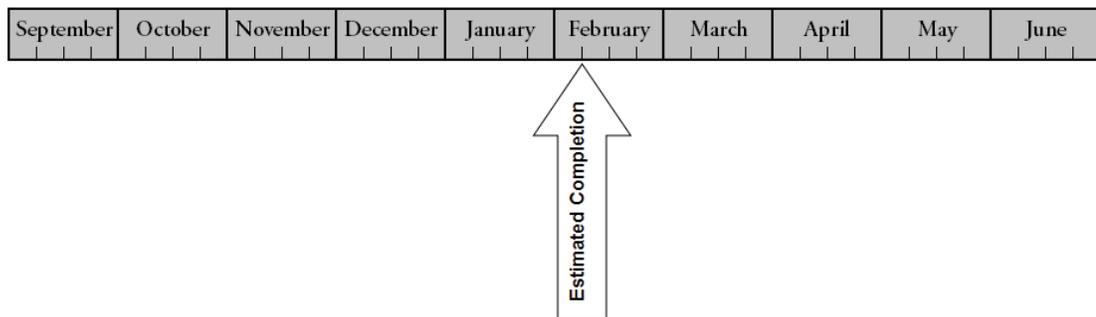
Try It Virtual Manipulatives

*This program allows you to create a graph, then interpolate and extrapolate values.*



# Polynomials

Suggested Time: 3 Weeks



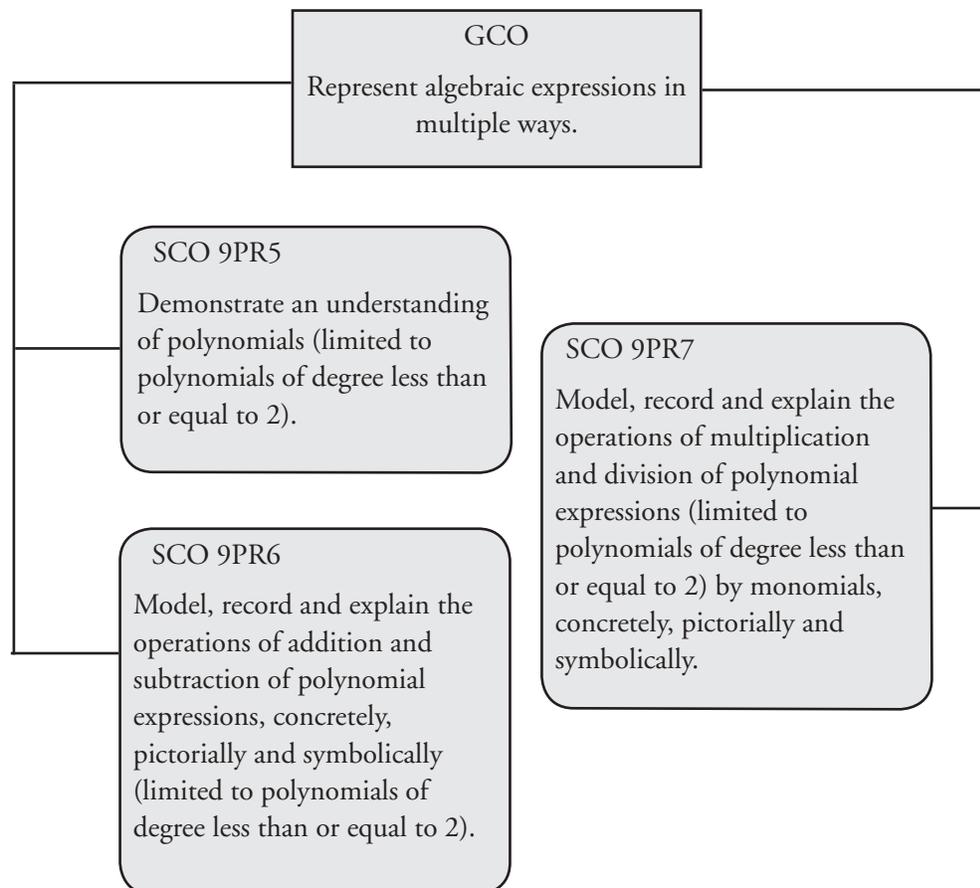
# Unit Overview

## Focus and Context

In this unit, students will explore polynomials of degree less than or equal to 2. They will become familiar with the language of polynomials and use concrete models such as algebra tiles to symbolically show their understanding of polynomials and to prepare them to work with pictorial and symbolic representations. Students will model, record and explain the operations of addition, subtraction, multiplication and division, concretely, pictorially and symbolically. Students will also write polynomials in simplified form. Perimeter and area will be used as applications of operations using polynomials.

Polynomials appear in a wide variety of areas of mathematics and science. They are used in problem solving from elementary word problems to multi-step problems in chemistry, physics, economics and the social sciences. They are also used in calculus, numerical analysis, abstract algebra and algebraic geometry.

## Outcomes Framework



## SCO Continuum

Mathematics 8	Mathematics 9	Mathematics Level I	
		1201	1202
Patterns and Relations			
<p>8N7. Demonstrate an understanding of multiplication and division of integers, concretely, pictorially and symbolically. [C, CN, PS, R, V]</p> <p>8PR2. Model and solve problems using linear equations of the form:</p> <ul style="list-style-type: none"> <li>• <math>ax = b</math></li> <li>• <math>\frac{x}{a} = b, a \neq 0</math></li> <li>• <math>ax + b = c</math></li> <li>• <math>\frac{x}{a} + b = c, a \neq 0</math></li> <li>• <math>a(x + b) = c</math></li> </ul> <p>concretely, pictorially and symbolically, where <math>a, b</math> and <math>c</math> are integers. [C, CN, PS, V]</p>	<p>9PR5. Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). [C, CN, R, V]</p> <p>9PR6. Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2). [C, CN, PS, R, V]</p> <p>9PR7. Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically. [C, CN, R, V]</p>	<p>AN4. Demonstrate an understanding of the multiplication of polynomial expressions (limited to monomials, binomials and trinomials), concretely, pictorially and symbolically. [CN, R, V]</p> <p>AN5. Demonstrate an understanding of common factors and trinomial factoring, concretely, pictorially and symbolically. [C, CN, R, V]</p>	not addressed

## Mathematical Processes

[C] Communication  
[CN] Connections  
[ME] Mental Mathematics and Estimation

[PS] Problem Solving  
[R] Reasoning  
[T] Technology  
[V] Visualization

## Strand: Patterns and Relations (Variables and Equations)

### Outcomes

*Students will be expected to*

**9PR5 Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2).**

[C, CN, R, V]

#### Achievement Indicators:

9PR5.1 *Identify the variables, degree, number of terms and coefficients, including the constant term, of a given simplified polynomial expression.*

9PR5.2 *Create a concrete model or a pictorial representation for a given polynomial expression.*

9PR5.3 *Write the expression for a given model of a polynomial.*

9PR5.4 *Match equivalent polynomial expressions given in simplified form.*

### Elaborations—Strategies for Learning and Teaching

Students are introduced to polynomials in this unit. They have been informally exposed to the language of algebra through their previous work with solving equations. Specific emphasis on the language of polynomials, however, is new.

Students have had some exposure to the language of polynomials through their work with equations. To increase their familiarity and comfort with the language, the use of proper terminology should permeate mathematics classes.

In discussing what polynomials are, it is important to provide examples of expressions that are not polynomials (e.g.,  $\sqrt{x}$  or  $\frac{5}{n}$ ). Students should also be introduced to different types of polynomials, and be able to distinguish between monomials, binomials and trinomials. It should be noted that  $1x^2$  is often written as  $x^2$  and has a coefficient of +1 whereas  $-x^2$  has a coefficient of -1. In discussing degree, it can be noted that terms with more than one variable have a degree equal to the sum of the exponents of the variables. The degree of  $3xy$ , for example, is 2. This is important for future work with polynomials.

Students should model polynomials using algebra tiles. Concrete models provide the necessary support to model polynomials symbolically. Students have used algebra tiles in previous grades. In Grade 7, the  $x$ -tiles and unit tiles were used when solving single variable linear equations involving integers (7PR6). They should now be introduced to the  $x^2$ -tile. Note that the variable used is not restricted to  $x$ , but could be any symbol. Using the colour tiles you have available, decide which colour will represent positive and which will represent negative. Throughout this curriculum guide, shaded tiles represent positive values and white tiles represent negative values. It is very important that students are aware that the  $x^2$ -tile and the  $x$ -tile represent unlike terms and, therefore, cannot be combined. This will be the first time students are exposed to like and unlike terms.

Provide students with a variety of polynomial expressions and ask them to model them using the appropriate tiles. Examples should include combinations of both positive and negative terms. They should sketch a diagram which represents the polynomial. Students should also represent a polynomial expression symbolically from a concrete or pictorial representation.

Students should realize that although polynomials are usually written in descending order, equivalent polynomials can be written by rearranging the terms. Stress that the signs of the terms have to remain the same (e.g.,  $4x - 3x^2 + 2$  is equivalent to  $-3x^2 + 4x + 2$ ).

## General Outcome: Represent algebraic expressions in multiple ways.

### Suggested Assessment Strategies

#### Paper and Pencil

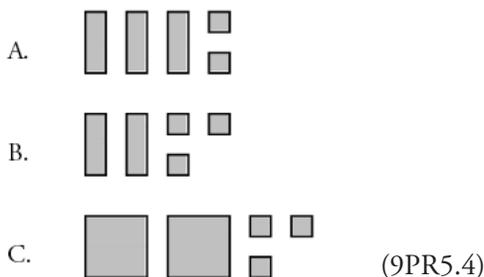
- Ask students to make a model of an algebraic expression that includes at least one  $x^2$ -tile, at least 3  $x$ -tiles, and two 1-tiles, and record the expression with a diagram and symbols. Ask students to identify the type of polynomial. (9PR5.1, 9PR5.2, 9PR5.3)

- Ask students to match the following polynomials to the appropriate diagram (shaded represents positive).

(i)  $2x^2 + 3$

(ii)  $3x + 2$

(iii)  $2x + 3$



#### Journal

- Sam rearranged the polynomial  $2x - 4 + 6x^2$  as  $6x^2 - 2x + 4$ . Ask students if Sam was correct. They should justify their answers using words, diagrams, pictures, models, etc. (9PR5.4)

#### Performance

- Students could play “I Have ... Who Has” to reinforce the language of polynomials. Provide them with a loop card as shown. A student starts and reads the “Who Has ...” part of the card aloud. A student will respond with “I have ...” answering with the correct polynomial. The student continues to read “Who Has ...”. This continues until all students have read their cards.

<i>I have ...</i> $2x^2 - x + 3$	<i>I have ...</i> $-3x^2 + 4x - 5$	<i>I have ...</i> $-3a^2 - 6a - 5$
<i>Who has ...</i> A polynomial with a constant term of -5	<i>Who has ...</i> A polynomial with all negative coefficients	<i>Who has ...</i> A polynomial with an $x$ coefficient of -1

(9PR5.1)

### Resources/Notes

#### Authorized Resource

##### *Math Makes Sense 9*

Prep Talk Video: Polynomials

Try It Virtual Manipulatives

*This program allows you to model polynomials with virtual algebra tiles.*

#### Lesson 5.1: Modelling Polynomials

ProGuide: pp. 4-10

Master 5.8

CD-ROM: Master 5.19

Student Book (SB): pp.210-216

Preparation and Practice Book (PB): pp. 180-185

#### Suggested Resource

<https://www.k12pl.nl.ca/curr/7-9/math/grade9/links/unit5.html>

- Templates for generating loop cards

## Strand: Patterns and Relations (Variables and Equations)

### Outcomes

*Students will be expected to*

**9PR6 Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2).**

[C, CN, PS, R, V]

### Achievement Indicators:

9PR6.1 *Identify equivalent polynomial expressions from a given set of polynomial expressions, including pictorial and symbolic representations.*

9PR6.2 *Model addition of two given polynomial expressions concretely or pictorially and record the process symbolically.*

### Elaborations—Strategies for Learning and Teaching

In previous grades, students used integer tiles or two-coloured counters to model addition and subtraction of integers (7N6). They will now extend this to modelling polynomials with algebra tiles. Working with concrete representations of polynomials will better prepare students to work with pictorial and symbolic representations. They will apply their understanding of addition and subtraction of integers to these operations with polynomial expressions.

The previous outcome (9PR5) exposed students to equivalent polynomials, already presented in simplified form. To begin addition and subtraction, they will simplify polynomials by combining like terms concretely, pictorially and symbolically. To simplify polynomials students will use tools such as algebra tiles, area models, drawings or sketches, and algebraic symbols. They should be encouraged to record the process symbolically whenever they use concrete materials or drawings and sketches. Simplifying polynomials should always be stressed. This reinforces the idea that the same polynomial can be written in many ways. Writing polynomials in descending order also makes them easier to compare.

Students should, with the aid of algebra tiles, be able to add and subtract to simplify expressions. They should know which terms can and cannot be combined. Students tend to grasp this notion readily with the use of algebra tiles. They should make the connection between combining like terms and combining polynomials. Students should be exposed to addition both horizontally and vertically.

Perimeter is a very useful application of addition and subtraction of polynomials. Students should recognize that, because it has linear units, perimeter can be represented by first degree polynomials.

## General Outcome: Represent algebraic expressions in multiple ways.

### Suggested Assessment Strategies

#### Observation

- Present students with models such as the following, and have them choose which ones result in the same simplified polynomial (shaded represents positive).

(i) 

(ii)   


(iii)   


(9PR6.1)

#### Paper and Pencil

- Ask students to complete the magic square. Rows, columns, and diagonals must have the same sum.

		$2x^2 - 3x + 24$
	$5x^2 - 4x + 15$	$9x^2 - 2x + 3$
$8x^2 - 5x + 6$		

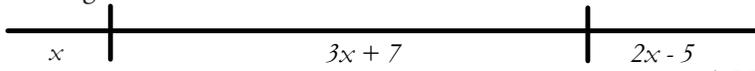
(9PR6.2, 9PR6.3)

- A rectangular flower garden has a length of  $8x$  cinder blocks and a width of  $9x$  bricks. Ask students to find a simplified expression for the perimeter of the flower garden. (9PR6.2)
- Ask students to write a simplified expression for the perimeter of the rectangle.



(9PR6.2)

- Ask students to write a simplified expression for the length of the string.



(9PR6.2)

### Resources/Notes

#### Authorized Resource

##### *Math Makes Sense 9*

##### Lesson 5.2: Like and Unlike Terms

ProGuide: pp. 11-18

Master 5.7a, 5.7b, 5.12

CD-ROM: Master 5.20

See It Videos and Animations:  
Like Terms and Unlike Terms

SB: pp. 217-224

PB: pp. 186-191

##### Lesson 5.3: Adding Polynomials

ProGuide: pp. 19-24

Master 5.9a, 5.9b, 5.9c

CD-ROM: Master 5.21

Classroom Video: Adding  
Polynomials, Part 1

Classroom Video: Adding  
Polynomials, Part 2

See It Videos and Animations:  
Adding Polynomials

SB: pp. 225-230

PB: pp. 192-197

## Strand: Patterns and Relations (Variables and Equations)

### Outcomes

*Students will be expected to*

9PR6 Continued...

#### Achievement Indicators:

9PR6.3 *Model subtraction of two given polynomial expressions concretely or pictorially and record the process symbolically.*

9PR6.4 *Apply a personal strategy for addition and subtraction of given polynomial expressions, and record the process symbolically.*

9PR6.5 *Identify the error(s) in a given simplification of a given polynomial expression.*

### Elaborations—Strategies for Learning and Teaching

While addition of polynomials is often straightforward, subtraction sometimes poses difficulty for students. Consideration should be given to the different representations of subtraction, including the following:

- comparison → comparing and finding the difference between two quantities
- taking away → starting with a quantity and removing a specified amount
- adding the opposite → subtracting by first changing the question to an addition and then adding the opposite of a quantity. For example, instead of subtracting  $2x - 1$ , one might add  $-(2x - 1)$  or  $-2x + 1$ . Students should model  $2x - 1$  and understand that the opposite is found by flipping the tiles.
- finding the missing addend → which asks the question, “What would be added to the number being subtracted to get the starting amount?”

All four of these meanings for subtraction have been developed in previous grades in the context of number.

Teachers may need to revisit the concept of zero pairs. Students have had exposure to this concept in Grade 7 (7N6) and Grade 8 (8N7).

When subtracting polynomials symbolically, students can apply the properties of integers. They should be cautioned about the use of brackets. A common student error is subtracting only the first term of the second polynomial.

Students should be encouraged to use algebra tiles, to use integer properties, and to use the horizontal and/or vertical methods to add and subtract polynomials. They may also want to use a combination of any of these methods.

It is beneficial to have students analyze solutions that contain errors. Along with providing the correct solutions, they should be able to identify incorrect solutions, including why errors might have occurred and how they can be corrected. This reinforces the importance of recording solution steps rather than only giving a final answer.

## General Outcome: Represent algebraic expressions in multiple ways.

### Suggested Assessment Strategies

#### *Paper and Pencil*

- Ask students to use at least two different methods to simplify:
  - (i)  $(3x^2 - 4x + 5) + (-2x^2 + x - 6)$
  - (ii)  $(5x^2 - 3x + 2) - (7x^2 - 2x + 5)$
 Ask them to indicate which method they prefer and why. (9PR6.4)
- The difference of two polynomials is  $-2x^2 - 4x + 3$ . Ask students to list three pairs of second degree polynomials that might have been subtracted to get that difference. (9PR6.3, 9PR6.4)
- Ask students to answer the following:
  - (i) Wayne was asked to write an expression equivalent to  $2x - 7 - 4x + 8$ . His solution was:
 
$$2x - 4x - 7 + 8 = 2x - 1$$
 Identify the errors and show the correct simplification. (9PR6.5)
  - (ii) Jennifer subtracted a polynomial from  $3x^2 - x - 1$ . The difference required eight algebra tiles to represent it. What polynomial could she have subtracted? (9PR6.3, 9PR6.4)

#### *Performance*

- Ask students to use algebra tiles to subtract the following polynomials:  $(2x^2 - 4x + 3) - (x^2 + x - 2)$ . (9PR6.3)

#### *Journal*

- Ask students to respond to the following:
  - (i) Use both the horizontal and vertical methods to simplify  $(3a^2 + 2) + (-4a^2 + 2a - 7)$ . Explain which method you prefer and why. (9PR6.2, 9PR6.4)
  - (ii) Your friend was absent today. Your homework requires you to subtract these two polynomials:  $(-3x + 5) - (4x - 3)$ . While chatting on the phone, explain to your friend how to find the solution. What would you say? Write a transcript of your conversation. (9PR6.3)

#### *Interview*

- Ask students how they would help Tim understand that his subtraction is incorrect:
 
$$(-2x^2 - 4x + 8) - (5x^2 + 8x - 2) = 3x^2 - 12x + 6.$$
 (9PR6.3, 9PR6.4)

### Resources/Notes

#### Authorized Resource

#### *Math Makes Sense 9*

Lesson 5.3: Adding Polynomials

Lesson 5.4: Subtracting Polynomials

ProGuide: pp. 19-24, 25-30

Master 5.13a, 5.13b, 5.14

CD-ROM: Master 5.22

See It Videos and Animations: Subtracting Polynomials

SB: pp. 225-230, 231-236

PB: pp. 192-197, 198-202

## Strand: Patterns and Relations (Variables and Equations)

### Outcomes

*Students will be expected to*

**9PR7 Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically.**

[C, CN, R, V]

### Achievement Indicator:

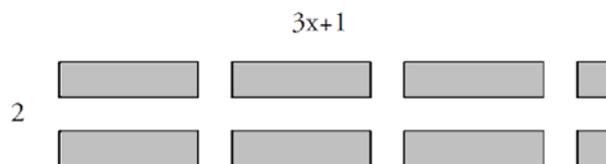
*9PR7.1 Model multiplication of a given polynomial expression by a given monomial concretely or pictorially and record the process symbolically.*

### Elaborations—Strategies for Learning and Teaching

When multiplying and dividing polynomial expressions, students should apply their understanding of concepts learned in previous grades, including multiplication and division of integers (8N7), the use of the distributive property (8PR2), and calculation of the area of rectangles (6SS3). They should be given the opportunity to connect these concepts with polynomials.

Students are expected to multiply a monomial by a monomial, a scalar by a polynomial, and a monomial by a polynomial. Multiplication of a polynomial by a scalar should be developed with concrete materials and diagrams, using repeated addition. Given  $2(3x + 1)$ , for example, students should recognize that it is the same as  $(3x + 1) + (3x + 1)$ , and therefore, model the binomial twice, combine the like terms and arrive at an answer.

The area model should also be explored so that students can relate results achieved through repeated addition with results achieved using the area model.



$$2(3x + 1) = 6x + 2$$

It would be useful to review the distributive property as well. In Grade 8, students applied the distributive property to solve linear equations (8PR2). For all numbers  $a$ ,  $b$  and  $c$ ,  $a(b + c)$  or  $(a + b)c = ac + bc$ .

This property can be demonstrated particularly well using algebra tiles.

When multiplying polynomial expressions, applications should be emphasized, especially in relation to area problems. Area can be represented by second degree polynomials because it has square units.

## General Outcome: Represent algebraic expressions in multiple ways.

### Suggested Assessment Strategies

#### Performance

- Ask students to model multiplication of polynomials such as the following using at least two different methods.

(i)  $2(4x^2 + 3x - 2)$

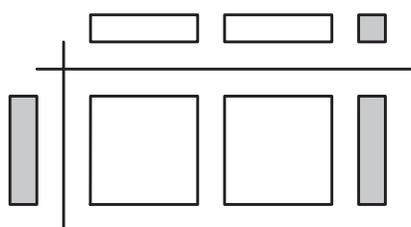
(ii)  $-3x(x - 4)$

(9PR7.1, 9PR7.3)

#### Paper and Pencil

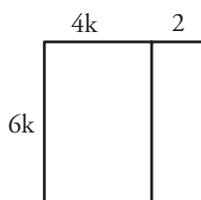
- Provide students with multiplication models and ask them to write a multiplication sentence for each model.

