Applied Mathematics 1202

Curriculum Guide 2015

Newfoundland
Labrador

Education and Early Childhood Development

Contents

Acknowledgements

INTRODUCTION

Background

The curriculum guide communicates high expectations for students.

The Mathematics curriculum guides for Newfoundland and Labrador have been derived from *The Common Curriculum Framework for 10-12 Mathematics: Western and Northern Canadian Protocol*, January 2008. These guides incorporate the conceptual framework for Grades 10 to 12 Mathematics and the general outcomes, specific outcomes and achievement indicators established in the common curriculum framework. They also include suggestions for teaching and learning, suggested assessment strategies, and an identification of the associated resource match between the curriculum and authorized, as well as recommended, resource materials.

Applied Mathematics 1202 was originally implemented in 2011.

Beliefs About Students and Mathematics

Mathematical understanding is fostered when students build on their own experiences and prior knowledge.

Students are curious, active learners with individual interests, abilities and needs. They come to classrooms with varying knowledge, life experiences and backgrounds. A key component in developing mathematical literacy is making connections to these backgrounds and experiences.

Students learn by attaching meaning to what they do, and they need to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. Through the use of manipulatives and a variety of pedagogical approaches, teachers can address the diverse learning styles, cultural backgrounds and developmental stages of students, and enhance within them the formation of sound, transferable mathematical understandings. Students at all levels benefit from working with a variety of materials, tools and contexts when constructing meaning about new mathematical ideas. Meaningful student discussions provide essential links among concrete, pictorial and symbolic representations of mathematical concepts.

The learning environment should value and respect the diversity of students' experiences and ways of thinking, so that students feel comfortable taking intellectual risks, asking questions and posing conjectures. Students need to explore problem-solving situations in order to develop personal strategies and become mathematically literate. They must come to understand that it is acceptable to solve problems in a variety of ways and that a variety of solutions may be acceptable.

Affective Domain

To experience success, students must learn to set achievable goals and assess themselves as they work toward these goals.

A positive attitude is an important aspect of the affective domain and has a profound impact on learning. Environments that create a sense of belonging, encourage risk taking and provide opportunities for success help develop and maintain positive attitudes and self-confidence within students. Students with positive attitudes toward learning mathematics are likely to be motivated and prepared to learn, participate willingly in classroom activities, persist in challenging situations and engage in reflective practices.

Teachers, students and parents need to recognize the relationship between the affective and cognitive domains, and attempt to nurture those aspects of the affective domain that contribute to positive attitudes. To experience success, students must learn to set achievable goals and assess themselves as they work toward these goals.

Striving toward success and becoming autonomous and responsible learners are ongoing, reflective processes that involve revisiting, asssessing and revising personal goals.

Goals For Students

Mathematics education must prepare students to use mathematics confidently to solve problems.

The main goals of mathematics education are to prepare students to:

- use mathematics confidently to solve problems
- communicate and reason mathematically
- appreciate and value mathematics
- make connections between mathematics and its applications
- commit themselves to lifelong learning
- become mathematically literate adults, using mathematics to contribute to society.

Students who have met these goals will:

- gain understanding and appreciation of the contributions of mathematics as a science, philosophy and art
- exhibit a positive attitude toward mathematics
- engage and persevere in mathematical tasks and projects
- contribute to mathematical discussions
- take risks in performing mathematical tasks
- exhibit curiosity.

CONCEPTUAL FRAMEWORK FOR 10-12 MATHEMATICS

The chart below provides an overview of how mathematical processes and the nature of mathematics influence learning outcomes.

Grade / Topics	10	11	12
The topics of study vary in the courses for grades 10–12 mathematics. Topics in the pathways include: Algebra Financial Mathematics Geometry Logical Reasoning Mathematics Research Project Measurement Number Permutations, Combinations and Binomial Theorem Probability Relations and Functions Statistics Trigonometry		GENERAL OUTCOMES, SPECIFIC OUTCOMES AND ACHIEVEMENT INDICATORS	

NATURE OF MATHEMATICS:

Change, Constancy, Number Sense, Patterns, Relationships, Spatial Sense, Uncertainty

MATHEMATICAL PROCESSES: Communication, Connections, Mental Mathematics and Estimation, Problem Solving, Reasoning, Technology, Visualization

Mathematical Processes

- *Communication [C]*
- *Connections [CN]*
- *Mental Mathematics and Estimation [ME]*
- *Problem Solving [PS]*
- *Reasoning [R]*
- *Technology [T]*
- *Visualization [V]*

There are critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and embrace lifelong learning in mathematics. Students are expected to:

- communicate in order to learn and express their understanding

- connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines

- demonstrate fluency with mental mathematics and estimation

- develop and apply new mathematical knowledge through problem solving

- develop mathematical reasoning

- select and use technologies as tools for learning and for solving problems

- develop visualization skills to assist in processing information, making connections and solving problems.