(i)



shaded → positive

(ii)



(9PR7.1)

#### Journal

- Ask students to respond to the following:  
Describe two methods you could use to multiply polynomials by monomials.

(9PR7.1, 9PR7.3)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 9*

Lesson 5.5: Multiplying and Dividing a Polynomial by a Constant

Lesson 5.6: Multiplying and Dividing a Polynomial by a Monomial

ProGuide: pp. 35-42, 43-51

Master 5.15, 5.16

CD-ROM: Master 5.23, 5.24

See It Videos and Animations:

Multiplying and Dividing a Polynomial by a Constant, Multiplying

Multiplying and Dividing a Polynomial by a Monomial, Multiplying

SB: pp. 241-248, 249-257

PB: pp. 206-213, 214-219

## Strand: Patterns and Relations (Variables and Equations)

### Outcomes

Students will be expected to

9PR7 Continued...

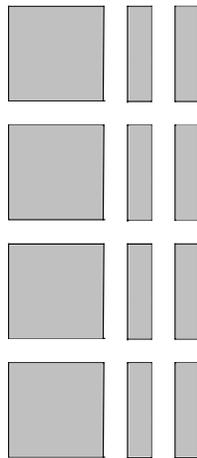
#### Achievement Indicator:

9PR7.2 Model division of a given polynomial expression by a given monomial concretely or pictorially and record the process symbolically.

### Elaborations—Strategies for Learning and Teaching

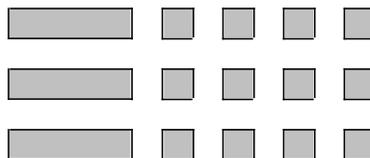
From previous work with number operations, students should be aware that division is the inverse of multiplication. This can be extended to divide polynomials by monomials. The study of division should begin with division of a monomial by a monomial, progress to a polynomial by a scalar, and then to division of a polynomial by any monomial.

Division of a polynomial by a monomial can be visualized using area models with algebra tiles. Students should be given situations where they have a specific collection of tiles and asked to create a rectangle with one dimension given. To model this, teachers could ask students to create a rectangle, using four  $x^2$ -tiles and eight  $x$ -tiles where  $4x$  is one of the dimensions.



From this model students should recognize  $x + 2$  as the other dimension.

The most commonly used symbolic method of dividing a polynomial by a monomial at this level is to divide each term of the polynomial by the monomial, and then use the exponent laws to simplify (e.g.,  $\frac{3x+12}{3} = \frac{3x}{3} + \frac{12}{3}$ ). This method can also be easily modelled using tiles, where students use the sharing model for division. They start with a collection of three  $x$ -tiles and 12 unit tiles and divide them into three groups.



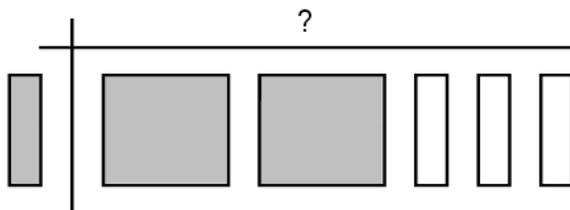
For this example,  $x + 4$  tiles will be a part of each group, so the quotient is  $x + 4$ .

## General Outcome: Represent algebraic expressions in multiple ways.

### Suggested Assessment Strategies

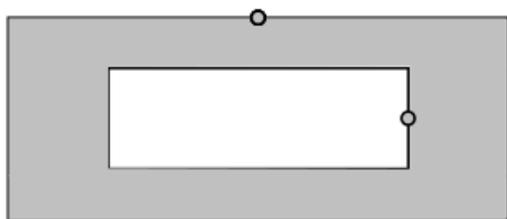
#### *Paper and Pencil*

- Ask students to write a division sentence to describe the following model and then to determine the quotient.  
(shaded represents positive)



(9PR7.2)

- Ask students to draw a rectangle with an area of  $36a^2 + 12a$  and list as many different dimensions as possible in two minutes. After the time is up, ask students to pass their list to another student. Then ask them, one at a time, to read an entry from the list in front of them. Everybody who has that entry on their list will cross it off. At the end, the list with the most remaining entries will be the winner. Ask students to explain why there are so many different solutions.  
(9PR7.2)
- The inside rectangle in the diagram below is a flower garden. The shaded area is a concrete walkway around it. The area of the flower garden is given by the expression  $x^2 + 5x$  and the area of the large rectangle, including the walkway and the flower garden, is  $2x^2 + 20x$ .



- Ask students to write an expression for the missing dimensions of each rectangle. Is there more than one possibility?
- Ask students to find the area of the walkway.  
(9PR7.2, 9PR7.3, 9PR6.3)

### Resources/Notes

#### Authorized Resource

##### *Math Makes Sense 9*

Lesson 5.5: Multiplying and Dividing a Polynomial by a Constant

Lesson 5.6: Multiplying and Dividing a Polynomial by a Monomial

ProGuide: pp. 35-42, 43-51

CD-ROM: Master 5.23, 5.24

See It Videos and Animations:

Multiplying and Dividing a Polynomial by a Constant, Dividing

Multiplying and Dividing a Polynomial by a Monomial, Dividing

SB: pp. 241-248, 249-257

PB: pp. 206-213, 214-219

## Strand: Patterns and Relations (Variables and Equations)

### Outcomes

*Students will be expected to*

9PR7 Continued...

#### Achievement Indicators:

9PR7.3 *Apply a personal strategy for multiplication and division of a given polynomial expression by a given monomial.*

9PR7.4 *Provide examples of equivalent polynomial expressions.*

9PR7.5 *Identify the error(s) in a given simplification of a given polynomial expression.*

### Elaborations—Strategies for Learning and Teaching

Because there are a variety of methods available to multiply or divide a polynomial by a monomial, students should be given the opportunity to apply their own personal strategies. They should be encouraged to use algebra tiles, area models, rules of exponents, the distributive property and repeated addition, or a combination of any of these methods, to multiply or divide polynomials. Regardless of the method used, students should be encouraged to record their work symbolically. Understanding the different approaches helps students develop flexible thinking.

Students should be encouraged to simplify all polynomials. They should realize that it is often difficult to compare polynomials for equivalency until they are presented in simplified form. This is also an opportunity to highlight the value of expressing solutions in descending order.

Questions requiring error analysis can be effective tools to assess students' understanding of simplifying polynomial expressions because it requires a deeper understanding than simply "doing the problem". Analyzing errors is a good way to focus discussion on "How did you get that?" rather than being limited to "Is my answer right?" This reinforces the idea that the process is as important as the solution.

Provide students with a variety of multiplication and division problems, such as the one below, which are not properly simplified. Ask them to identify and circle the errors in the solutions and to write the correct solution.

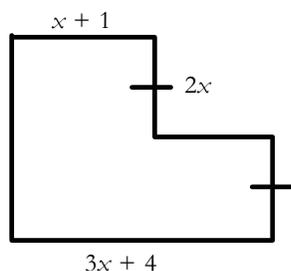
$$\begin{aligned}
 &(12x^2 - 4x) \div (-2x) \\
 &= \frac{12x^2}{-2x} - \frac{4x}{-2x} \\
 &= -6x - 2 \\
 &= -8x
 \end{aligned}$$

## General Outcome: Represent algebraic expressions in multiple ways.

### Suggested Assessment Strategies

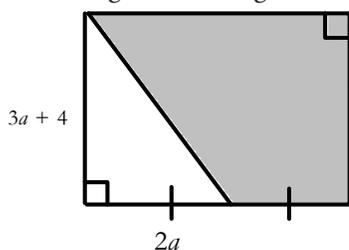
#### Paper and Pencil

- Ask students to write a simplified expression for the area of this figure.



(9PR6.4, 9PR7.3)

- Ask students to write a simplified expression for the area of the shaded region in the figure below:



(9PR6.4, 9PR7.3)

#### Performance

- Students could work in groups to play the *Domino Game*. Provide each group with 10 domino cards. One side of the card should contain a polynomial expression, while the other side contains a simplification of the polynomial expression. The object is to lay the dominos out such that the simplification of the polynomial expression on one card will match with the correct polynomial expression on another. They will eventually form a complete loop with the first card matching with the last card. A sample is shown below:

$3(2x+5)$	$4x^2 + 20x$
-----------	--------------

$4x(x+5)$	$-6x+8$
-----------	---------

$\frac{18x^2 - 24x}{-3x}$	$5x(3-x)$
---------------------------	-----------

(9PR7.4)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 9*

Lesson 5.5: Multiplying and Dividing a Polynomial by a Constant

Lesson 5.6: Multiplying and Dividing a Polynomial by a Constant

ProGuide: pp. 35-42, 43-51

CD-ROM: Master 5.23, 5.24

SB: pp. 241-248, 249-257

PB: pp. 206-213, 214-219

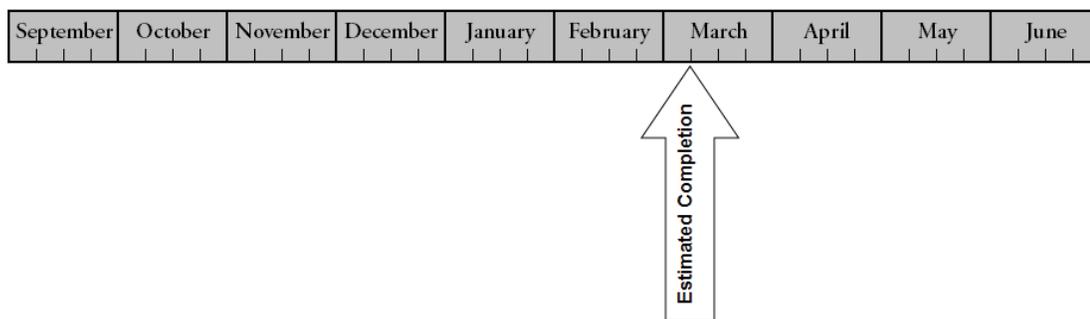
#### Suggested Resource

“Using Error Analysis to Teach Equation Solving” *Mathematics Teaching in the Middle School* 12, 5 (December 2006/January 2007), pp. 238-242



# Linear Equations and Inequalities

Suggested Time: 4 Weeks



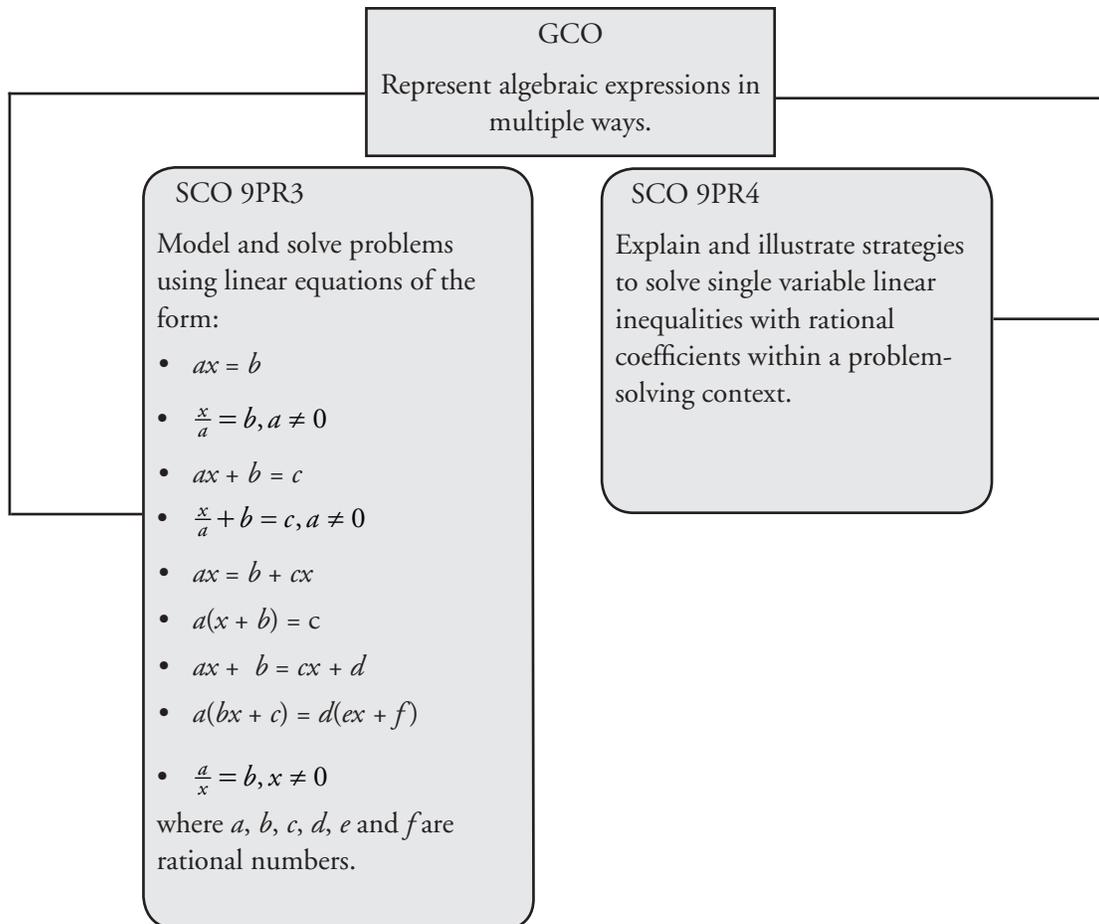
# Unit Overview

## Focus and Context

In this unit, students will model and solve problems using one-step and multi-step linear equations involving rational numbers, variables on both sides of the equation, and the distributive property. There should be a progression from the use of concrete materials to solving equations symbolically. In addition, students will verify solutions to show a better understanding of the processes involved. A student explanation and illustration of strategies to solve equations within a problem-solving context will also be expected. Finally, students will explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context and will graph the solutions.

Solving equations and inequalities and representing the solutions on number lines is an important prerequisite for problem solving and graphing in high school mathematics. The study of algebra assists with the development of logical thinking and problem solving. Algebraic skills are needed in occupations such as computer science, electronics, engineering, medicine and commerce.

## Outcomes Framework



## SCO Continuum

Mathematics 8	Mathematics 9	Mathematics Level I	
		1201	1202
Patterns and Relations			
<p>8PR2. Model and solve problems using linear equations of the form:</p> <ul style="list-style-type: none"> <li>• <math>ax = b</math></li> <li>• <math>\frac{x}{a} = b, a \neq 0</math></li> <li>• <math>ax + b = c</math></li> <li>• <math>\frac{a}{x} = b, x \neq 0</math></li> <li>• <math>a(x + b) = c</math></li> </ul> <p>concretely, pictorially and symbolically, where <math>a, b</math> and <math>c</math> are integers. [C, CN, PS, V]</p>	<p>9PR3. Model and solve problems using linear equations of the form:</p> <ul style="list-style-type: none"> <li>• <math>ax = b</math></li> <li>• <math>\frac{x}{a} = b, a \neq 0</math></li> <li>• <math>ax + b = c</math></li> <li>• <math>\frac{x}{a} + b = c, a \neq 0</math></li> <li>• <math>ax = b + cx</math></li> <li>• <math>a(x + b) = c</math></li> <li>• <math>ax + b = cx + d</math></li> <li>• <math>a(bx + c) = d(ex + f)</math></li> <li>• <math>\frac{a}{x} = b, x \neq 0</math></li> </ul> <p>where <math>a, b, c, d, e</math> and <math>f</math> are rational numbers. [C, CN, PS, V]</p> <p>9PR4. Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context. [C, CN, PS, R, V]</p>	<p>RF6. Relate linear relations expressed in:</p> <ul style="list-style-type: none"> <li>• slope–intercept form (<math>y = mx + b</math>)</li> <li>• general form (<math>Ax + By + C = 0</math>)</li> <li>• slope–point form (<math>y - y_1 = m(x - x_1)</math>)</li> </ul> <p>to their graphs. [CN, R, T, V]</p> <p>RF7. Determine the equation of a linear relation, given:</p> <ul style="list-style-type: none"> <li>• a graph</li> <li>• a point and the slope</li> <li>• two points</li> <li>• a point and the equation of a parallel or perpendicular line</li> </ul> <p>to solve problems. [CN, PS, R, V]</p> <p>RF9. Solve problems that involve systems of linear equations in two variables, graphically and algebraically. [CN, PS, R, T, V]</p>	not addressed

## Mathematical Processes

[C] Communication  
[CN] Connections  
[ME] Mental Mathematics and Estimation

[PS] Problem Solving  
[R] Reasoning  
[T] Technology  
[V] Visualization

**Strand: Patterns and Relations (Variables and Equations)**

**Outcomes**

*Students will be expected to*

**9PR3 Model and solve problems using linear equations of the form:**

- $ax = b$
- $\frac{x}{a} = b, a \neq 0$
- $ax + b = c$
- $\frac{x}{a} + b = c, a \neq 0$
- $ax = b + cx$
- $a(x + b) = c$
- $ax + b = cx + d$
- $a(bx + c) = d(ex + f)$
- $\frac{a}{x} = b, x \neq 0$

where  $a, b, c, d, e$  and  $f$  are rational numbers.

[C, CN, PS, V]

**Achievement Indicators:**

9PR3.1 *Model the solution of a given linear equation using concrete or pictorial representations, and record the process.*

9PR3.2 *Determine, by substitution, whether a given rational number is a solution to a given linear equation.*

9PR3.3 *Solve a given linear equation symbolically.*

**Elaborations—Strategies for Learning and Teaching**

In Grades 7 and 8, students solved one- and two-step equations including  $a(x + b) = c$ , where  $a, b$  and  $c$  are integers (7PR6, 8PR2). This unit builds on previous experience to include rational numbers and equations with variables on both sides.

Students have also worked extensively with concrete and pictorial representations. Instruction should start with concrete materials and pictorial models and then move to symbolic representation. Ultimately students should be able to solve equations without concrete or pictorial support.

Research has shown that the use of concrete models is critical in mathematics because most mathematical ideas are abstract. Students must move from the concrete to the pictorial to the symbolic, and part of instructional planning involves making informed decisions about where students are on the continuum of concrete to symbolic thinking. When solving equations, a balance scale is an appropriate model when the coefficients and constants are positive integers. Arrow diagrams can be used to solve equations with a variable on one side. Algebra tiles can be used to represent equations with any integer coefficient and constant. As students use a concrete model, they should also record the steps symbolically. Work with models leads to solving equations using inverse operations to gather like terms and balance the equation. Equations with rational numbers such as fractions or decimals cannot be solved easily using concrete models. Students, therefore, need to be able to solve equations symbolically. It is expected that students will initially show all the steps as they solve equations. As they develop these skills, they may be able to reduce the number of steps.

**General Outcome: Represent algebraic expressions in multiple ways.**

**Suggested Assessment Strategies**

*Journal*

- Ask students to respond to the following:  
What are the advantages and disadvantages of using concrete models to solve an equation? (9PR3.1)

*Performance*

- Quiz-Quiz-Trade: Each student is given a card with an equation. The answer is written on the back of the card. In groups of two, partner A asks the question and partner B answers. They switch roles and repeat. Students move around the classroom until every student has had a chance to solve all equations. Sample cards are shown below.

Solve: $6x = 2x - 8$	Solve: $\frac{1}{2} = 2 + 5x$	Solve: $-\frac{3}{x} = 1$
Solve: $-\frac{3}{2}(x + 2) = 7$	Solve: $3(x - 2) = -4(2x + 5)$	

(9PR3.3)

*Paper and Pencil*

- Ask students to determine the solution to the following equations:  
(i)  $\frac{-10}{x} = 2$   
(ii)  $-\frac{1}{2}(x + 3) = 8$   
They should verify their answers by substitution. (9PR3.2, 9PR3.3)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 9*

Prep Talk Video: Linear Equations and Inequalities

Try It Virtual Manipulatives

*This program allows you to solve equations using balance strategies.*

**Lesson 6.1: Solving Equations by Using Inverse Operations**

**Lesson 6.2: Solving Equations by Using Balance Strategies**

ProGuide: pp. 4-12, 13-21

Master 6.6a, 6.6b, 6.7a, 6.7b, 6.7c, 6.8a, 6.8b

CD-ROM: Master 6.18, 6.19

See It Videos and Animations:

Solving Equations by Using Balance Strategies, Balance-Scales Model and Algebra Tiles Model

Student Book (SB): pp.266-274, 275-283

Preparation and Practice Book (PB): pp. 228-232, 233-241

**Strand: Patterns and Relations (Variables and Equations)**

**Outcomes**

*Students will be expected to*

9PR3 Continued...

**Achievement Indicators:**

9PR3.1, 9PR3.2, 9PR3.3  
*Continued*

**Elaborations – Strategies for Learning and Teaching**

Students may use different strategies when solving equations involving fractions. When solving an equation such as  $\frac{x}{4} + \frac{1}{2} = \frac{x}{3}$ , some students may eliminate the denominators by multiplying each term by the lowest common denominator. As they proceed to solve the equation, ask them the following questions:

- What is the lowest common denominator of 4, 2 and 3?
- What would happen if the lowest common denominator was multiplied on both sides of the equation? Why is this mathematically correct?
- What is the simplified equation?
- What is the solution?

Another strategy that can be used centers around the idea of undoing the operations that are being done to the variable. When students are provided with an equation such as  $2x = 8$ , they divide both sides of the equation by 2 since the inverse of multiplication is division. Similarly, if students are provided with the equation  $\frac{x}{2} = 8$ , they multiply both sides of the equation by 2 since the inverse of division is multiplication.

When solving the previous equation  $\frac{x}{4} + \frac{1}{2} = \frac{x}{3}$ , students may decide to solve this equation using the “undo” process.

$$\frac{x}{4} + \frac{1}{2} = \frac{x}{3}$$

$$4\left(\frac{x}{4}\right) + 4\left(\frac{1}{2}\right) = 4\left(\frac{x}{3}\right)$$

$$x + 2 = \frac{4x}{3}$$

$$3(x) + 3(2) = 3\left(\frac{4x}{3}\right)$$

$$3x + 6 = 4x$$

$$x = 6$$

After solving several examples, discuss with students the connection between this method and the process of multiplying each term by the lowest common denominator.

Students should consider in advance what might be a reasonable solution. They should be reminded that once they acquire a solution, it can be checked for accuracy by substitution into the original equation. Always encourage students to verify solutions, as this will lead to a better understanding of the process involved.

**General Outcome: Represent algebraic expressions in multiple ways.**

**Suggested Assessment Strategies**

*Performance*

- Ask students to work in pairs to solve the following equations using algebra tiles. Students should take turns doing the following: Decide who will be the scribe and who will model the algebra tiles. The partner modelling with the tiles will tell the other person the steps to solve the equation. The scribe writes down the procedure algebraically.

(i)  $2a + 7 = 12$

(ii)  $\frac{1}{3} = 3 + 4x$

(iii)  $9 - 3c = 15$

(9PR3.1, 9PR3.3)

- Pass the Pen: Write a multi-step problem on the board and call on one student to come up and complete the first step. The student should explain how to complete this step to the class. The student then calls on another student to complete the next step and “passes the pen”. This continues until the problem is finished. When a question arises, the student holding the marker must answer the question, call on another student to help, or “pass the pen” to a different student.

<u>Written on board</u>	<u>Student explanation</u>
$2(x + 1) = x - 4(x - 2)$	Teacher presents problem
$2x + 2 = x - 4x + 8$ (cont')	Student 1 uses distributive property

(9PR3.3, 9PR3.6)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 9*

Prep Talk Video: Linear Equations and Inequalities

Try It Virtual Manipulatives

*This program allows you to solve equations using balance strategies.*

**Lesson 6.1: Solving Equations by Using Inverse Operations**

**Lesson 6.2: Solving Equations by Using Balance Strategies**

ProGuide: pp. 4-12, 13-21

Master 6.6a, 6.6b, 6.7a, 6.7b, 6.7c, 6.8a, 6.8b

CD-ROM: Master 6.18, 6.19

See It Videos and Animations:

Solving Equations by Using Balance Strategies, Balance-Scales Model and Algebra Tiles Model

SB: pp.266-274, 275-283

PB: pp. 228-232, 233-241

**Note**

Equations of the form  $\frac{a}{x} = b$ ,  $x \neq 0$  are limited in *Math Makes Sense 9*.

**Strand: Patterns and Relations (Variables and Equations)**

**Outcomes**

*Students will be expected to*  
**9PR3 Continued...**

**Achievement Indicators:**

*9PR3.4 Identify and correct an error in a given incorrect solution of a linear equation.*

*9PR3.5 Represent a given problem using a linear equation.*

*9PR3.6 Solve a given problem using a linear equation and record the process.*

**Elaborations—Strategies for Learning and Teaching**

Students should be provided with worked solutions of linear equations to verify. Along with providing the correct answers, they should identify any errors they find in solutions, and correct those errors. Common student errors include mistakes in using the distributive property, using sign rules incorrectly for multiplication and division, and errors in preserving equality. Error analysis reinforces the importance of verifying solutions and recording steps, rather than only producing the final answer.

To solve problems, it is necessary for students to make connections to previous work with linear equations. Students should be given the opportunity to solve problems with variables on both sides. Many problems can be solved using methods other than algebra, such as guess-and-check and systematic trial. It may, therefore, be necessary to specify the strategy to ensure that algebraic problem solving is being used. Students should be able to solve equations using rational numbers. Algebra can be used to solve problems that might otherwise be tedious to solve using methods such as guess-and-check. It would be manageable for students to solve a problem such as the following using guess-and-check:

John and Judy work part time. John earns \$10 per day plus \$6 per hour. Judy earns \$8 per hour. Determine how many hours they have to work to earn the same daily pay.

It is more tedious to use guess-and-check to solve a problem such as:

A cell phone company offers two different plans:

Plan A: monthly fee of \$45 and \$0.20 per minute

Plan B: monthly fee of \$28 and \$0.45 per minute

Determine how many minutes result in the same monthly cost for both plans.

In this case, students should realize that it is more efficient to solve the equation  $0.2x + 45 = 0.45x + 28$  to determine the number of minutes.

Problem solving also requires communication through the application of a four-stage process which was referenced in Grade 8:

- understand the problem by identifying given information
- make a plan to solve the problem
- carry out the plan and record the solution
- verify that the solution is correct for the information given in the problem

**General Outcome: Represent algebraic expressions in multiple ways.**

**Suggested Assessment Strategies**

*Journal*

- Ask students to answer the following:  
Your class had to solve the equation  $4(x - 2) = -3(2x + 6)$  on a recent math test. The question was worth 4 marks. Below are two student solutions. If you were the teacher, how many marks would you give each student? Justify your answer by indicating the mistake the student made. (9PR3.4)

<u>Devon's solution</u>	<u>Alison's solution</u>
$4(x - 2) = -3(2x + 6)$	$4(x - 2) = -3(2x + 6)$
$4x - 2 = -6x + 6$	$4x - 8 = -6x - 18$
$4x + 6x = 6 + 2$	$4x - 6x = -18 - 8$
$10x = 8$	$-2x = -26$
$\frac{10x}{10} = \frac{8}{10}$	$\frac{-2x}{-2} = \frac{-26}{-2}$
$x = \frac{8}{10}$ or $\frac{4}{5}$	$x = 13$

*Paper and Pencil*

- Ask students to complete the following:  
Two computer technicians both charge a fee for a home visit, plus an hourly rate for their work. Dawn charges a \$64.95 fee plus \$45 per hour. Brandon charges a \$79.95 fee plus \$40 per hour. For what length of service call do Dawn and Brandon charge the same amount? (9PR3.6)

*Performance*

- Create a scavenger hunt around the school using QR codes. The questions should involve solving linear equations. Place a variety of QR codes around the classroom. Each group will select a question, scan it and then write the steps of the solution. Provide each group with a scavenger hunt key where their solution informs them where to find the next code. The group is required to return all their solutions and workings to the teacher when they are finished with the activity.

*QR Code Scavenger Hunt Key #1*

If your answer is:	Go to:
7/5	Art Room
-3	Office
-1	Music Room
7/8	Bulletin Board by Library
-3/10	Library
4	Front Door of the Gym
10	Front Door of the Cafeteria



(9PR3.3)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 9*

Lesson 6.1: Solving Equations by Using Inverse Operations

Lesson 6.2: Solving Equations by Using Balance Strategies

ProGuide: pp. 4-12, 13-21

Master 6.6a, 6.6b, 6.7a, 6.7b, 6.7c, 6.8a, 6.8b

CD-ROM: Master 6.18, 6.19

SB: pp.266-274, 275-283

PB: pp. 228-232, 233-241

**Suggested Resource**

<https://www.k12pl.nl.ca/curr/7-9/math/grade9/links/unit6.html>

- Link to a barcode scanner for iPhone, iPad and iPod Touch

**Strand: Patterns and Relations (Variables and Equations)**

**Outcomes**

*Students will be expected to*

**9PR4** Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.

[C, CN, PS, R, V]

**Achievement Indicators:**

9PR4.1 *Translate a given problem into a single variable linear inequality using the symbols  $\geq$ ,  $>$ ,  $<$  or  $\leq$ .*

9PR4.2 *Determine if a given rational number is a possible solution of a given linear inequality.*

9PR4.3 *Graph the solution of a given linear inequality on a number line.*

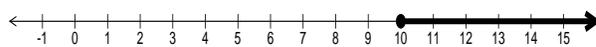
**Elaborations—Strategies for Learning and Teaching**

Students are familiar with the symbols  $<$  and  $>$  from their work with comparing two integers in Grade 6 (6N7). The symbols  $\leq$  and  $\geq$  will have to be introduced. So far in this unit students have worked with algebraic equations where the solution is a single number. Work with inequalities should help students understand what the answer represents; that is, a set of values rather than a single number. If a container can hold no more than 45 kg, for example, different masses can be put in the container as long as they are less than or equal to 45,  $x \leq 45$ . Students should also recognize that the same inequality can be written in two different ways. For example,  $x \leq 45$  and  $45 \geq x$  represent the same set of numbers.

Similarly, graphing inequalities on a number line results in graphing part of the line rather than one specific point. Since there are too many points to graph when you have to consider rational numbers, shading of the number line is necessary. Ensure that students understand the difference between  $<$  and  $\leq$  and the effect this would have on the graph.

Although the majority of the work with inequalities includes rational numbers, some applications will involve discrete data. It is necessary to discuss how this affects the graph. The concept of continuous and discrete data was discussed in the Linear Relations unit.

For example, the solution to the inequality  $x \geq 10$  results in the following graph:



An application of this same inequality can modify this graph.

*Chantal’s mom said she had to invite at least 10 people to the pool party.*

This would result in the following graph for  $x \geq 10$ :



---

**General Outcome: Represent algebraic expressions in multiple ways.**


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**Suggested Assessment Strategies**
*Paper and Pencil*

- Ask students to identify an inequality sign that best describes the following terms:  
*at least*                      *fewer than*  
*maximum*                      *must exceed*  
 Ask them to provide an example that helps explain their choice.  
 (9PR4.1)
- In many provinces, you must be at least 16 years of age to get a driver's licence.
  - (i) Ask students to represent this situation on a number line.
  - (ii) Ask students to represent the situation algebraically.  
 (9PR4.1, 9PR4.3)

*Journal*

- Ask students to respond to the following:  
 Tanya and Jackie have each written an inequality to represent numbers that are not more than 8. Their teacher says that both are correct. Explain why.  
 Tanya:  $8 \geq x$   
 Jackie:  $x \leq 8$   
 (9PR4.1)

**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 9*
**Lesson 6.3: Introduction to Linear Inequalities**

ProGuide: pp. 26-31

CD-ROM: Master 6.20

SB: pp. 288-293

PB: pp. 242-245

**Strand: Patterns and Relations (Variables and Equations)**

**Outcomes**

*Students will be expected to*  
 9PR4 Continued...

**Achievement Indicators:**

9PR4.4 *Generalize and apply a rule for adding or subtracting a positive or negative number to determine the solution of a given inequality.*

9PR4.5 *Generalize and apply a rule for multiplying or dividing by a positive or negative number to determine the solution of a given inequality.*

9PR4.6 *Solve a given linear inequality algebraically and explain the process orally or in written form.*

9PR4.7 *Compare and explain the process for solving a given linear equation to the process for solving a given linear inequality.*

9PR4.8 *Compare and explain the solution of a given linear equation to the solution of a given linear inequality.*

**Elaborations—Strategies for Learning and Teaching**

An understanding of how various operations affect the truth of an inequality should be developed before introducing variables. Students can start with true sentences such as  $-2 < 4$  and  $5 > 1$ . They can make a chart which shows each inequality and investigate how the truth of each is affected when the following operations are performed on both sides of the inequality:

- add a positive number
- subtract a positive number
- multiply by a positive number
- divide by a positive number
- add a negative number
- subtract a negative number
- multiply by a negative number
- divide by a negative number

Through investigation, students should recognize that adding or subtracting a number from both sides of the inequality has no effect on the truth of the inequality. Likewise, multiplying or dividing by a positive number results in a true inequality. Multiplying or dividing by a negative number, however, results in a false inequality. In this case, the inequality sign must be reversed to keep the truth of the inequality.

Once students have generalized these rules, they can apply them to solving inequalities. The process for solving equations is very similar to the process for solving inequalities. Both need to be balanced, through the use of inverse operations, to isolate the variable. When solving inequalities, however, emphasis should be placed on applying the generalized rule of reversing the sign when multiplying or dividing by a negative number. Provide students with ample practice to reinforce this concept.

As with equations, when solving inequalities the variable can be isolated on the left hand side or the right hand side. When students solved equations, they could easily see that this did not affect the meaning of their solution (i.e.,  $x = 3$  and  $3 = x$  are equivalent solutions). Students may be confused with the different ways of writing the solution for a linear inequality. Some work may need to be done so that students understand that the solution  $x > 3$  is the same as  $3 < x$ .

Students should note that the main difference in the solution of an equation as compared to an inequality is the value of the variable. A linear equation has only one value of the variable that makes it true. There may be many values of the variable that satisfy an inequality.

**General Outcome: Represent algebraic expressions in multiple ways.****Suggested Assessment Strategies***Performance*

- Students should work with a partner to complete the following activity:
  - (i) Each partner chooses a different number.
  - (ii) Decide who has the greatest number and write an inequality that compares both numbers.
  - (iii) Choose the same mathematical operation to perform on each number.
  - (iv) Decide whose resulting number is greater and record an inequality that compares these new numbers.
  - (v) Repeat this process with different mathematical operations.
  - (vi) Try different operations until you are able to predict which operations will reverse an inequality symbol and which ones will keep it the same.
  - (vii) Organize your observations and results.

(9PR4.4, 9PR4.5)

*Journal*

- Ask students to respond to the following:
  - (i) Jason says you can solve an inequality by replacing the inequality sign with an equal sign and putting it back in after solving the equation. Do you agree? Explain.

(9PR4.7)

- (ii) Explain why  $3n - 2 > 8$  and  $3n + 4 < 14$  do not have any solutions in common. Modify one of the inequalities so that they have exactly one solution in common.

(9PR4.6)

*Paper and Pencil*

- Ask students to verify whether  $\{-2, +3, +5, -1, +9, -9, -14\}$  are solutions to the inequality  $-2x - 5 > 7$ . Ask them to solve the inequality and graph the solution on a number line. They should check to determine how many of the numbers from the set above would be part of the graphical solution.

(9PR4.6, 9PR4.9)

- Ask students to explain how solving an inequality is similar to solving an equation. How is it different?

(9PR4.7)

**Resources/Notes****Authorized Resource***Math Makes Sense 9***Lesson 6.4: Solving Linear Inequalities by Using Addition and Subtraction****Lesson 6.5: Solving Linear Inequalities by Using Multiplication and Division**

ProGuide: pp. 32-37, 38-44

Master 6.9a, 6.9b, 6.9c

CD-ROM: Master 6.21, 6.22

See It Videos and Animations:  
Solving Linear Inequalities by  
Using Addition and SubtractionSolving Linear Inequalities by  
Using Multiplication and Division

SB: pp. 294-299, 300-306

PB: pp. 246-249, 250-254

**Strand: Patterns and Relations (Variables and Equations)****Outcomes**

*Students will be expected to*

9PR4 Continued...

**Achievement Indicators:**

9PR4.9 *Verify the solution of a given linear inequality using substitution for multiple elements in the solution.*

9PR4.10 *Solve a given problem involving a single variable linear inequality and graph the solution.*

**Elaborations—Strategies for Learning and Teaching**

As with equations, students should be aware that once they acquire a solution for an inequality, it can be checked for accuracy by substitution into the original inequality. To reinforce the concept that solutions to inequalities are sets of numbers, students should verify the solution by substituting multiple elements into the original inequality. Students are also required to apply the skills learned earlier in this unit to graph their solutions.

It is important that problem solving situations where students have to solve and graph inequalities are included. At this level, problems should include only situations that have an upper or lower limit. Students should be reminded to distinguish between discrete and continuous data and the effect these have on graphing solutions.

**General Outcome: Represent algebraic expressions in multiple ways.****Suggested Assessment Strategies***Journal*

- Wayne and Nancy are discussing the inequality  $2x > 10$ .  
Wayne says, “*The solution to the inequality is 6. When I substitute 6 for  $x$ , a true statement results*”.  
Nancy says, “*I agree that 6 is a solution, but it is not the whole solution*”.  
Ask students to explain what Nancy means. (9PR4.9)

*Paper and Pencil*

- Ask students to use the inequalities  $x > -1$  and  $x < 5$  to answer the following:
  - (i) Identify three possible values for  $x$  that satisfy both inequalities.
  - (ii) Identify a number that is a possible value for  $x$  in one but not in both inequalities.
  - (iii) How are possible values for inequalities involving  $>$  or  $<$  different than for inequalities involving  $\geq$  or  $\leq$ ? Give an example to support your answer. (9PR4.9)
- Ask students to answer the following:  
Christy downloads music from two online companies. Tunes4U charges \$1.50 per download plus a one-time membership fee of \$15. YRTunes charges \$2.25 per download with no membership fee.
  - (i) Write an expression for the cost to download  $n$  songs from Tunes4U.
  - (ii) Write an expression for the cost to download  $n$  songs from YRTunes.
  - (iii) Write and solve an inequality to determine when it costs more to download songs from Tunes4U than from YRTunes. Verify and graph the solution.
  - (iv) Which site should Christy use to download music? (9PR4.10)
- Ask students to answer the following for a regular octagon where each side measures  $x + 3$ .
  - (i) Write an expression for the perimeter.
  - (ii) The perimeter of this octagon must be less than 52 cm. What inequality would you solve to determine the possible perimeter?
  - (iii) Solve the inequality in part (ii). Verify and graph the solution.
  - (iv) What is the length of one side of the octagon? (9PR4.10)

**Resources/Notes****Authorized Resource***Math Makes Sense 9*

**Lesson 6.4: Solving Linear Inequalities by Using Addition and Subtraction**

**Lesson 6.5: Solving Linear Inequalities by Using Multiplication and Division**

ProGuide: pp. 32-37, 38-44

Master 6.9a, 6.9b, 6.9c

CD-ROM: Master 6.21, 6.22

See It Videos and Animations:  
Solving Linear Inequalities by Using Addition and Subtraction

Solving Linear Inequalities by Using Multiplication and Division

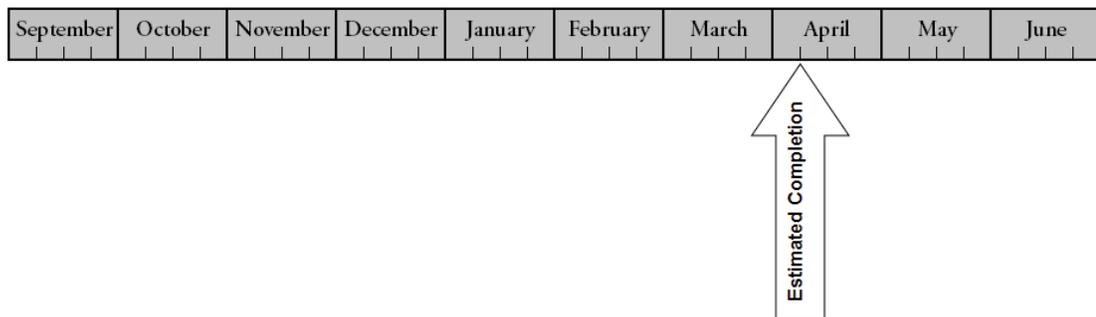
SB: pp. 294-299, 300-306

PB: pp. 246-249, 250-254



# Similarity and Transformations

Suggested Time: 4 Weeks



# Unit Overview

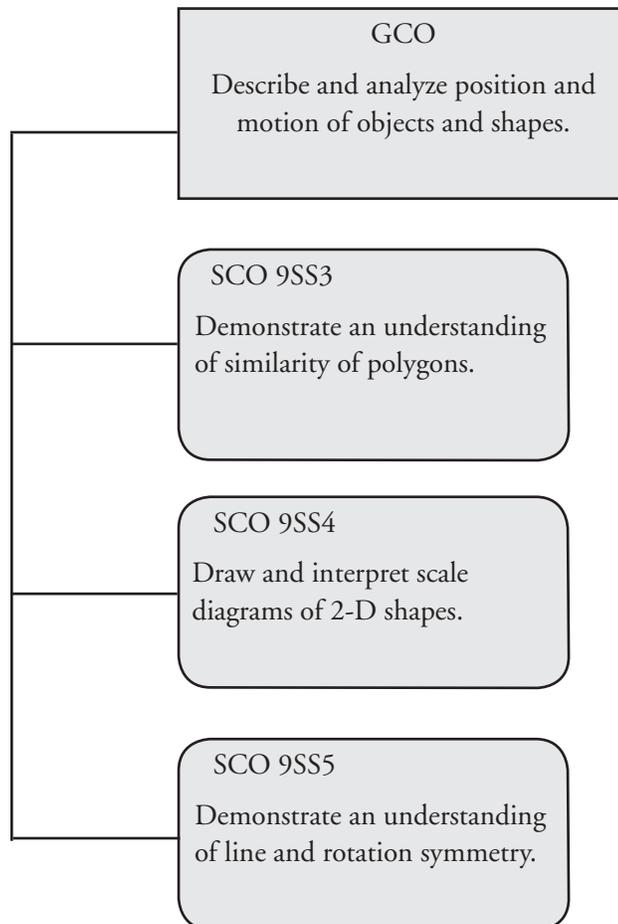
## Focus and Context

In this unit, students will draw two-dimensional diagrams to scale and determine the scale factor. They will identify and draw similar polygons, and solve problems using the properties of similar polygons.

In transformational geometry, students will complete translations, reflections, and rotations and record the resulting coordinates. Finally, they will consider figures that have line symmetry and rotational symmetry.

The concept of symmetry in mathematics can be connected to science. Line symmetry, for example, is similar to reflection or bilateral symmetry and rotational symmetry is the same as radial symmetry. In science, students find symmetry in humans, plants and the environment whereas in mathematics, it would be with polygons. The concept is the same, regardless of its application.

## Outcomes Framework



## SCO Continuum

Mathematics 8	Mathematics 9	Mathematics Level I	
		1201	1202
Patterns and Relations			
<p>8SS6. Demonstrate an understanding of tessellation by:</p> <ul style="list-style-type: none"> <li>explaining the properties of shapes that make tessellating possible</li> <li>creating tessellations</li> <li>identifying tessellations in the environment.</li> </ul> <p>[C, CN, PS, T, V]</p>	<p>9SS3. Demonstrate an understanding of similarity of polygons.</p> <p>[C, CN, PS, R, V]</p> <p>9SS4 Draw and interpret scale diagrams of 2-D shapes.</p> <p>[CN, R, T, V]</p> <p>9SS5 Demonstrate an understanding of line and rotation symmetry.</p> <p>[C, CN, PS, V]</p>	not addressed	<p>G3. Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons.</p> <p>[C, CN, PS, V]</p>

## Mathematical Processes

[C] Communication  
 [CN] Connections  
 [ME] Mental Mathematics and Estimation

[PS] Problem Solving  
 [R] Reasoning  
 [T] Technology  
 [V] Visualization

## Strand: Shape and Space (Transformations)

### Outcomes

Students will be expected to

**9SS4 Draw and interpret scale diagrams of 2-D shapes.**

[CN, R, T, V]

### Achievement Indicators:

9SS4.1 Identify an example in print and electronic media, e.g., newspapers, the Internet, of a scale diagram and interpret the scale factor.

9SS4.2 Draw a diagram to scale that represents an enlargement or reduction of a given 2-D shape.

9SS4.3 Determine the scale factor for a given diagram drawn to scale.

9SS4.4 Determine if a given diagram is proportional to the original 2-D shape and, if it is, state the scale factor.

### Elaborations—Strategies for Learning and Teaching

Students were introduced to angles and determining angle measures in Grade 6 (6SS1). Investigations in this unit will involve measuring segments and angles, where accuracy of measurement is important. Access to a math set will be necessary. Students will be responsible for the correct use of language, symbols and conventions throughout the unit. There should be opportunities to use dynamic geometry software such as Geometer's Sketchpad®, Geometria or FX Draw.

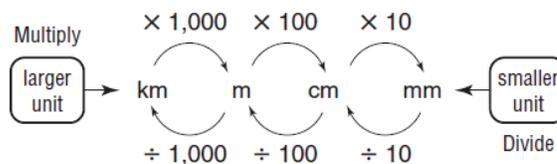
Students should be provided with an opportunity to explore real world examples of scale diagrams. Through these investigations, students should come to understand the concept of a scale factor, be able to determine scale factor, and use it to create enlargements and reductions. Students should also be aware of the effect of the magnitude of a scale factor (i.e., What happens when a scale factor is greater than 1? Less than 1?). A common student error is interchanging the numerator and denominator while calculating scale factor. Understanding that for an enlargement the scale factor is greater than 1, and for a reduction the scale factor is less than 1, should help students avoid making that mistake.

Students should work with scale factors as fractions, decimals and percents.

Since like units are necessary when finding the scale factor, a review of conversion may be necessary. Remind students they can either multiply or divide by powers of ten when changing from one unit to another within the metric system. Provide them with the following chart to illustrate that each place value is 10 times the place value to its right.

1,000	100	10	1	0.1	0.01	0.001
thousands	hundreds	tens	ones	tenths	hundredths	thousandths
				•		
kilo	hecto	deka	basic unit	deci	centi	milli

Students should notice that multiplication changes larger units to smaller units and division changes smaller units to larger units.



**General Outcome: Describe and analyze position and motion of objects and shapes.****Suggested Assessment Strategies***Paper and Pencil*

- Provide students with a table which requires them to record missing measurements or scale factor.

Scale Factor	Original Length	Image Length
4	6 cm	
0.6		30 cm
25%	160 m	
	18 m	6 m

(9SS4.2, 9SS4.3)

- Ask students to answer questions such as the following:
  - (i) The flying distance from St. John's to Montreal is 1650 km. If this distance on a map is 5 cm, what is the scale factor? (9SS4.3)
  - (ii) Some viruses measure 0.0001 mm in diameter. An artist's diagram of a virus shows the diameter as 5 mm. What is the scale factor used? (9SS4.3)

**Resources/Notes****Authorized Resource***Math Makes Sense 9*

Prep Talk Video: Similarity and Transformations

Try It Virtual Manipulatives

*This program allows you to produce enlargements and reductions and transform shapes.*

**Lesson 7.1: Scale Diagrams and Enlargements****Lesson 7.2: Scale Diagrams and Reductions**

ProGuide: pp.6-12, 13-21

Master 7.17

CD-ROM: Master 7.23, 7.24

See It Videos and Animations:

Solving Equations by Using Balance Strategies, Balance-Scales Model and Algebra Tiles Model

Student Book (SB): pp.318-324, 325-333

Preparation and Practice Book (PB): 262-266, 267-270, 228-232, 233-241

**Strand: Shape and Space (Transformations)****Outcomes***Students will be expected to*

9SS4 Continued...

**Achievement Indicators:**

9SS4.1, 9SS4.2, 9SS4.3

9SS4.4 *Continued***Elaborations – Strategies for Learning and Teaching**

It would be useful to collect town and provincial maps to study the scales used. Students could be asked to find the actual distance, using the scale, or to convert the scale provided in one form to a different form. For example:

If the scale is given as the ratio 1 : 500 000, how many kilometres does 7.5 cm represent?

To solve this problem students might recognize that they need to multiply 500 000 by 7.5 cm and do this calculation directly, or they might set up the proportion  $\frac{1}{500000} = \frac{7.5}{x}$ . They should initially determine that  $x = 3\,750\,000$ . Since 7.5 is in cm, then 3 750 000 is also in cm, and when converted equals 37.5 km.

Unit analysis could be introduced here as a way to verify that units in a conversion are correct. To convert 3 750 000 cm to kilometres:

$$3750000 \text{ cm} \times \frac{1 \text{ km}}{100000 \text{ cm}}$$

$$37.5 \text{ km}$$

Since 1 km = 100 000 cm, the ratio  $\frac{1 \text{ km}}{100\,000 \text{ cm}} = 1$ .

Students should recognize that when a numerical value is multiplied by one, the value remains the same. This is the basis of unit conversion. Students will work with unit analysis in Mathematics 1201 and Mathematics 1202.

Reading, interpreting and constructing scale diagrams provides a good introduction to similarity.

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**General Outcome: Describe and analyze position and motion of objects and shapes.**


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**Suggested Assessment Strategies**
*Journal*

- A billboard picture is 8 m wide and 12 m long and needs to be recreated as a sign that is only 1 m wide. Ask students to explain how they would determine the length of the sign. (9SS4.2)

*Paper and Pencil*

- Provide students with a simple diagram on a grid and ask them to reduce or enlarge the diagram using a given scale factor. (9SS4.2)
- Provide students with the coordinates of the vertices of an object and the coordinates of the vertices of the image. Ask them to plot the two, and then determine the type of dilation and the scale factor. (9SS4.2, 9SS4.3)

*Performance*

- Students could design a logo that includes geometric shapes. Once they have created a design, they should:
  - Decide on the dimensions of an enlargement of the logo which would fit on a banner or billboard.
  - Determine the scale factor.
  - Create a business card using the logo by repeating the process for a reduction. (9SS4.2, 9SS4.3)

**Resources/Notes**
**Authorized Resource**
*Math Makes Sense 9*

Prep Talk Video: Similarity and Transformations

Try It Virtual Manipulatives

*This program allows you to produce enlargements and reductions and transform shapes.*

**Lesson 7.1: Scale Diagrams and Enlargements**
**Lesson 7.2: Scale Diagrams and Reductions**

ProGuide: pp.6-12, 13-21

Master 7.17

CD-ROM: Master 7.23, 7.24

See It Videos and Animations:

Solving Equations by Using Balance Strategies, Balance-Scales Model and Algebra Tiles Model

SB: pp.318-324, 325-333

PB: 262-266, 267-270, 228-232, 233-241

**Strand: Shape and Space (3-D Objects and 2-D Shapes)****Outcomes**

*Students will be expected to*

**9SS3 Demonstrate an understanding of similarity of polygons.**

[C, CN, PS, R, V]

**9SS4 Continued...**

**Achievement Indicators:**

9SS3.1 *Determine if the polygons in a given set are similar and explain the reasoning.*

9SS3.2 *Draw a polygon similar to a given polygon and explain why the two are similar.*

9SS3.3 *Solve a given problem using the properties of similar polygons.*

9SS4.5 *Solve a given problem that involves a scale diagram by applying the properties of similar triangles.*

**Elaborations—Strategies for Learning and Teaching**

Students are introduced to the concept of similarity in this unit. Through investigation, they should recognize that polygons which have equal corresponding angle measures and have corresponding proportional sides are similar.

The use of technology (e.g., overhead projectors, printers, photocopiers, design/publishing software) can enhance the study of similarity.

When shapes are similar, corresponding angles are congruent. This alone, however, is not enough to determine similarity. For example, a square and a long, thin rectangle both have four right angles, but they are not similar shapes. To determine similarity, side lengths must also be considered. Corresponding side lengths are all enlarged or reduced by the same factor if the polygons are similar. To determine similarity, students can compare the side lengths to determine if they have been increased or decreased by the same proportion.

When first constructing similar shapes, students can construct a shape on a square grid, and then copy the shape onto another grid with larger or smaller squares. Rulers and protractors can also be used to construct similar polygons. Students can use dynamic geometry software to construct a shape and then reduce or enlarge it.

Students should be given a wide variety of problem-solving situations that involve similarity. They should be encouraged to use a variety of strategies, such as observation, actual measurement, proportional reasoning and scale factor, to determine unknown measures. Accuracy of measurement, proper labelling, and correct notation are important. When finding unknown measures using the properties of similar figures, an emphasis should be placed on the relationship between similarity and scale factor.

The connection between proportional reasoning and similarity is very important. Similar figures provide a visual representation of proportions, and proportional thinking enhances the understanding of similarity. When discussing similarity, ratios should be explored. It is through using ratios of sides within triangles that the trigonometric ratios will be developed in senior high.

**General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.**

### Suggested Assessment Strategies

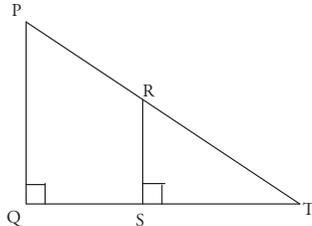
#### Journal

- Ask students to respond to the following:  
One triangle has two  $50^\circ$  angles. Another triangle has a  $50^\circ$  angle and an  $80^\circ$  angle. Could the triangles be similar? Explain your thinking. (9SS3.1)

#### Paper and Pencil

- A software program offers these preset paper sizes for printing:  
A4 (210 mm by 297 mm)  
A5 (148 mm by 210 mm)  
B5 (182 mm by 257 mm)  
Ask students to use scale factors to determine if the paper sizes are similar. (9SS3.3)

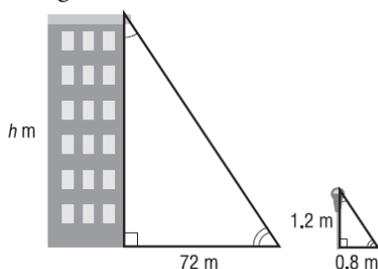
- Ask students to answer the questions that follow based on the diagram given.



- Which triangles are similar?
- Measure the sides and determine the ratios of  
(a)  $\frac{PQ}{QT}, \frac{RS}{ST}$     (b)  $\frac{PQ}{PT}, \frac{RS}{RT}$     (c)  $\frac{QT}{PT}, \frac{ST}{RT}$

What do you notice about the values?

- If  $PQ = 8.2$  cm,  $QS = 5.3$  cm, and  $ST = 7.3$  cm, use one of the ratio pairs in part (ii) to find  $RS$ . (9SS3.3, 9SS4.5)
- A building casts a shadow 72 meters long. At the same time, a parking meter that is 1.2 meters tall casts a shadow that is 0.8 meter long. Ask students to determine the height of the building.



(9SS3.3, 9SS4.5)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 9*

Lesson 7.3: Similar Polygons

Lesson 7.4: Similar Triangles

ProGuide: pp. 22-30, 31-39

Master 7.6a, 7.6b, 7.7a, 7.7b

CD-ROM: Master 7.25, 7.26

SB: pp.334-342, 343-352

PB: pp. 271-277, 278-288

## Strand: Shape and Space (Transformations)

### Outcomes

*Students will be expected to*

**9SS5 Demonstrate an understanding of line and rotation symmetry.**

[C, CN, PS, V]

### Achievement Indicators:

9SS5.1 *Classify a given set of 2-D shapes or designs according to the number of lines of symmetry.*

9SS5.2 *Complete a 2-D shape or design given one half of the shape or design and a line of symmetry.*

### Elaborations—Strategies for Learning and Teaching

A famous example of symmetry in architecture is the Taj Mahal in India. Many parts of the building and grounds were designed and built to be perfectly symmetrical. Symmetry creates a sense of balance.

Students will work with both types of symmetry in 2-D geometry: reflective and rotational. When 2-D shapes are divided along one or more lines of symmetry, and the opposite sides are mirror images, the shapes have reflective, or line symmetry. Rotation symmetry refers to the number of times a 2-D shape fits over an image of itself when it is rotated a full rotation. Students will examine both line and rotation symmetry in tessellations and artwork, as well as transformations on the Cartesian plane. They have had previous experience with lines of symmetry in Grade 4 (4SS5) and with tessellations in Grade 8 (8SS6). They have also done significant work with the Cartesian plane.

A 2-D shape has line symmetry if one half of the shape is a reflection of the other half. The reflection occurs across a line. The line of symmetry, or line of reflection, can be horizontal, vertical or oblique, and may or may not be part of the diagram itself.

To help explain line symmetry, students should view examples and non-examples. Another approach is to have students fold a sheet of paper in half and cut out a shape of their choosing. When they open the paper, the fold line will be a line of symmetry. A third way is to use transparent mirrors. If the shape is symmetrical where the Mira™ has been placed, the image of one side of the shape will fall right on top of the other side of the shape.

Students should be given an opportunity to investigate the number of lines of symmetry that exist in various 2-D shapes. These shapes could include polygons, letters, pictures, logos, etc. In examining shapes to be classified based on the number of lines of symmetry, it is important to include shapes that are asymmetrical as well. Students should conclude that the number of lines of symmetry in a regular polygon is equal to the number of vertices.

Students can complete shapes and designs with line symmetry using square tiles or pattern blocks, folded paper, a Mira™, grid paper, or technology tools such as a drawing program or dynamic geometry software. Relating a line of symmetry to a line of reflection should enable students to complete a figure, describe the completed shape, and describe the reflection. This should include working with and without a coordinate plane.

**General Outcome: Describe and analyze position and motion of objects and shapes.****Suggested Assessment Strategies***Observation*

- Ask students to use a geoboard to create a shape with one rubber band. Students can use other bands to make as many lines of symmetry as possible. As a class, students can classify each figure according to the number of lines of symmetry. (9SS5.1)

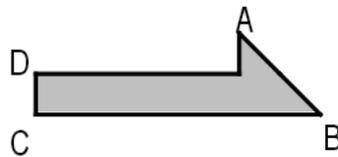
*Journal*

- Ask students to explain how the number of lines of symmetry of a regular polygon relates to the number of sides it has. (9SS5.1)
- Ask students to respond to the following journal prompt:  
Any rectangle has only two lines of symmetry.  
Do you agree or disagree with this statement? Explain. Use drawings to support your argument. (9SS5.1)

*Paper and Pencil*

- Students can sketch, or use a geoboard, to create the following figure. This figure represents half of another shape. Ask students to create the final shape by constructing the missing half, given each of the following cases:

- Line of symmetry is  $\overline{CB}$
- Line of symmetry is  $\overline{CD}$
- Line of symmetry is  $\overline{AB}$



(9SS5.2)

- For this activity, students need to use either isometric or rectangular dot grid paper. Students should draw a horizontal, vertical or oblique line through several dots. Have them make a design completely on one side of the drawn line that touches the line in some way. The task is to make a mirror image of their design on the other side of the line. Students can exchange designs and make the mirror image of a classmate's design. When finished, they can use a mirror to check their work. You can also challenge them to make designs with more than one line of symmetry. (9SS5.2)

**Resources/Notes****Authorized Resource***Math Makes Sense 9***Lesson 7.5: Reflections and Line Symmetry**

ProGuide: pp. 41-47

CD-ROM: Master 7.27

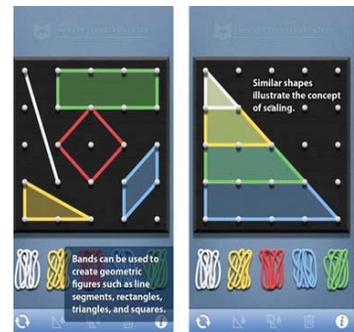
SB: pp. 353-359

PB: pp. 289-296

**Suggested Resource**

<https://www.k12pl.nl.ca/curr/7-9/math/grade9/links/unit7.html>

- Links for a Geoboard web app and an iPhone / iPad app



The Geoboard is a tool for exploring topics such as perimeter, area, angles, congruence, and fractions.

## Strand: Shape and Space (Transformations)

### Outcomes

*Students will be expected to*

9SS5 Continued...

### Achievement Indicators:

9SS5.3 Determine if a given 2-D shape or design has rotation symmetry about the point at the centre of the shape or design and, if it does, state the order and angle of rotation.

9SS5.4 Rotate a given 2-D shape about a vertex and draw the resulting image.

### Elaborations—Strategies for Learning and Teaching

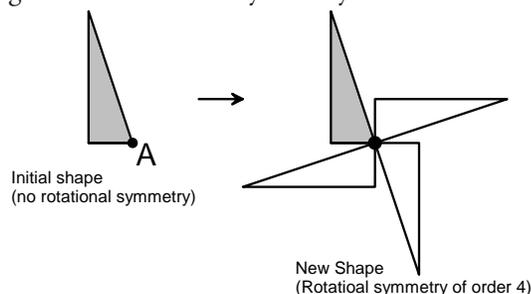
Students have worked with rotations of 2-D shapes in earlier grades (5SS7, 5SS8, 6SS6, 6SS7, 7SS5). The concept of rotation symmetry, however, is new to them.

A 2-D shape has rotation symmetry if, when turned around its centre point, it fits into an outline of itself at least once before it has completed a full rotation. One way to test for this is to trace the shape, and then turn the tracing over the original shape around a pencil point to see whether it fits over itself. For example, a rectangle fits over itself twice – once after a half turn, and once again after a complete turn. Students should use grid paper to investigate the rotation symmetry of various objects.

Students should determine the order of rotation and the angle of rotation. For example, a square has rotation symmetry of order 4, an equilateral triangle has rotation symmetry of order 3, and the order of rotation symmetry for a circle is infinite. A shape with order 2 symmetry has an angle of rotation of  $180^\circ$ , one with order 3 symmetry has an angle of rotation of  $120^\circ$ , and a shape with order 4 symmetry has a  $90^\circ$  angle of rotation. The number of degrees refers to the smallest angle through which the shape must be rotated to lie on itself. Students should be able to express the angle of rotation in both degrees and fractions of a turn (e.g.,  $90^\circ$  angle of rotation is a one-quarter turn). Students can determine the angle of rotation by dividing the number of degrees in a circle by the order of rotation. Make students aware that, because every shape always coincides with itself after a  $360^\circ$  rotation, rotation symmetry of order 1 is not identified.

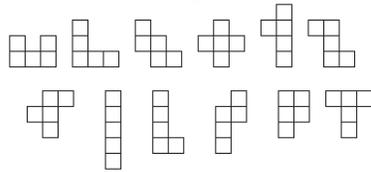
Students should be able to rotate objects in both a clockwise and counter-clockwise direction. They will need to use various types of grid paper/dot paper for these activities. It may be useful to provide tracing paper for students who have difficulty visualizing a rotation.

Students should identify rotation symmetry and order of rotation for a new shape formed by rotating an initial shape about a vertex. The result of a number of rotations about vertex A below, for example, is a new figure with a rotation symmetry of order 4.



**General Outcome: Describe and analyze position and motion of objects and shapes.****Suggested Assessment Strategies***Paper and Pencil*

- Ask students to answer the following:
  - Determine which objects have rotation symmetry. If they have rotation symmetry identify the center of rotation, and then state the order and the angle of rotation.

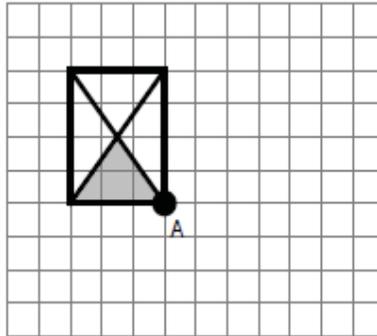


(9SS5.3)

- A shape has rotation symmetry of order infinity. What is the shape and what does this mean?

(9SS5.3)

- Use point A as the center of rotation and create a design that has an order of rotation equal to 4.



(9SS5.4)

*Observation*

- Ask students to find a photograph or drawing of each item.
  - a 2-D shape with line and rotation symmetry
  - a 2-D shape with line symmetry but without rotation symmetry
  - a 2-D shape with rotation symmetry but without line symmetry

(9SS5.1, 9SS5.3)

**Resources/Notes****Authorized Resource***Math Makes Sense 9***Lesson 7.6: Rotations and Rotational Symmetry**

ProGuide: pp. 49-55

CD-ROM: Master 7.28

See It Videos and Animations:  
Rotations and Rotational  
Symmetry

SB: pp. 361-367

PB: pp. 297-303

**Note**

Patty paper could be used as tracing paper to assist with transformations.

## Strand: Shape and Space (Transformations)

### Outcomes

*Students will be expected to*  
9SS5 Continued...

#### Achievement Indicators:

9SS5.5 *Identify a line of symmetry or the order and angle of rotation symmetry in a given tessellation.*

9SS5.6 *Identify and describe the types of symmetry created in a given piece of artwork.*

9SS5.7 *Create or provide a piece of artwork that demonstrates line and rotation symmetry, and identify the line(s) of symmetry and the order and angle of rotation.*

### Elaborations—Strategies for Learning and Teaching

A tessellation is a pattern or arrangement that covers an area without overlapping or leaving gaps. It is also known as a tiling pattern. In Grade 8, students were expected to explain the properties of shapes that make tessellating possible, create tessellations, and identify tessellations in the environment (8SS6). Now they will explore line and rotation symmetry in tessellations.

Students should be provided with samples of tessellations that have:

- line symmetry
- rotation symmetry
- both
- neither

They should be able to describe how the properties of line symmetry and rotation symmetry apply to each tessellation. Students could be asked to create a tessellation and then analyze the resulting composite shape for symmetry. They should conclude that if they can identify line symmetry in one part of a tessellation, the same symmetry can be found elsewhere in the tessellation. Similarly, if they identify rotation symmetry around one point in a tessellation, the same rotation symmetry can be identified around every like point.

Students should be given an opportunity to explore symmetry in the context of everyday experiences. They should explore different types of art, such as paintings, jewellery, quilts, tiles, murals, and cultural artwork. Students could analyze pictures, logos, flags, signs, playing cards, kaleidoscopes, etc. They may use computer assisted software to experiment with symmetry of designs or photos.

In small groups, students could search for company logos that contain various geometric shapes. They should copy them and draw in any lines of symmetry. They should then discuss the following:

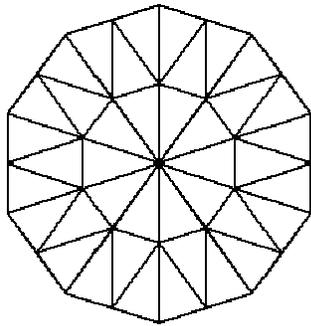
- Are logos that contain lines of symmetry more pleasing to the eye than those without lines of symmetry?
- Are logos that contain lines of symmetry easier to remember than those without lines of symmetry?

This topic also provides a good opportunity to collaborate with the Art teacher on a cross-curricular project or activity.

**General Outcome: Describe and analyze position and motion of objects and shapes.****Suggested Assessment Strategies***Paper and Pencil*

- Give students a variety of tessellation patterns and ask them to identify:
  - (i) the number of lines of symmetry
  - (ii) the order of rotation
  - (iii) the angle of rotation

Sample pattern:



(9SS5.5)

*Performance*

- Ask students to use the Internet (e.g., Google Images) to find examples of both line symmetry and rotation symmetry in artwork. Have them print various samples and determine:
  - (i) the number of lines of symmetry
  - (ii) the order of rotation
  - (iii) the angle of rotation

(9SS5.6, 9SS5.7)

*Observation*

- Ask students to use a digital camera to take photographs of their faces. Instruct them to look directly at the camera and avoid tilting their heads. Using a drawing program, such as Paint Shop Pro or Adobe Photoshop, students can then follow the steps outlined below:
  - (1) Crop the right side of the face
  - (2) Copy the remaining left side
  - (3) Paste a mirror image of the left side in the right-hand position
  - (4) This will create a perfectly symmetrical face which can be compared with the original picture.
  - (5) Repeat the procedure to produce a mirror image of the right side of the face.
  - (6) Print the three photos.
  - (7) Compare the “symmetrical” photos with the original.

(9SS5.2, 9SS5.6, 9SS5.7)

**Resources/Notes****Authorized Resource**

*Math Makes Sense 9*

**Lesson 7.5: Reflections and Line Symmetry**

**Lesson 7.6: Rotations and Rotational Symmetry**

ProGuide: pp. 41-47, 49-55

SB: pp. 353-359, 361-367

PB: pp. 291-296

**Lesson 7.7: Identifying Types of Symmetry on the Cartesian Plane**

ProGuide: pp. 56-63

SB: pp. 368-375

**Unit Problem: Designing a Flag**

ProGuide: p. 69

SB: p. 381

**Suggested Resource**

<https://www.k12pl.nl.ca/curr/7-9/math/grade9/links/unit7.html>

- A website featuring tessellations in artwork

## Strand: Shape and Space (Transformations)

### Outcomes

*Students will be expected to*

**9SS5 Continued...**

#### Achievement Indicators:

9SS5.8 *Determine whether or not two given 2-D shapes on the Cartesian plane are related by either rotation or line symmetry.*

9SS5.9 *Identify the type of symmetry that arises from a given transformation on the Cartesian plane.*

9SS5.10 *Complete, concretely or pictorially, a given transformation of a 2-D shape on a Cartesian plane and describe the type of symmetry that results.*

9SS5.11 *Draw, on a Cartesian plane, the translation image of a given shape using a given translation rule, such as  $R2$ ,  $U3$  or  $\rightarrow\rightarrow$ ,  $\uparrow\uparrow\uparrow$ , label each vertex and its corresponding ordered pair and determine why the translation may or may not result in line or rotation symmetry.*

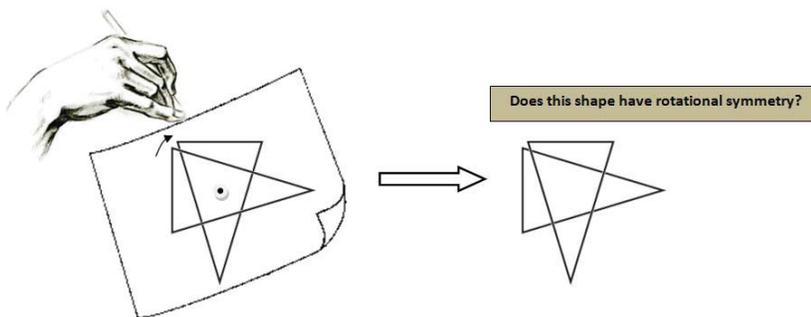
### Elaborations—Strategies for Learning and Teaching

Students will now explore line and rotation symmetry for transformations on the Cartesian plane. They have previously worked with transformations (5SS7, 5SS8, 6SS6, 6SS7). In Grade 7, they worked with translations, rotations and reflections of a 2-D shape in all four quadrants of a Cartesian plane (7SS5). Transformations were revisited again in Grade 8 in the context of tessellations (8SS6).

Students should be able to identify symmetries in the combined shape of an object and its resulting image. They should be provided with examples of transformations that have already been graphed, as well as be expected to perform the transformations themselves. Students should be encouraged to use proper conventions when labelling axes, vertices, coordinates, etc. Accuracy in drawings is important.

Students should discover that when a figure is reflected, the combined shape will have line symmetry. It may or may not have rotation symmetry. When a figure is rotated, the combined shape may have rotation symmetry. It may or may not have line symmetry. The symmetry of a combined shape created when a figure is translated depends on the type of translation and also on the original shape's symmetry.

The intent of this outcome is to apply transformations to create new shapes. Those shapes are then examined to determine if symmetry exists. Traditional methods such as tracing paper and Miras™ could be used.

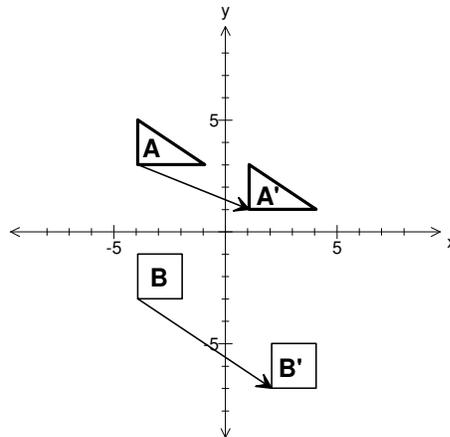


**General Outcome: Describe and analyze position and motion of objects and shapes.**

### Suggested Assessment Strategies

#### *Paper and Pencil*

- Ask students to identify the type of transformation presented in each case below.



- Ask them to determine if the object and image for each situation are related by line symmetry and/or rotation symmetry. (9SS5.8, 9SS5.9)
- Ask students to complete several assigned transformations and analyze their own drawings for symmetry. (9SS5.10)

#### *Journal*

- Ask students to answer the following:  
Some regular shapes, such as an equilateral triangle, a square, or a regular hexagon, appear to show line symmetry when they are translated in one direction.  
Do you agree or disagree with this statement? Give examples to support your argument. Discuss your answer with a partner. (9SS5.10, 9SS5.11)

#### *Performance*

- Create a grid on the floor with masking tape. Use rope or coloured tape to place the axes. One student chooses a spot. Another student directs him/her to translate the position. This activity could progress to more than three students on the grid holding elastic to form a 2-D shape. A different student directs them (as vertices) to “walk through” various transformations.

(9SS5.11)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 9*

#### Lesson 7.7: Identifying Types of Symmetry on the Cartesian Plane

ProGuide: pp. 56-63

CD-ROM: Master 7.29

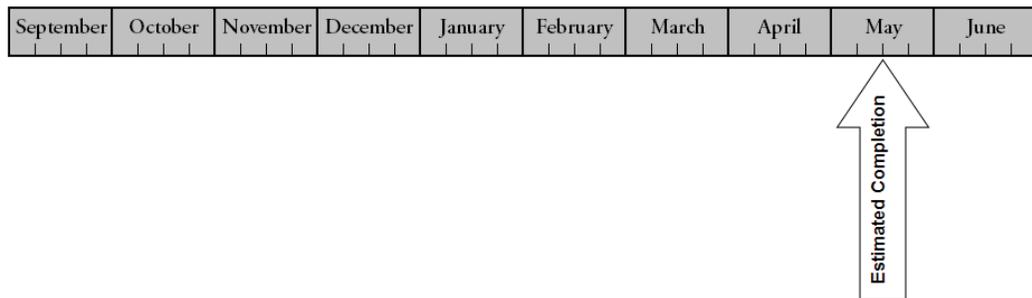
SB: pp. 368-375

PB: pp. 304-310



# Circle Geometry

Suggested Time: 3 Weeks



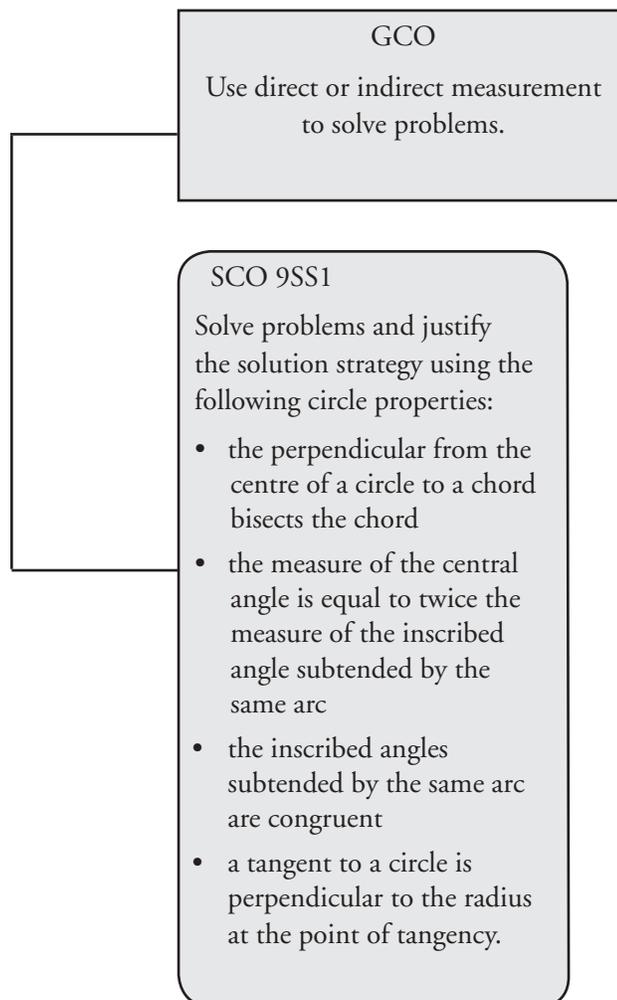
# Unit Overview

## Focus and Context

In this unit, students will explore properties of circles. They will discover the relationship between a tangent and a radius, and apply the tangent radius property to solve related problems. The relationship between a chord, its perpendicular bisector, and the centre of a circle will be developed. Finally, students will develop the relationship between inscribed angles and a central angle subtended by the same arc.

It is important to connect the study of geometry to meaningful situations. Whether determining the correct location for handles on a bucket, finding the centre of a circle in an irrigation project, or determining the length of a tangent to the earth from an orbiting satellite, properties of circles come into play.

## Outcomes Framework



## SCO Continuum

Mathematics 8	Mathematics 9	Mathematics Level I	
		1201	1202
Shape and Space (Measurement)			
<p>8SS1 Develop and apply the Pythagorean theorem to solve problems. [CN, PS, R, T, V]</p>	<p>9SS1 Solve problems and justify the solution strategy using the following circle properties:</p> <ul style="list-style-type: none"> <li>the perpendicular from the centre of a circle to a chord bisects the chord</li> <li>the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc</li> <li>the inscribed angles subtended by the same arc are congruent</li> <li>a tangent to a circle is perpendicular to the radius at the point of tangency.</li> </ul> <p>[C, CN, PS, R, T, V]</p>	not addressed	<p>G2. Demonstrate an understanding of the Pythagorean theorem by:</p> <ul style="list-style-type: none"> <li>identifying situations that involve right triangles</li> <li>verifying the formula</li> <li>applying the formula</li> <li>solving problems.</li> </ul> <p>[C, CN, PS, V]</p>

## Mathematical Processes

[C] Communication  
 [CN] Connections  
 [ME] Mental Mathematics and Estimation

[PS] Problem Solving  
 [R] Reasoning  
 [T] Technology  
 [V] Visualization

**Strand: Shape and Space (Measurement)****Outcomes**

*Students will be expected to*

**9SS1 Solve problems and justify the solution strategy using the following circle properties:**

- the perpendicular from the centre of a circle to a chord bisects the chord
- the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
- the inscribed angles subtended by the same arc are congruent
- a tangent to a circle is perpendicular to the radius at the point of tangency.

[C, CN, PS, R, T, V]

**Achievement Indicators:**

9SS1.1 *Explain the relationship between the tangent of a circle and the radius at the point of tangency.*

9SS1.2 *Solve a given problem involving application of one or more of the circle properties.*

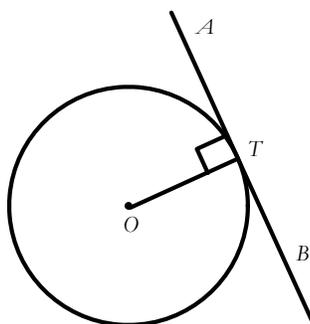
**Elaborations—Strategies for Learning and Teaching**

In this unit, chord properties in circles, inscribed and central angle relationships, and tangents to circles will be explored. Students' previous exposure to circles includes the terminology radius, diameter, circumference and pi. They have explored relationships that exist between these values (7SS1, 7SS2, 7SS3). Students are also familiar with constructing circles and central angles. While problem solving in this unit, the Pythagorean theorem developed in Grade 8 (8SS1) will be used, and should be reviewed in context.

The outcome develops properties of circles and will introduce students to new terminology. Each property should be developed through a geometric exploration, which brings out the new terminology and then applies it to real life situations. Once all properties have been developed, students can solve problems involving a combination of properties. The use of technology is encouraged. Programs such as FX Draw, Geometer's Sketchpad®, or any dynamic geometry software package can help students explore the relationships.

As students use circle properties to determine angle measures, it will sometimes be necessary to apply previously learned concepts. A circle may contain an isosceles triangle, for example, whose legs are radii of the circle. Students must recognize that the angle opposite the congruent sides have equal measures. This was introduced in Grade 6 (6SS4). Another commonly used property is that the sum of interior angles in a triangle is  $180^\circ$  (6SS2).

The properties of a circle can be introduced in any order. By starting with the property "*A tangent to a circle is perpendicular to the radius at the point of tangency*", students are introduced to only one new term. This provides the opportunity for contextual problem solving before any other properties are developed. All properties should be developed in this manner so that students make connections with real life situations.



$O$  is the center of the circle

$\overline{OT}$  is the radius

$T$  is a point of tangency

$\overline{AB}$  is a tangent line

The tangent-radius property states that under the given conditions  $\angle ATO = 90^\circ$ .

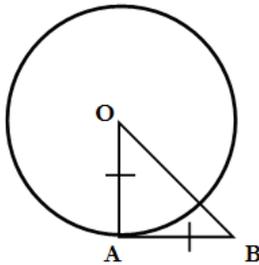
**General Outcome: Use direct or indirect measurement to solve problems.**

### Suggested Assessment Strategies

#### *Paper and Pencil*

- Ask students to answer the following:

If  $\overline{OA} = \overline{BA}$ , and  $\overline{BA}$  is tangent to the circle at A, determine the measure of  $\angle ABO$ .



(9SS1.1, 9SS1.2)

- Students can complete a problem such as the following.

Mike has a rock tied to the end of a 5 m rope and is swinging it over his head to form a circle with him at the center. The rock comes free of the rope and flies along a tangent from the circle until it hits the side of a building that is 14 m away from Mike. How far along the tangent did the rock travel? Determine the answer to the nearest meter.

(9SS1.1, 9SS1.2)

### Resources/Notes

#### Authorized Resource

##### *Math Makes Sense 9*

Prep Talk Video: Circle Geometry

Try It Virtual Manipulatives

*This program allows you to verify the tangent and chord properties, and the properties of angles in a circle.*

#### Lesson 8.1: Properties of Tangents to a Circle

ProGuide: pp. 4-11

Master 8.6, 8.7

CD-ROM: Master 8.17

See It Videos and Animations:

Properties of Tangents to a Circle

Student Book (SB): pp. 384-391

Preparation and Practice Book

(PB): pp. 318-322

**Strand: Shape and Space (Measurement)****Outcomes**

*Students will be expected to*

9SS1 Continued...

**Achievement Indicators:**

9SS1.3 *Explain the relationship between the perpendicular from the centre of the circle and a chord.*

9SS1.2 *Continued*

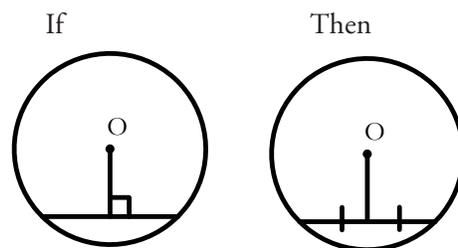
**Elaborations—Strategies for Learning and Teaching**

Paper folding provides a good means of exploring the relationship between a perpendicular drawn from the centre of the circle to a chord. It can also be used to locate the centre of the circle. Students should come to realize that if any two of the following three conditions are in place, then the third condition is true for a given line and a given chord in a circle:

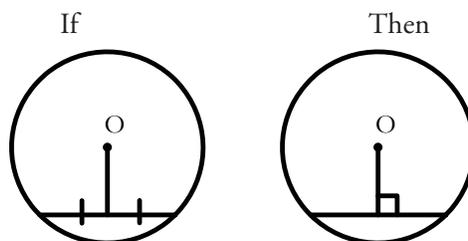
- the line bisects the chord
- the line passes through the centre of the circle
- the line is perpendicular to the chord

Illustrate the properties using the following diagrams:

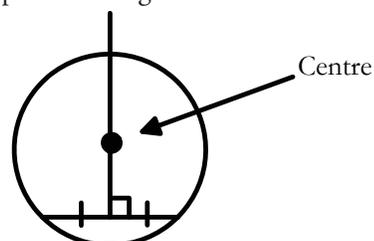
**Property 1:** A line from the centre of the circle that is perpendicular to a chord will bisect the chord.



**Property 2:** A line from the centre that bisects a chord is perpendicular to the chord.



**Property 3:** If a line is a perpendicular bisector of a chord, then the line passes through the centre of the circle.



**General Outcome: Use direct or indirect measurement to solve problems.**

### Suggested Assessment Strategies

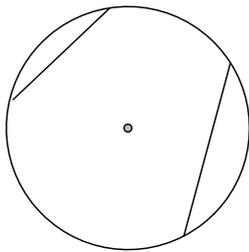
#### Performance

- Ask students to complete the following paper folding activity to develop the relationship between the perpendicular from the centre of the circle and a chord.
  - (i) Construct a large circle on tracing paper and draw two different chords.
  - (ii) Construct the perpendicular bisector of each chord.
  - (iii) Label the point inside the circle where the two perpendicular bisectors intersect.
  - (iv) What do you notice about the point of intersection of the two perpendicular bisectors?

(9SS1.3)

#### Journal

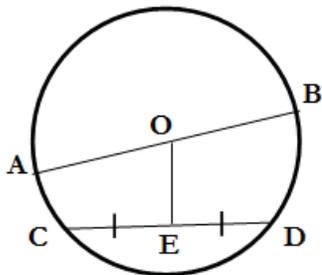
- Ask students to explain how they could locate the center of a circle if they were given any two chords in the circle that are not parallel.



(9SS1.3)

#### Paper and Pencil

- Ask students to answer the following :  
In the circle with center O, the diameter is 40 cm and chord CD is 34 cm. What is the length of  $\overline{OE}$ ?



(9SS1.2, 9SS1.3)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 9*

#### Lesson 8.2: Properties of Chords in a Circle

ProGuide: pp. 12-21

Master 8.8a, 8.8b

CD-ROM: Master 8.18

Classroom Videos:

Properties of Chords in a Circle, Part 1 and 2

SB: pp. 392-399

PB: pp. 323-331

## Strand: Shape and Space (Measurement)

### Outcomes

Students will be expected to

9SS1 Continued...

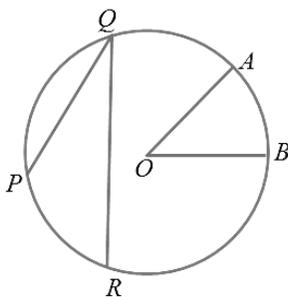
#### Achievement Indicators:

9SS1.4 Explain the relationship between the measure of the central angle and the inscribed angle subtended by the same arc.

9SS1.2 Continued

### Elaborations—Strategies for Learning and Teaching

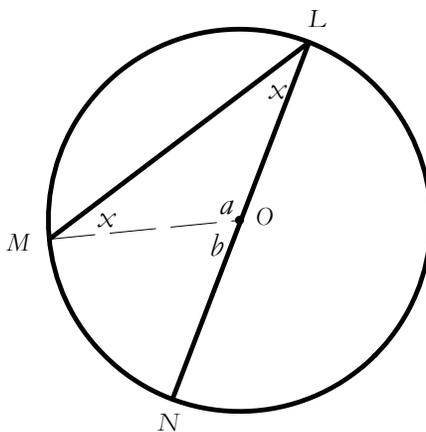
Students will also discover relationships between central and inscribed angles. Circle geometry is very visual, and students should be encouraged to draw diagrams. Some students may have difficulty identifying the arc that subtends an inscribed or central angle. They may benefit from using different colours to outline and label different lines that make angles. Reinforce the idea that an angle subtended by an arc is an angle that has common endpoints with the arc.



$\angle PQR$  is an inscribed angle subtended by arc PR

$\angle AOB$  is a central angle subtended by arc AB

Students should discover the relationship between inscribed and central angles that are subtended on the same arc. One way to demonstrate the relationship is indicated below.



Notice  $a + 2x = 180^\circ$

Also,  $a + b = 180^\circ$

Therefore  $b = 2x$

Since  $b$  represents a central angle and  $x$  represents an inscribed angle, students should conclude that inscribed angles are equal to half the measure of the central angle subtended by the same arc.

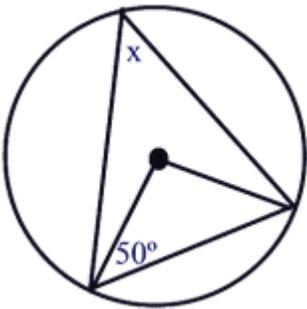
**General Outcome: Use direct or indirect measurement to solve problems.**

### Suggested Assessment Strategies

#### Performance

- The activity *Commit and Toss* gives students an opportunity to anonymously commit to an answer they selected. Give students a selected response question, as shown below. Students write their answer, crumble their solutions into a ball, and toss the paper into a basket. Once all papers are in the basket, ask students to reach in and take one out. They then move to the corner of the room designated to match the selected response on the paper they have taken. In their respective corners, they should discuss the similarities and differences in the explanation provided and report back to the class.

In the circle with centre indicated, what is the value of  $x$ ?



(A)  $40^\circ$   
 (B)  $50^\circ$   
 (C)  $80^\circ$   
 (D)  $100^\circ$

Explain your reasoning:

(9SS1.2, 9SS1.4)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 9*

#### Lesson 8.3: Properties of Angles in a Circle

ProGuide: pp. 24-34

Master 8.9

CD-ROM: Master 8.19

SB: pp. 404-414

PB: pp. 332-337

## Strand: Shape and Space (3-D Objects and 2-D Shapes)

### Outcomes

*Students will be expected to*

9SS1 Continued...

#### Achievement Indicators:

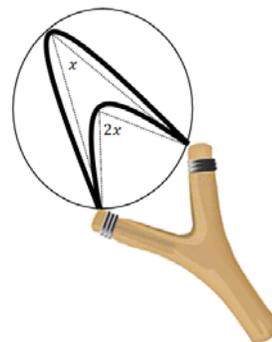
9SS1.2, 9SS1.4

*Continued*

9SS1.5 *Determine the measure of a given angle inscribed in a semicircle using the circle properties.*

### Elaborations—Strategies for Learning and Teaching

A common error occurs when students double the measure of the central angle to determine the inscribed angle. The use of diagrams is a good visual tool to show the impossibility of an inscribed angle being larger than a central angle subtending the same arc. Students could think about the act of drawing back a slingshot and measuring the angle that is formed by the elastic. The further the slingshot is pulled back the more acute (smaller) the angle becomes. This mental exercise will reinforce the notion that the inscribed angle is smaller than the central angle subtended on the same arc.



Students could complete the following paper folding activity to show that an inscribed angle subtended by a diameter is  $90^\circ$ . Ask students to:

- Draw a large circle on a piece of paper.
- Fold the paper to form a diameter and mark endpoints A and B.
- Mark a point C on the circumference. Fold to form chord  $\overline{AC}$ .
- Fold to form chord  $\overline{BC}$ .
- Measure angle C. What do you notice?

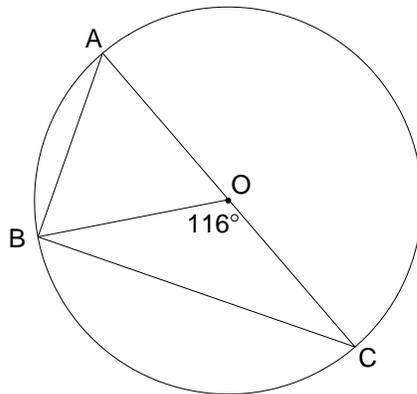
Alternatively, if students understand that the diameter is a central angle measuring  $180^\circ$ , they should conclude that an inscribed angle is half of the central angle subtended by the same arc and, since the central angle is  $180^\circ$ , the inscribed angle must be  $90^\circ$ .

**General Outcome: Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.**

### Suggested Assessment Strategies

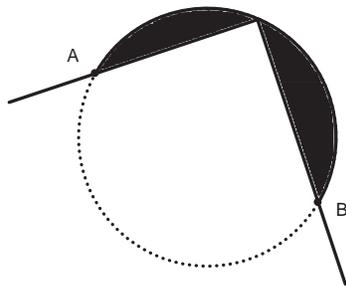
#### *Paper and Pencil*

- Ask students to answer the following:
  - (i) In the circle with centre O shown,  $\angle BOC = 116^\circ$ . What is the measure, in degrees, of  $\angle ABO$  and  $\angle BCO$ ?



(9SS1.2, 9SS1.4, 9SS1.5)

- (ii) The corner of a piece of paper is a  $90^\circ$  angle and is placed on a circle as shown.



- (a) Why is  $\overline{AB}$  the diameter?
- (b) How can the corner of the paper be used to find the centre of the circle?

(9SS1.2, 9SS1.3, 9SS1.5)

### Resources/Notes

#### Authorized Resource

*Math Makes Sense 9*

Lesson 8.3: Properties of Angles in a Circle

ProGuide: pp. 24-34

Master 8.9

CD-ROM: Master 8.19

SB: pp. 404-414

PB: pp. 332-337

**Strand: Shape and Space (Measurement)**

**Outcomes**

Students will be expected to  
9SS1 Continued...

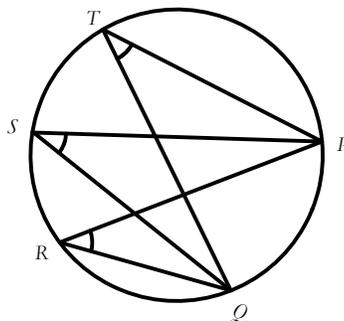
**Achievement Indicators:**

9SS1.6 *Explain the relationship between inscribed angles subtended by the same arc.*

9SS1.2 *Continued*

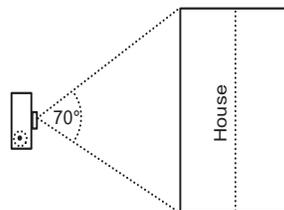
**Elaborations—Strategies for Learning and Teaching**

Students should also have an opportunity to discover that inscribed angles subtended by the same arc are equal.

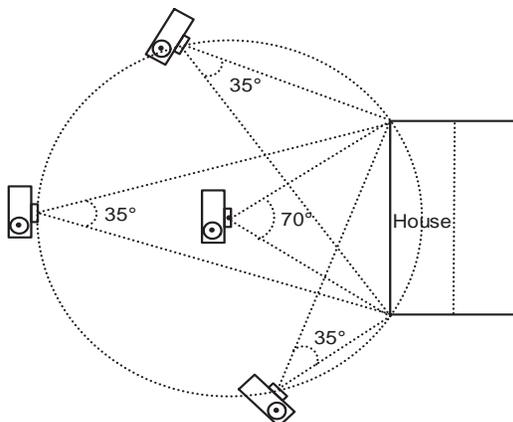


Work through the following example with students to help them develop the relationship between angles in a circle:

Jackie works for a realtor photographing houses that are for sale. She photographed a house two months ago using a camera lens that has a  $70^\circ$  field of view. She has returned to the house to update the photo, but she has forgotten her lens. Today she only has a telephoto lens with a  $35^\circ$  field of view. From what location(s) could Jackie photograph the house with the telephoto lens, so that the entire house still fills the width of the picture? Explain your choices.



A possible solution is shown here. This also illustrates that inscribed angles subtended by the same arc are congruent.



**General Outcome: Use direct or indirect measurement to solve problems.**

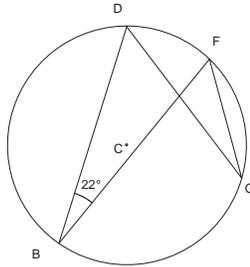
**Suggested Assessment Strategies**

**Resources/Notes**

*Paper and Pencil*

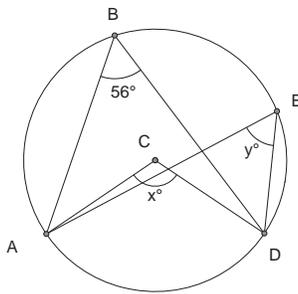
- Ask students to determine the indicated angle measures in the circles with centre C, as shown.

(i) What is the measure of  $\angle DGF$ ?



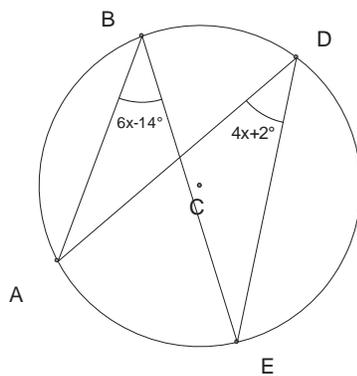
(9SS1.6)

(ii) What are the measures of  $x$  and  $y$ ?



(9SS1.4, 9SS1.6)

(iii) What is the value of  $x$ ? the measure of  $\angle ABE$ ?



(9SS1.2, 9SS1.6)

- A city is building a pedestrian tunnel under a street using a large culvert. The culvert has a diameter of 5 meters. The city is going to fill the bottom of the culvert with concrete to create a surface for walking. Regulations state that there must be 4.2 m of space between the top of the culvert and the walking surface. Ask students:

- How deep must the city pour the concrete in the bottom of the culvert?
- How wide will the walking surface be when it is completed?

(9SS1.2, 9SS1.6)

**Authorized Resource**

*Math Makes Sense 9*

**Lesson 8.3: Properties of Angles in a Circle**

ProGuide: pp. 24-34

Master 8.9

CD-ROM: Master 8.19

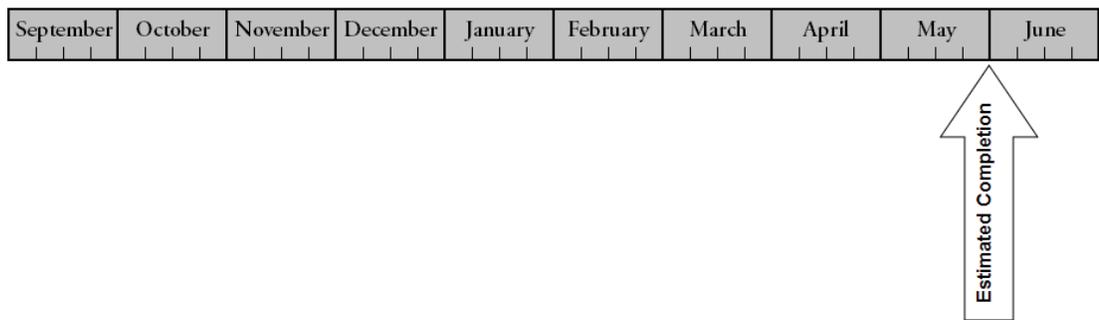
SB: pp.404-414

PB: pp. 332-337



# Probability and Statistics

Suggested Time: 2 Weeks



# Unit Overview

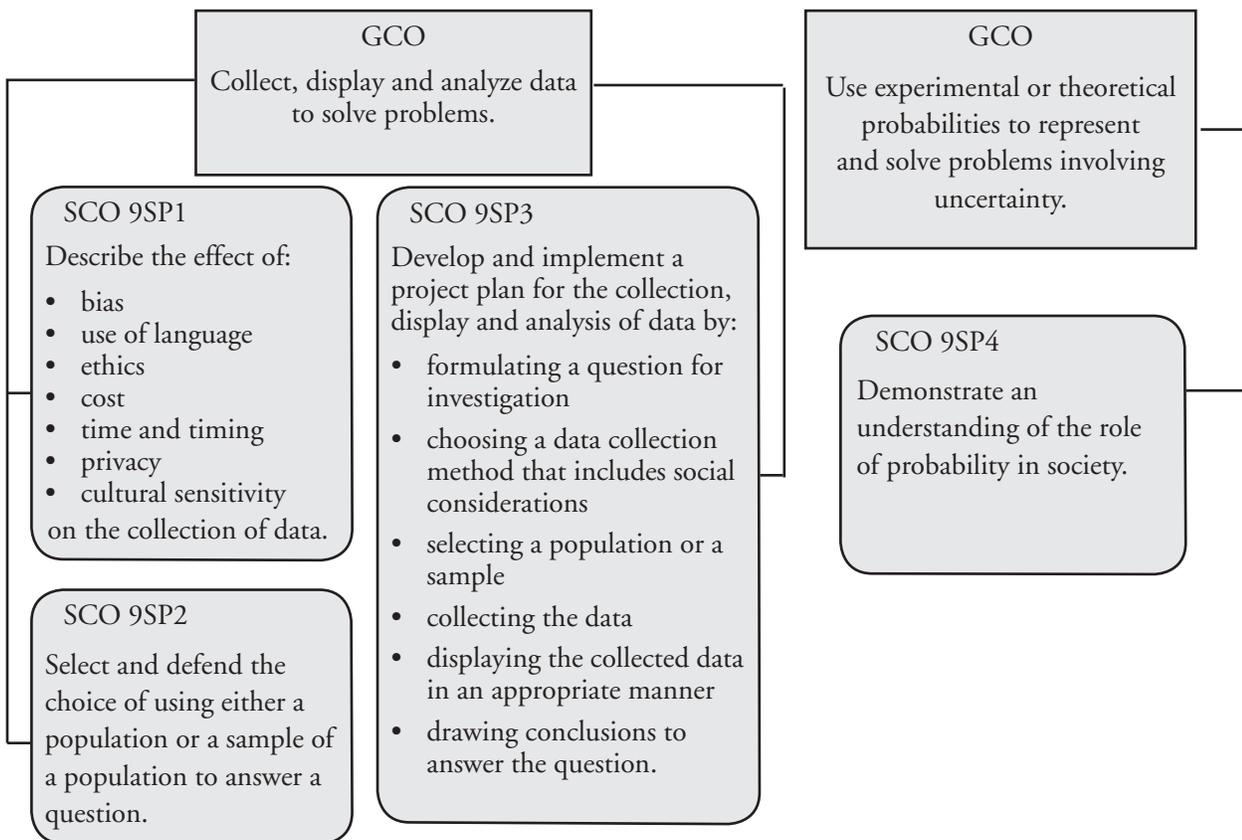
## Focus and Context

In this unit, the data collection process will be analyzed and critiqued. Students will develop and implement a project plan for the collection, display and analysis of data. They will consider such factors as the method used, the reliability and usefulness of data, and the ability to make generalizations about the population from a sample. Students will describe the effect of bias, language use, ethics, cost, time and timing, privacy, and cultural sensitivity on the collection of data. They will also create a rubric that can be used to assess the project.

Students will also explore the role of probability in society. They will explain how decisions based on probability may be a combination of theoretical probability, experimental probability and subjective judgement. To complete the unit, students will examine the validity of using calculated probability to make decisions.

The concepts taught in data analysis and statistics are used to make important decisions in industries such as marketing, research, sports, medicine, law-making, law enforcement, business, and government. Being familiar with these ideas will equip students to make intelligent and informed decisions in the future.

## Outcomes Framework



## SCO Continuum

Mathematics 8	Mathematics 9	Mathematics Level I	
		1201	1202
Statistics and Probability (Data Analysis)			
8SP1 Critique ways in which data is presented. [C, R, T, V]	9SP1 Describe the effect of: <ul style="list-style-type: none"> <li>• bias</li> <li>• use of language</li> <li>• ethics</li> <li>• cost</li> <li>• time and timing</li> <li>• privacy</li> <li>• cultural sensitivity</li> </ul> on the collection of data. [C, CN, R, T]  9SP2 Select and defend the choice of using either a population or a sample of a population to answer a question. [C, CN, PS, R]  9SP3 Develop and implement a project plan for the collection, display and analysis of data by: <ul style="list-style-type: none"> <li>• formulating a question for investigation</li> <li>• choosing a data collection method that includes social considerations</li> <li>• selecting a population or a sample</li> <li>• collecting the data</li> <li>• displaying the collected data in an appropriate manner</li> <li>• drawing conclusions to answer the question.</li> </ul> [C, PS, R, T, V]	not addressed	not addressed
Statistics and Probability (Chance and Uncertainty)			
8SP2 Solve problems involving the probability of independent events. [C, CN, PS, T]	9SP4 Demonstrate an understanding of the role of probability in society. [C, CN, R, T]	not addressed	not addressed

## Mathematical Processes

[C] Communication  
 [CN] Connections  
 [ME] Mental Mathematics and Estimation

[PS] Problem Solving  
 [R] Reasoning  
 [T] Technology  
 [V] Visualization

**Strand: Statistics and Probability (Chance and Uncertainty)**

**Outcomes**

*Students will be expected to*

**9SP4 Demonstrate an understanding of the role of probability in society.**

[C, CN, R, T]

**Achievement Indicator:**

9SP4.1 *Provide an example from print and electronic media, e.g., newspapers, the Internet, where probability is used.*

**Elaborations—Strategies for Learning and Teaching**

Students are familiar with terminology related to probability and have previously solved problems involving the probability of independent events (6SP4, 7SP4, 7SP5, 7SP6 and 8SP2). The concepts taught in data analysis and probability are used every day to make important decisions in various industries. Being familiar with these ideas will equip students to make intelligent and informed decisions.

An example such as the following could be presented to illustrate one use of probability in society.



Reference: [www.theweathernetwork.com](http://www.theweathernetwork.com)

To produce a probability forecast, the forecaster studies the current weather situation, including wind and moisture patterns and determines how these patterns will change over time. The probability of precipitation in a weather forecast is an estimate of the chance of a measurable amount of rain or snow falling anywhere in a given forecast region over the forecast period. For P.O.P. of 40%, for example, forecasters have calculated that in 100 similar weather situations, rain has fallen 40 times in the forecast area. When discussing this example, ask students what assumptions a weather forecaster might make when making a probability forecast. This can lead into a discussion of other societal uses of probability.

Students should realize that within some print or electronic media, probability is often implied without the use of specific terminology. This can be illustrated through discussion of a news story similar to the following:

*The Premier says he's confident there is big oil to be found off the west coast of the island and he's pleased to see NALCOR taking steps to move the industry forward. The Opposition Leader raised concerns this week after the energy corporation invested 20 million dollars in such exploration; she compared it to a risky poker game. The Premier says some minor discoveries bode well for NALCOR's recent investment, but he's hoping for more. He admits the oil business is risky, but he says the Opposition is just playing politics.*

Reference: VOXM, Aug14, 2009

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**General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.**

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**Suggested Assessment Strategies**

*Journal*

- Ask students to provide a report on examples of where probability is used in print and electronic media. (9SP4.1)
  
- Ask students to think of a TV game show where players consider probabilities when deciding how to proceed, and then explain the extent to which probability is involved. (9SP4.1)
  
- Ask students to look through print media and the Internet to find examples of cases such as the following:
  - (i) a situation where decisions affecting your community were made that might have been based on probabilities
  - (ii) a situation where a medical organization might make a decision based on probabilities
 Ask them to describe how probabilities were involved. (9SP4.1)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 9*

Prep Talk Video: Probability and Statistics

Try It Virtual Manipulatives

*This program allows you to display data on graphs using spreadsheets.*

**Lesson 9.1: Probability in Society**

ProGuide: pp. 4-9

Master: 9.6a, 9.6b

CD-ROM: Master 9.16

Student Book (SB): pp. 424-429

Preparation and Practice Book (PB): pp. 347-352

## Strand: Statistics and Probability (Chance and Uncertainty)

### Outcomes

*Students will be expected to*

9SP4 Continued...

#### Achievement Indicators:

9SP4.2 *Identify the assumptions associated with a given probability and explain the limitations of each assumption.*

9SP4.3 *Explain how a single probability can be used to support opposing positions.*

### Elaborations—Strategies for Learning and Teaching

From the previous news story, the investing of 20 million dollars is justified based on previous smaller oil finds and the assumed probability of even greater finds. Students need to realize that predictions based on probability are affected by many factors and the assumption that these factors are constant. The article assumed that the investment in oil is appropriate based on previous oil finds, as well as the price and demand for oil. These assumptions would change as factors such as price and demand for oil changed.

Calculations of probability are always based on assumptions. Students should be encouraged to identify and examine the assumptions to help them determine whether the calculated probability is meaningful when making a decision.

Since probabilities involve assumptions and personal decisions about risks, it is possible to come to different probability conclusions when using the same information. A *Think-Pair-Share* activity could be used to discuss how a single probability, such as the following, could be used to support opposing positions.

*The Weather Network says there is a 90% chance that 40 cm of snow will fall within the next 24 hours.*

**Think:** Individually, students brainstorm different reactions to this probability statement.

**Pair:** Students link up with a partner to discuss ideas.

**Share:** The class compiles all the ideas.

**General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.**

**Suggested Assessment Strategies**

*Paper and Pencil*

- Ask students to answer the following:  
Wayne thinks that a good way to model the performance of a baseball player who gets a hit 1 time in 4 at bats is to use a spinner with 4 sections. What assumptions is he making? Are his assumptions valid?  
(9SP4.2)

*Journal*

- Ask students to find an article that includes probabilities and discuss the possible opposing viewpoints.  
(9SP4.3)

*Performance*

- Provide students with a card containing a misleading probability statement. Ask their partner to explain the limitations of the statement.

1. I've spun an unbiased coin 3 times and got 3 heads. It is more likely to be tails than heads if I spin it again.	2. The Rovers Team play the Shooters Team. Rovers can win, lose or draw, so the probability that Rovers will win is $\frac{1}{3}$ .
3. There are 3 red beads and 5 blue beads in a bag. I pick a bead at random. The probability that it is red is $\frac{3}{5}$ .	4. I roll 2 dice and add the results. The probability of getting a total of 6 is $\frac{1}{12}$ because there are 12 different possibilities and 6 is one of them.
5. It is more difficult to throw a 6 than a 3 with a die.	6. Tomorrow it will either rain or not rain, so the probability that it will rain is 0.5.
7. Mr. Brown has to have a major operation. 90% of the people who have this operation make a complete recovery. There is a 90% chance that Mr. Brown will make a complete recovery if he has this operation.	8. If 6 fair dice are thrown at the same time, I am less likely to get 1, 1, 1, 1, 1, 1 than 1, 2, 3, 4, 5, 6.

(9SP4.2)

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 9*

**Lesson 9.1: Probability in Society**

ProGuide: pp. 4-9

Master: 9.6a, 9.6b

CD-ROM: Master 9.16

SB: pp. 424-429

PB: pp. 347-352

**Strand: Statistics and Probability (Chance and Uncertainty)****Outcomes**

*Students will be expected to*

9SP4 Continued...

**Achievement Indicator:**

9SP4.4 *Explain, using examples, how decisions based on probability may be a combination of theoretical probability, experimental probability and subjective judgment.*

**Elaborations—Strategies for Learning and Teaching**

A review of how to determine probability and the difference between theoretical and experimental probabilities may be necessary.

Students should relate to how decision making is affected by the combination of probability and subjective judgements. Ask them to think about the variety of strategies people use when choosing their lottery numbers. Some use the same numbers for repeated lotteries, others use past frequencies to select their numbers, while others allow their numbers to be randomly selected.

A discussion of games of chance could lead to an explanation of how theoretical probability, experimental probability and subjective judgement would play a role in decisions made when playing such games. Students could play a game such as SKUNK. The goal in this game is to accumulate points by rolling dice. Points are accumulated by making several good rolls in a row but making the choice to stop before a bad roll wipes out all the points. As students reflect on whether or not to continue in the game, they should gain a better understanding of probability.

Students should engage in evaluating situations that lend themselves to reasonably accurate predictions, those that are questionable, and those for which the unknowns are not quantifiable. Injury as a result of road accidents with/without seatbelts is a good example for safe prediction. Health professionals predicting that people of lower socioeconomic status will have more health problems is a more questionable situation. There are many situations where the unknowns are too great to make probabilistic arguments. Attempting to find the probability of someone having the same name, age and birthdate as yourself, for example, involves too many unknowns to make an accurate prediction. Teachers could discuss with students:

- What are the reasons for the uncertainty?
- What are the important questions to ask regarding a situation in order to reduce it to probabilistic form?

## General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.

### Suggested Assessment Strategies

#### *Paper and Pencil*

- Students could be asked to give a written or oral report on scenarios such as the following:
  - (i) Jolene’s mother has an important presentation to make in the morning at a conference 200 km away. She has an evening meeting at work tonight. The weather network has reported a 50% probability of snow in the morning. The company she works for would pay for her hotel. What are the probabilities Jolene’s mother has to consider when deciding whether to make the drive tonight or in the morning? Which probability do you think would have the most impact on her decision? Explain.
  - (ii) What probabilities might a government consider when deciding whether to turn a two-lane highway into a four-lane highway?  
(9SP4.4)
- Many insurance companies charge drivers under the age of 25 higher insurance premiums based on the probability of accidents. Ask students to find an article about car insurance costs based on the probability of collision and answer the following questions.
  - (i) In the article, what are the assumptions associated with each probability? Explain.
  - (ii) In your opinion, is there a bias against young drivers?
  - (iii) “Discussions about car insurance costs are based on a combination of experimental probability, theoretical probability, and subjective judgement.” Do you agree or disagree with this statement? Explain.  
(9SP4.1, 9SP4.2, 9SP4.4)

#### *Journal*

- Odette knows that theoretically she has a 1 in 2 chance of getting a head when she flips a coin. Claude had a particular coin that, when flipped 50 times, came up heads 40 of the 50 times. Ingrid feels that even if there is an equal chance of getting heads, heads will appear more often because she feels it is her lucky choice. Ask students to categorize whether or not the three individual’s decisions are based on subjective, experimental, or theoretical probabilities, and describe how each can play a part in decision making.  
(9SP4.4)

### Resources/Notes

#### Authorized Resource

##### *Math Makes Sense 9*

#### Lesson 9.1: Probability in Society

ProGuide: pp. 4-9

Master: 9.6a, 9.6b

CD-ROM: Master 9.16

SB: pp. 424-429

PB: pp. 347-352

#### Suggested Resource

<https://www.k12pl.nl.ca/curr/7-9/math/grade9/links/unit9.html>

- Instructions for a version of the game SKUNK, along with follow-up activities, can be found on the NCTM website.

## Strand: Statistics and Probability (Data Analysis)

### Outcomes

*Students will be expected to*

9SP1 Describe the effect of:

- bias
- use of language
- ethics
- cost
- time and timing
- privacy
- cultural sensitivity

on the collection of data.

[C, CN, R, T]

### Achievement Indicators:

9SP1.1 *Analyze a given case study of data collection, and identify potential problems related to bias, use of language, ethics, cost, time and timing, privacy or cultural sensitivity.*

9SP1.2 *Provide examples to illustrate how bias, use of language, ethics, cost, time and timing, privacy or cultural sensitivity may influence the data.*

### Elaborations—Strategies for Learning and Teaching

In Grade 8, the focus of instruction was to critique ways in which data was presented (8SP1). The emphasis now is to analyze and critique the data collection process.

There are many factors within the data collection process that have the potential to influence the results. Students should consider factors such as the method used, the reliability and usefulness of data, and the ability to make generalizations about the population from a sample. To critically analyze data collection, students must have an understanding of the factors that might lead to problems in the data collection process. A good way to approach this would be for students to analyze survey questions showing only one problem. The following situation, for example, illustrates how timing can affect data collection:

*Free samples of sunscreen are sent to every home in the fall and winter. A mail reply card asks people if they would use the product again.*

When preparing to collect data, appropriate questions are important. Students should consider the following:

- appropriate questions are clearly written, easy to answer, and effective in generating the desired data
- multiple choice questions are useful for identifying respondents' preferences
- questions should be ordered appropriately

Students should analyze how the phrasing of questions might affect the data collected. For a given case study, they should ask questions such as:

- Does asking this question collect the required information?
- Does the question make one response sound right and another one wrong (i.e., does it have bias)?
- Is the question respectful?

When wording survey questions, factors that may influence the responses should be considered.

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**General Outcome: Collect, display and analyze data to solve problems.**


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**Suggested Assessment Strategies**
*Journal*

- Ask students to write about the following:  
Your friend is unclear what the term *bias* means. Develop an example to help explain the term.  
(9SP1.2)

*Paper and Pencil*

- Students could be given a case study such as the following and asked to determine the factor(s) that might affect the data collection. Ask students to rewrite the scenario without any bias.

*A marketing agency wants to determine how Canadians spend their clothing dollars. Jody wrote this question to determine how much is spent on imported clothing.*

*What does your closet contain more of?*

*A. less expensive, foreign made clothes.*

*B. high-quality, made-in Canada clothes.*

- (i) What specific information is Jody trying to obtain?
  - (ii) Rewrite the question to avoid bias and sensitivity issues.  
(9SP1.1)
- Students could be asked to develop their own survey question that involves factor(s) that affect data collection. They could then identify the factor(s) involved and rewrite the question to collect accurate data.  
(9SP1.2)

**Resources/Notes**
**Authorized Resource**

*Math Makes Sense 9*

**Lesson 9.2: Potential Problems with Collecting Data**

ProGuide: pp. 11-16

CD-ROM: Master 9.17

SB: pp. 431-436

PB: pp. 353-357

**Suggested Resource**

<https://www.k12pl.nl.ca/curr/7-9/math/grade9/links/unit9.html>

- Census at School is an international classroom project for students aged 8 to 18. It provides a real survey project which engages students in data collection, graphing and data analysis.

## Strand: Statistics and Probability (Data Analysis)

### Outcomes

*Students will be expected to*

**9SP2 Select and defend the choice of using either a population or a sample of a population to answer a question.**

[C, CN, PS, R]

#### Achievement Indicators:

9SP2.1 *Identify whether a given situation represents the use of a sample or a population.*

9SP2.2 *Provide an example of a situation in which a population may be used to answer a question and justify the choice.*

9SP2.3 *Provide an example of a question where a limitation precludes the use of a population and describe the limitation, e.g., too costly, not enough time, limited resources.*

9SP2.4 *Identify and critique a given example in which a generalization from a sample of a population may or may not be valid for the population.*

### Elaborations—Strategies for Learning and Teaching

To analyze whether a given situation represents a sample or a population, students must clearly understand these terms in the context of data collection and analysis.

Students may think that the term population only refers to a group of people. In fact, the term population can refer to a complete group of anything, such as all of the light bulbs produced by a factory.

Students may not recognize that a group of people referred to as a population could also represent a sample. Examples such as the following could clarify this: all the people living in Gander would be considered a population if a survey only required responses from residents of Gander. If a survey were conducted involving the residents of Newfoundland, then Gander residents would only represent a sample.

Different types of sampling may be examined here. This will be beneficial to students when they are developing a project plan (9SP3) and have to consider the choice of a data collection method.

Students should also be able to determine when it is best to use a population versus a sample when limitations are present. Suppose students want to conduct a survey to find out where to go on a class trip. It would be appropriate to ask everyone in the class. With a large population, however, it is impractical to survey everyone, so students need to use a representative sample group. Suppose they want to conduct a survey to determine if people in their community support year-round schooling. They would have to carefully consider whom to ask and how many people to ask. There are many factors that affect the feasibility of using the entire population.

Students should be encouraged to carefully consider any generalizations made from a sample to a population, as sometimes they may not be valid. For example, students could consider the following scenario: All Grade 9 students in the province were surveyed to determine the start time for the school day. 90% of the students in the St. John's area wanted school to start at 7:50 a.m. as they wanted to finish up early. This sample might not represent the majority of students outside St. John's because they would be considering different factors, such as length of time spent on bus travel to school, when completing their survey. On the other hand, students should realize that if proper sampling techniques are used, the survey results will be valid.

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**General Outcome: Collect, display and analyze data to solve problems.**


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**Suggested Assessment Strategies**
*Paper and Pencil*

- Ask students to identify whether a sample or a population is used in each of the following situations:
    - (i) All residents of a town are asked where a new school for the town should be located.
    - (ii) One out of every 100 iPads is tested for defects.
    - (iii) A student from each junior high class is questioned about removing chocolate milk from the lunch menu.
- (9SP2.1)

*Journal*

- Students could be asked to explain why data collection should include the entire population for situations such as the following:
  - (i) Jet engines, produced by a factory, should be tested before use.
  - (ii) A government official is elected.
  - (iii) Determine whether or not a junior high school should have a uniform.

(9SP2.2)
- Students could be asked to explain the factors that would determine using a sample rather than a population in the following scenarios:
  - (i) Is there a need in Newfoundland and Labrador for a mass vaccination for the flu virus?
  - (ii) Is there a need to check each light bulb coming off an assembly line for defects?
  - (iii) Is there a need to survey all people in an electoral district before an election to predict the winner?

(9SP2.3)

*Performance*

- Each student is given a cut-out of a fish and is asked to label the fish as male or female. Ask students to determine the percentage of male and female in the populations. Place all fish in a fishbowl and depending on class size draw a:
  - (i) small sample
  - (ii) medium sample
  - (iii) large sample

Ask students to calculate the percentage of male/female in the sample and compare this value to the experimental results of the population.

(9SP2.1 9SP2.2)

**Resources/Notes**
**Authorized Resource**

*Math Makes Sense 9*

**Lesson 9.3: Using Samples and Populations to Collect Data**

**Lesson 9.4: Selecting a Sample**

ProGuide: pp. 17-23, 25-29

Master 9.7, 9.8a, 9.8b

CD-ROM: Master 9.18, 9.19

SB: pp. 437-443, 445-449

PB: pp. 358-362, 365-370

**Note**

In *Math Makes Sense 9*, the term “census” refers to population.

## Strand: Statistics and Probability (Data Analysis)

### Outcomes

*Students will be expected to*

**9SP3 Develop and implement a project plan for the collection, display and analysis of data by:**

- **formulating a question for investigation**
- **choosing a data collection method that includes social considerations**
- **selecting a population or a sample**
- **collecting the data**
- **displaying the collected data in an appropriate manner**
- **drawing conclusions to answer the question.**

[C, PS, R, T, V]

### Elaborations—Strategies for Learning and Teaching

Students will plan and carry out a data project to answer a question, and create a rubric to assess the project. The project will include formulating an appropriate question, collecting data from a sample or population, displaying the data, and drawing conclusions. Students will, individually or as a group, design a rubric to assess the project. Proper planning should identify potential problems with questions or data collection methods. Problem solving should permeate the whole process, as students decide on interesting topics, formulate questions, plan the collection of data, implement plans, and analyze results. This outcome should incorporate many of the other Statistics and Probability outcomes. This outcome is meant to be assessed based on the development and implementation of an individualized or group project.

The following are useful guidelines for project-based learning:

- Create teams to work on the project or students may work independently.
- Allow students choice on the topic and on the project presentation.
- Plan the project with drafts and timely benchmarks.
- Students should be familiar with the assessment plan.
- Choose to begin a stage of the project at various times throughout the year.

The following represents a list of ideas for use in the development of statistics projects. Each can be shaped by students to better reflect their interests.

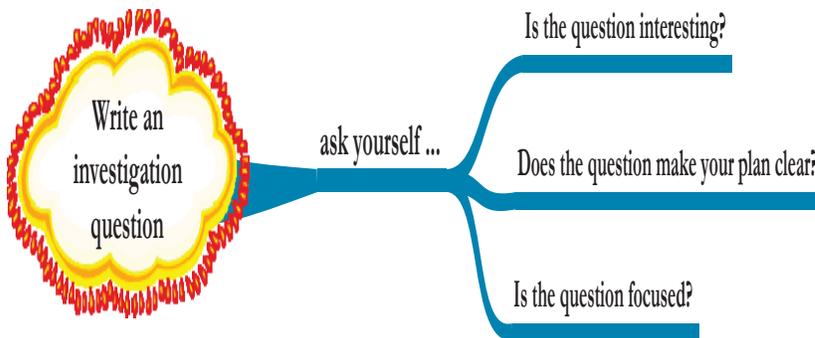
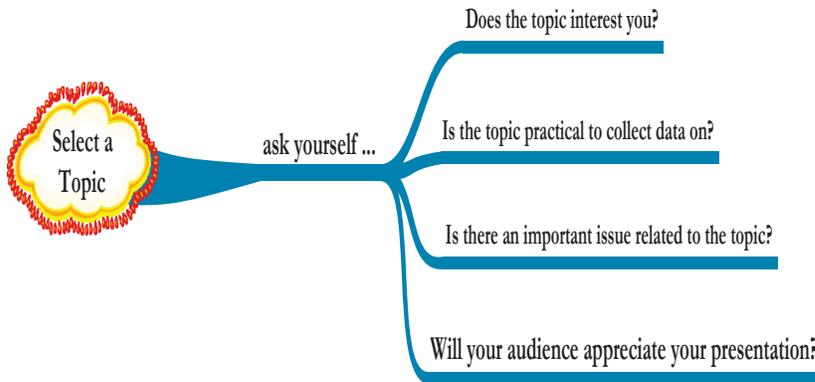
- Determine what type of transportation students in their school use to get to school. Does it differ with the time of year? Does it differ by grade level?
- Determine the most popular types of after-school activities of students in their school. Does it differ by grade level?
- Approach the student council, school council, or community council to suggest issues they would like investigated. Use this as a source for project work.
- Survey or interview Grade 9 students to determine preferred part-time jobs and the amount of money typically earned. They may wish to include jobs such as babysitting, lawn mowing, and paper routes.
- Conduct a survey to find out information related to:
  - (i) their favourite sports team
  - (ii) the effects of social media on sleep patterns, bullying, interests
  - (iii) online versus in-store shopping

**General Outcome: Collect, display and analyze data to solve problems.**

**Suggested Assessment Strategies**

*Paper and Pencil*

- Ask students to brainstorm possible questions, ideas, and/or issues that could be investigated. They could organize their thoughts with mind maps such as the ones shown here.



**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 9*

**Lesson 9.5: Designing a Project Plan**

ProGuide: pp. 34-39

SB: pp. 452-453, 454-456

PB: pp. 371-373

**Suggested Resources**

<https://www.k12pl.nl.ca/curr/7-9/math/grade9/links/unit9.html>

- Creating a Rubric
- Rubric Maker Tools
- Teachnology: Why Rubrics?

**Strand: Statistics and Probability (Chance and Uncertainty)**

**Outcomes**

Students will be expected to  
9SP3 Continued...

**Achievement Indicator:**

- 9SP3.1 *Create a rubric to assess a project that includes the assessment of:*
- *a question for investigation*
  - *the choice of a data collection method that includes social considerations*
  - *the selection of a population or a sample and justifying the choice*
  - *the display of the collected data*
  - *the conclusions to answer the question.*

**Elaborations—Strategies for Learning and Teaching**

In the mathematics curriculum, this would be the first exposure students have had developing rubrics. Individually or as a group, students will design a rubric to assess the project. The rubric should be developed before completing the project plan so they can reflect on appropriate data collection and analysis strategies from previous outcomes. A rubric can help focus students as they create the plan. One approach to preparing students to develop and carry out their own plan would be through a guided example. Students could use a rubric developed by the class to develop, carry out and assess their own project.

- List the criteria in column 1. Students may find it useful to order the criteria according to the sequence of the project.
- For each criterion, record an indicator for each of the four levels of performance. The first row in the example is completed to provide suggestions when students develop their own rubric.
  - Level 1 reflects work that shows little evidence of expected results.
  - Level 2 reflects work that meets minimum expected standards.
  - Level 3 reflects work that meets the expected standards.
  - Level 4 reflects work that is beyond the expected standards.

An example of what a rubric for a data analysis project could look like follows.

Criteria	Level 1	Level 2	Level 3	Level 4
The survey question	<i>- not clear and not related - limited or missing</i>	<i>- fairly clear but not related - some description</i>	<i>- mostly clear and related - adequate description</i>	<i>- very clear, concise, and related - detailed description</i>
The choice of data collection method				
Appropriate choice of sample or population				
The data collection process				
Appropriate data display				
Appropriate conclusions made from results				

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**General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.**

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**Suggested Assessment Strategies**

**Resources/Notes**

**Authorized Resource**

*Math Makes Sense 9*

Lesson 9.5: Designing a Project Plan

ProGuide: pp. 34-39

SB: pp. 452-453, 454-456

PB: pp. 371-373

**Strand: Statistics and Probability (Chance and Uncertainty)**

**Outcomes**

*Students will be expected to*  
 9SP3 Continued...

**Achievement Indicators:**

9SP3.1 *Continued*

9SP3.2 *Develop a project plan that describes:*

- *a question for investigation*
- *the method of data collection that includes social considerations*
- *the method for selecting a population or a sample*
- *the method to be used for collection of the data*
- *the methods for analysis and display of the data.*

9SP3.3 *Complete the project according to the plan, draw conclusions and communicate findings to an audience.*

9SP3.4 *Self-assess the completed project by applying the rubric.*

**Elaborations—Strategies for Learning and Teaching**

Rubrics do not necessarily need four levels of achievement. The following shows a section of a rubric that has three levels of achievement.

Criteria	3 Good	2 Acceptable	1 Not Acceptable
Survey/Interview Question (1)	<i>Appropriate questions are asked; questions should generate all needed data.</i>	<i>Fairly appropriate questions are asked; questions should generate most of the needed data.</i>	<i>Inappropriate questions are asked to gather needed data.</i>
Survey/Interview Question (2)	<i>Questions are sensitive and do not create bias.</i>	<i>Questions are fairly sensitive; some may create bias.</i>	<i>Questions are biased and/or likely to offend respondents.</i>

In some situations, projects may be such that each group of 3 or 4 students takes a different project topic, and tasks are sub-divided to individual members of this smaller group.

Alternatively, it may be desirable to develop a whole-class project in which small groups work on components of a larger question. Later the parts are combined to answer the larger question. For example, a large group wishing to study a common issue may split into smaller groups, with each assigned to study one of the following:

- parental or community opinions
- student views
- teacher or administration views

You may wish to have students present their findings. They could do a written or oral report, incorporating technology if desired. The presentation should outline the project plan and conclusions. To judge if conclusions are reasonable, communication about how data was collected should describe the method of collection, the sample or population used, and why the survey was conducted. The report should also include:

- the question(s) asked in the survey
- appropriate display of the data
- valid conclusions based on the data.

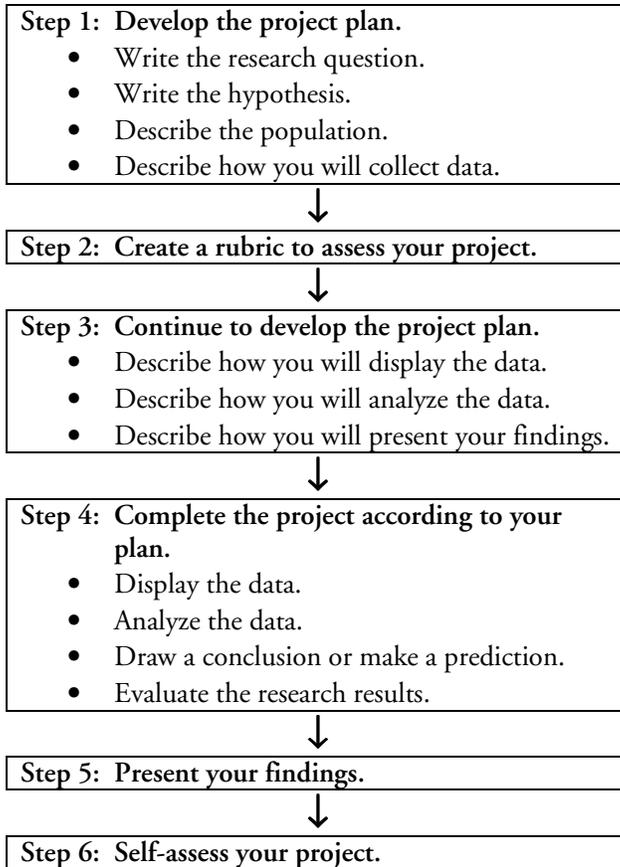
Completing the project will allow students to revisit the various display methods developed in previous grades.

**General Outcome: Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.**

### Suggested Assessment Strategies

#### *Paper and Pencil*

- Ask students to create an organizer, such as the flow chart below, to help organize the research project and carry out the plan.



### Resources/Notes

#### Authorized Resource

*Math Makes Sense 9*

Lesson 9.5: Designing a Project Plan

ProGuide: pp. 34-39

SB: pp. 454-456

PB: pp. 371-373



# Appendix:

## Outcomes with Achievement Indicators Organized by Topic (With Curriculum Guide References)

Topic: Number	General Outcome: Develop number sense.	
Specific Outcomes	Achievement Indicators	Page Reference
It is expected that students will:	<i>The following sets of indicators help determine whether students have met the corresponding specific outcome</i>	
<p>9N1. Demonstrate an understanding of powers with integral bases (excluding base 0) and whole number exponents by:</p> <ul style="list-style-type: none"> <li>• representing repeated multiplication using powers</li> <li>• using patterns to show that a power with an exponent of zero is equal to one</li> <li>• solving problems involving powers.</li> </ul> <p>[C, CN, PS, R]</p>	<p>9N1.1 Demonstrate the difference between the exponent and the base by building models of a given power, such as <math>2^3</math> and <math>3^2</math>.</p> <p>9N1.2 Explain, using repeated multiplication, the difference between two given powers in which the exponent and base are interchanged, e.g., <math>10^3</math> and <math>3^{10}</math>.</p> <p>9N1.3 Express a given power as a repeated multiplication.</p> <p>9N1.4 Express a given repeated multiplication as a power.</p> <p>9N1.5 Explain the role of parentheses in powers by evaluating a given set of powers, e.g., <math>(-2)^4</math>, <math>(-2^4)</math> and <math>-2^4</math>.</p> <p>9N1.6 Demonstrate, using patterns, that <math>a^0</math> is equal to 1 for a given value of <math>a</math> (<math>a \neq 0</math>).</p> <p>9N1.7 Evaluate powers with integral bases (excluding base 0) and whole number exponents.</p>	<p>p. 40</p> <p>p. 40</p> <p>p. 40</p> <p>p. 40</p> <p>p. 42</p> <p>p. 42</p> <p>p. 42</p>
<p>9N2. Demonstrate an understanding of operations on powers with integral bases (excluding base 0) and whole number exponents:</p> <ul style="list-style-type: none"> <li>• <math>(a^m)(a^n) = a^{m+n}</math></li> <li>• <math>a^m \div a^n = a^{m-n}</math>, <math>m &gt; n</math></li> <li>• <math>(a^m)^n = a^{mn}</math></li> <li>• <math>(ab)^m = a^m b^m</math></li> <li>• <math>\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}</math>, <math>b \neq 0</math>.</li> </ul> <p>[C, CN, PS, R, T]</p>	<p>9N2.1 Explain, using examples, the exponent laws of powers with integral bases (excluding base 0) and whole number exponents:</p> <ul style="list-style-type: none"> <li>• <math>(a^m)(a^n) = a^{m+n}</math></li> <li>• <math>a^m \div a^n = a^{m-n}</math>, <math>m &gt; n</math></li> <li>• <math>(a^m)^n = a^{mn}</math></li> <li>• <math>(ab)^m = a^m b^m</math></li> <li>• <math>\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}</math>, <math>b \neq 0</math>.</li> </ul> <p>9N2.2 Evaluate a given expression by applying the exponent laws.</p> <p>9N2.3 Determine the sum of two given powers, e.g., <math>5^2 + 5^3</math>, and record the process.</p> <p>9N2.4 Determine the difference of two given powers, e.g., <math>4^3 - 4^2</math>, and record the process.</p> <p>9N2.5 Identify the error(s) in a given simplification of an expression involving powers.</p>	<p>pp. 44-46</p> <p>p. 46</p> <p>p. 48</p> <p>p. 48</p> <p>p. 48</p>

Topic: Number	General Outcome: Develop number sense.	
Specific Outcomes	Achievement Indicators	Page Reference
It is expected that students will:	<i>The following sets of indicators help determine whether students have met the corresponding specific outcome</i>	
<p>9N3. Demonstrate an understanding of rational numbers by:</p> <ul style="list-style-type: none"> <li>• comparing and ordering rational numbers</li> <li>• solving problems that involve arithmetic operations on rational numbers.</li> </ul> <p>[C, CN, PS, R, T, V]</p>	<p>9N3.1 Order a given set of rational numbers, in fraction and decimal form, by placing them on a number line, e.g., <math>\frac{3}{5}</math>, <math>-0.666\dots</math>, <math>0.5</math>, <math>-\frac{5}{8}</math>.</p> <p>9N3.2 Identify a rational number that is between two given rational numbers.</p> <p>9N3.3 Solve a given problem involving operations on rational numbers in fraction form and decimal form.</p>	<p>p. 54</p> <p>p. 56</p> <p>p. 58</p>
<p>9N4. Explain and apply the order of operations, including exponents, with and without technology.</p> <p>[PS, T]</p>	<p>9N4.1 Solve a given problem by applying the order of operations without the use of technology.</p> <p>9N4.2 Solve a given problem by applying the order of operations with the use of technology.</p> <p>9N4.3 Identify the error in applying the order of operations in a given incorrect solution.</p>	<p>p. 60</p> <p>p. 60</p> <p>p. 60</p>
<p>9N5. Determine the square root of positive rational numbers that are perfect squares.</p> <p>[C, CN, PS, R, T]</p>	<p>9N5.1 Determine whether or not a given rational number is a square number and explain the reasoning.</p> <p>9N5.2 Determine the square root of a given positive rational number that is a perfect square.</p> <p>9N5.3 Identify the error made in a given calculation of a square root, e.g., Is 3.2 the square root of 6.4?</p> <p>9N5.4 Determine a positive rational number given the square root of that positive rational number.</p>	<p>p. 22</p> <p>pp. 24-26</p> <p>p. 26</p> <p>p. 26</p>
<p>9N6. Determine an approximate square root of positive rational numbers that are non-perfect squares.</p> <p>[C, CN, PS, R, T]</p>	<p>9N6.1 Estimate the square root of a given rational number that is not a perfect square using the roots of perfect squares as benchmarks.</p> <p>9N6.2 Determine an approximate square root of a given rational number that is not a perfect square using technology, e.g., calculator, computer.</p> <p>9N6.3 Explain why the square root of a given rational number as shown on a calculator may be an approximation.</p> <p>9N6.4 Identify a number with a square root that is between two given numbers.</p>	<p>p. 28</p> <p>p. 28</p> <p>p. 28</p> <p>p. 30</p>

Topic: Patterns and Relations (Patterns)	General Outcome: Use patterns to describe the world and to solve problems.	
Specific Outcomes It is expected that students will:	Achievement Indicators <i>The following sets of indicators help determine whether students have met the corresponding specific outcome</i>	Page Reference
9PR1. Generalize a pattern arising from a problem-solving context, using linear equations, and verify by substitution. [C, CN, PS, R, V]	9PR1.1 Write an expression representing a given pictorial, oral or written pattern. 9PR1.2 Write a linear equation to represent a given context. 9PR1.3 Write a linear equation representing the pattern in a given table of values and verify the equation by substituting values from the table. 9PR1.4 Solve, using a linear equation, a given problem that involves pictorial, oral and written linear patterns. 9PR1.5 Describe a context for a given linear equation.	p. 66  p. 66  pp. 66-68  p. 68  p. 68
9PR2. Graph a linear relation, analyze the graph, and interpolate or extrapolate to solve problems. [C, CN, PS, R, T, V]	9PR2.1 Describe the pattern found in a given graph. 9PR2.2 Graph a given linear relation, including horizontal and vertical lines. 9PR2.3 Match given equations of linear relations with their corresponding graphs. 9PR2.4 Interpolate the approximate value of one variable on a given graph given the value of the other variable. 9PR2.5 Extrapolate the approximate value of one variable from a given graph given the value of the other variable. 9PR2.6 Solve a given problem by graphing a linear relation and analyzing the graph.	pp. 70-74 pp. 70-74  p. 74  p. 76  p. 76  p. 76

Topic: Patterns and Relations (Variables and Equations)	General Outcome: Represent algebraic expressions in multiple ways.	
Specific Outcomes	Achievement Indicators	Page Reference
It is expected that students will:	<i>The following sets of indicators help determine whether students have met the corresponding specific outcome</i>	
<p>9PR3. Model and solve problems using linear equations of the form:</p> <ul style="list-style-type: none"> <li>• <math>ax = b</math></li> <li>• <math>\frac{a}{x} = b, x \neq 0</math></li> <li>• <math>ax + b = c</math></li> <li>• <math>\frac{x}{a} + b = c, a \neq 0</math></li> <li>• <math>ax = b + cx</math></li> <li>• <math>a(x + b) = c</math></li> <li>• <math>ax + b = cx + d</math></li> <li>• <math>a(bx + c) = d(ex + f)</math></li> <li>• <math>\frac{a}{x} = b, x \neq 0</math></li> </ul> <p>where <math>a, b, c, d, e</math> and <math>f</math> are rational numbers.</p> <p>[C, CN, PS, V]</p>	<p>9PR3.1 Model the solution of a given linear equation using concrete or pictorial representations, and record the process.</p> <p>9PR3.2 Determine, by substitution, whether a given rational number is a solution to a given linear equation.</p> <p>9PR3.3 Solve a given linear equation symbolically.</p> <p>9PR3.4 Identify and correct an error in a given incorrect solution of a linear equation.</p> <p>9PR3.5 Represent a given problem using a linear equation.</p> <p>9PR3.6 Solve a given problem using a linear equation and record the process.</p>	<p>pp. 98-100</p> <p>pp. 98-100</p> <p>pp. 98-100</p> <p>p. 102</p> <p>p. 102</p> <p>p. 102</p>
<p>9PR4. Explain and illustrate strategies to solve single variable linear inequalities with rational coefficients within a problem-solving context.</p> <p>[C, CN, PS, R, V]</p>	<p>9PR4.1 Translate a given problem into a single variable linear inequality using the symbols <math>\geq, &gt;, &lt;</math> or <math>\leq</math>.</p> <p>9PR4.2 Determine if a given rational number is a possible solution of a given linear inequality.</p> <p>9PR4.3 Graph the solution of a given linear inequality on a number line.</p> <p>9PR4.4 Generalize and apply a rule for adding and subtracting a positive or negative number to determine the solution of a given inequality.</p> <p>9PR4.5 Generalize and apply a rule for multiplying or dividing by a positive or negative number to determine the solution of a given inequality.</p> <p>9PR4.6 Solve a given linear inequality algebraically and explain the process orally or in written form.</p> <p>9PR4.7 Compare and explain the process for solving a given linear equation to the process for solving a given linear inequality.</p> <p>9PR4.8 Compare and explain the solution of a given linear equation to the solution of a given linear inequality.</p> <p>9PR4.9 Verify the solution of a given linear inequality using substitution for multiple elements in the solution.</p> <p>9PR4.10 Solve a given problem involving a single variable linear inequality and graph the solution.</p>	<p>p. 104</p> <p>p. 104</p> <p>p. 104</p> <p>p. 106</p> <p>p. 106</p> <p>p. 106</p> <p>p. 106</p> <p>p. 106</p> <p>p. 106</p> <p>p. 108</p> <p>p. 108</p>

Topic: Patterns and Relations (Variables and Equations)	General Outcome: Represent algebraic expressions in multiple ways.	
Specific Outcomes	Achievement Indicators	Page Reference
It is expected that students will:	<i>The following sets of indicators help determine whether students have met the corresponding specific outcome</i>	
9PR5. Demonstrate an understanding of polynomials (limited to polynomials of degree less than or equal to 2). [C, CN, R, V]	9PR5.1 Identify the variables, degree, number of terms and coefficients, including the constant term, of a given simplified polynomial expression. 9PR5.2 Create a concrete model or a pictorial representation for a given polynomial expression. 9PR5.3 Write the expression for a given model of a polynomial. 9PR5.4 Match equivalent polynomial expressions given in simplified form.	p. 82  p. 82  p. 82  p. 82
9PR6. Model, record and explain the operations of addition and subtraction of polynomial expressions, concretely, pictorially and symbolically (limited to polynomials of degree less than or equal to 2). [C, CN, PS, R, V]	9PR6.1 Identify equivalent polynomial expressions from a given set of polynomial expressions, including pictorial and symbolic representations. 9PR6.2 Model addition of two given polynomial expressions concretely or pictorially and record the process symbolically. 9PR6.3 Model subtraction of two given polynomial expressions concretely or pictorially and record the process symbolically. 9PR6.4 Apply a personal strategy for addition and subtraction of given polynomial expressions, and record the process symbolically. 9PR6.5 Identify the error(s) in a given simplification of a given polynomial expression.	p. 84  p. 84  p. 86  p. 86  p. 86
9PR7. Model, record and explain the operations of multiplication and division of polynomial expressions (limited to polynomials of degree less than or equal to 2) by monomials, concretely, pictorially and symbolically. [C, CN, R, V]	9PR7.1 Model multiplication of a given polynomial expression by a given monomial concretely or pictorially and record the process symbolically. 9PR7.2 Model division of a given polynomial expression by a given monomial concretely or pictorially and record the process symbolically. 9PR7.3 Apply a personal strategy for multiplication and division of a given polynomial expression by a given monomial. 9PR7.4 Provide examples of equivalent polynomial expressions. 9PR7.5 Identify the error(s) in a given simplification of a given polynomial expression.	p. 88  p. 90  p. 92  p. 92  p. 92

<b>Topic:</b> Shape and Space (Measurement)	<b>General Outcome:</b> Use direct or indirect measurement to solve problems.	
<b>Specific Outcomes</b> It is expected that students will:	<b>Achievement Indicators</b> <i>The following sets of indicators help determine whether students have met the corresponding specific outcome</i>	<b>Page Reference</b>
<p>9SS1. Solve problems and justify the solution strategy using the following circle properties:</p> <ul style="list-style-type: none"> <li>• the perpendicular from the centre of a circle to a chord bisects the chord</li> <li>• the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc</li> <li>• the inscribed angles subtended by the same arc are congruent</li> <li>• a tangent to a circle is perpendicular to the radius at the point of tangency.</li> </ul> <p>[C, CN, PS, R, T, V]</p>	<p>9SS1.1 Explain the relationship between the tangent of a circle and the radius at the point of tangency.</p> <p>9SS1.2 Solve a given problem involving application of one or more of the circle properties.</p> <p>9SS1.3 Explain the relationship between the perpendicular from the centre of the circle and a chord.</p> <p>9SS1.4 Explain the relationship between the measure of the central angle and the inscribed angle subtended by the same arc.</p> <p>9SS1.5 Determine the measure of a given angle inscribed in a semicircle using the circle properties.</p> <p>9SS1.6 Explain the relationship between inscribed angles subtended by the same arc.</p>	<p>p. 132</p> <p>pp. 132-140</p> <p>p. 134</p> <p>pp. 136-138</p> <p>p. 138</p> <p>p. 140</p>

<b>Topic:</b> Shape and Space (3-D Objects and 2-D Shapes)	<b>General Outcome:</b> Describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.	
<b>Specific Outcomes</b> It is expected that students will:	<b>Achievement Indicators</b> <i>The following sets of indicators help determine whether students have met the corresponding specific outcome</i>	<b>Page Reference</b>
9SS2. Determine the surface area of composite 3-D objects to solve problems. [C, CN, PS, R, V]	9SS2.1 Determine the overlap in a given concrete composite 3-D object, and explain its effect on determining the surface area (limited to right cylinders, right rectangular prisms and right triangular prisms).	pp. 32-34
	9SS2.2 Determine the surface area of a given concrete composite 3-D object (limited to right cylinders, right rectangular prisms and right triangular prisms).	pp. 32-34
	9SS2.3 Solve a given problem involving surface area.	pp. 32-34
9SS3. Demonstrate an understanding of similarity of polygons. [C, CN, PS, R, V]	9SS3.1 Determine if the polygons in a given set are similar and explain the reasoning.	p. 118
	9SS3.2 Draw a polygon similar to a given polygon and explain why the two are similar.	p. 118
	9SS3.3 Solve a given problem using the properties of similar polygons.	p. 118

<b>Topic:</b> Shape and Space (Transformations)	<b>General Outcome:</b> Describe and analyze position and motion of objects and shapes.	
<b>Specific Outcomes</b> It is expected that students will:	<b>Achievement Indicators</b> <i>The following sets of indicators help determine whether students have met the corresponding specific outcome</i>	<b>Page Reference</b>
9SS4. Draw and interpret scale diagrams of 2-D shapes. [CN, R, T, ,V]	9SS4.1 Identify an example in print and electronic media, e.g., newspapers, the Internet, of a scale diagram and interpret the scale factor.	pp.114-116
	9SS4.2 Draw a diagram to scale that represents an enlargement or reduction of a given 2-D shape.	pp.114-116
	9SS4.3 Determine the scale factor for a given diagram drawn to scale.	pp.114-116
	9SS4.4 Determine if a given diagram is proportional to the original 2-D shape and, if it is, state the scale factor.	pp.114-116
	9SS4.5 Solve a given problem that involves a scale diagram by applying the properties of similar triangles.	p. 118

Topic: Shape and Space (Transformations)	General Outcome: Describe and analyze position and motion of objects and shapes.	
Specific Outcomes It is expected that students will:	Achievement Indicators	Page Reference
9SS5. Demonstrate an understanding of line and rotation symmetry. [C, CN, PS, V]	<p><i>The following sets of indicators help determine whether students have met the corresponding specific outcome</i></p> <p>9SS5.1 Classify a given set of 2-D shapes or designs according to the number of lines of symmetry.</p> <p>9SS5.2 Complete a 2-D shape or design given one half of the shape or design and a line of symmetry.</p> <p>9SS5.3 Determine if a given 2-D shape or design has rotation symmetry about the point at the centre of the shape or design and, if it does, state the order and angle of rotation.</p> <p>9SS5.4 Rotate a given 2-D shape about a vertex and draw the resulting image.</p> <p>9SS5.5 Identify a line of symmetry or the order and angle of rotation symmetry in a given tessellation.</p> <p>9SS5.6 Identify and describe the types of symmetry created in a given piece of artwork.</p> <p>9SS5.7 Create or provide a piece of artwork that demonstrates line and rotation symmetry, and identify the line(s) of symmetry and the order and angle of rotation.</p> <p>9SS5.8 Determine whether or not two given 2-D shapes on the Cartesian plane are related by either rotation or line symmetry.</p> <p>9SS5.9 Identify the type of symmetry that arises from a given transformation on the Cartesian plane.</p> <p>9SS5.10 Complete, concretely or pictorially, a given transformation of a 2-D shape on a Cartesian plane and describe the type of symmetry that results.</p> <p>9SS5.11 Draw, on a Cartesian plane, the translation image of a given shape using a given translation rule, such as <math>R_2</math>, <math>U_3</math> or <math>\rightarrow\rightarrow</math>, <math>\uparrow\uparrow\uparrow</math>, label each vertex and its corresponding ordered pair and determine why the translation may or may not result in line or rotation symmetry.</p>	<p>p. 120</p> <p>p. 120</p> <p>p. 122</p> <p>p. 122</p> <p>p. 124</p> <p>p. 124</p> <p>p. 124</p> <p>p. 126</p> <p>p. 126</p> <p>p. 126</p>

<b>Topic:</b> Statistics and Probability (Data Analysis)	<b>General Outcome:</b> Collect, display and analyze data to solve problems.	
<b>Specific Outcomes</b> It is expected that students will:	<b>Achievement Indicators</b> <i>The following sets of indicators help determine whether students have met the corresponding specific outcome</i>	<b>Page Reference</b>
9SP1. Describe the effect of: <ul style="list-style-type: none"> <li>• bias</li> <li>• use of language</li> <li>• ethics</li> <li>• cost</li> <li>• time and timing</li> <li>• privacy</li> <li>• cultural sensitivity</li> </ul> on the collection of data. [C, CN, R, T]	9SP1.1 Analyze a given case study of data collection, and identify potential problems related to bias, use of language, ethics, cost, time and timing, privacy or cultural sensitivity. 9SP1.2 Provide examples to illustrate how bias, use of language, ethics, cost, time and timing, privacy or cultural sensitivity may influence the data.	p. 152  p. 152
9SP2. Select and defend the choice of using either a population or a sample of a population to answer a question. [C, CN, PS, R]	9SP2.1 Identify whether a given situation represents the use of a sample or a population. 9SP2.2 Provide an example of a situation in which a population may be used to answer a question and justify the choice. 9SP2.3 Provide an example of a question where a limitation precludes the use of a population and describe the limitation, e.g., too costly, not enough time, limited resources. 9SP2.4 Identify and critique a given example in which a generalization from a sample of a population may or may not be valid for the population.	p. 154  p. 154  p. 154  p. 154



<b>Topic:</b> Statistics and Probability (Chance and Uncertainty)	<b>General Outcome:</b> Use experimental or theoretical probabilities to represent and solve problems involving uncertainty.	
<b>Specific Outcomes</b> It is expected that students will:	<b>Achievement Indicators</b> <i>The following sets of indicators help determine whether students have met the corresponding specific outcome</i>	<b>Page Reference</b>
9SP4. Demonstrate an understanding of the role of probability in society. [C, CN, R, T]	<p>9SP4.1 Provide an example from print and electronic media, e.g., newspapers, the Internet, where probability is used.</p> <p>9SP4.2 Identify the assumptions associated with a given probability and explain the limitations of each assumption.</p> <p>9SP4.3 Explain how a single probability can be used to support opposing positions.</p> <p>9SP4.4 Explain, using examples, how decisions based on probability may be a combination of theoretical probability, experimental probability and subjective judgement.</p>	<p>p. 146</p> <p>p. 148</p> <p>p. 148</p> <p>p. 150</p>

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