This curriculum guide incorporates these seven interrelated mathematical processes that are intended to permeate teaching and learning.

Communication [C]

Students must be able to communicate mathematical ideas in a variety of ways and contexts.

Students need opportunities to read about, represent, view, write about, listen to and discuss mathematical ideas. These opportunities allow students to create links between their own language and ideas, and the formal language and symbols of mathematics.

Communication is important in clarifying, reinforcing and modifying ideas, attitudes and beliefs about mathematics. Students should be encouraged to use a variety of forms of communication while learning mathematics. Students also need to communicate their learning using mathematical terminology.

Communication helps students make connections among concrete, pictorial, symbolic, oral, written and mental representations of mathematical ideas.

Connections [CN]

Through connections, students begin to view mathematics as useful and relevant.

Contextualization and making connections to the experiences of learners are powerful processes in developing mathematical understanding. When mathematical ideas are connected to each other or to real-world phenomena, students begin to view mathematics as useful, relevant and integrated.

Learning mathematics within contexts and making connections relevant to learners can validate past experiences and increase student willingness to participate and be actively engaged.

The brain is constantly looking for and making connections. "Because the learner is constantly searching for connections on many levels, educators need to *orchestrate the experiences* from which learners extract understanding … Brain research establishes and confirms that multiple complex and concrete experiences are essential for meaningful learning and teaching" (Caine and Caine, 1991, p.5).

Mental Mathematics and Estimation [ME]

Mental mathematics and estimation are fundamental components of number sense.

Mental mathematics is a combination of cognitive strategies that enhance flexible thinking and number sense. It is calculating mentally without the use of external memory aids.

Mental mathematics enables students to determine answers without paper and pencil. It improves computational fluency by developing efficiency, accuracy and flexibility.

"Even more important than performing computational procedures or using calculators is the greater facility that students need—more than ever before—with estimation and mental math" (National Council of Teachers of Mathematics, May 2005).

Students proficient with mental mathematics "... become liberated from calculator dependence, build confidence in doing mathematics, become more flexible thinkers and are more able to use multiple approaches to problem solving" (Rubenstein, 2001, p. 442).

Mental mathematics "... provides the cornerstone for all estimation processes, offering a variety of alternative algorithms and nonstandard techniques for finding answers" (Hope, 1988, p. v).

Estimation is used for determining approximate values or quantities or for determining the reasonableness of calculated values. It often uses benchmarks or referents. Students need to know when to estimate, how to estimate and what strategy to use.

Estimation assists individuals in making mathematical judgements and in developing useful, efficient strategies for dealing with situations in daily life.

Problem Solving [PS]

Learning through problem solving should be the focus of mathematics at all grade levels.

Learning through problem solving should be the focus of mathematics at all grade levels. When students encounter new situations and respond to questions of the type, "How would you know?" or "How could you ...?", the problem-solving approach is being modelled. Students develop their own problem-solving strategies by listening to, discussing and trying different strategies.

A problem-solving activity requires students to determine a way to get from what is known to what is unknown. If students have already been given steps to solve the problem, it is not a problem, but practice. A true problem requires students to use prior learning in new ways and contexts. Problem solving requires and builds depth of conceptual understanding and student engagement.

Problem solving is a powerful teaching tool that fosters multiple, creative and innovative solutions. Creating an environment where students openly seek and engage in a variety of strategies for solving problems empowers students to explore alternatives and develops confident, cognitive mathematical risk takers.

Reasoning [R]

Mathematical reasoning helps students think logically and make sense of mathematics.

Mathematical reasoning helps students think logically and make sense of mathematics. Students need to develop confidence in their abilities to reason and justify their mathematical thinking. High-order questions challenge students to think and develop a sense of wonder about mathematics.

Mathematical experiences in and out of the classroom provide opportunities for students to develop their ability to reason. Students can explore and record results, analyze observations, make and test generalizations from patterns, and reach new conclusions by building upon what is already known or assumed to be true.

Reasoning skills allow students to use a logical process to analyze a problem, reach a conclusion and justify or defend that conclusion.

Technology [T]

Technology contributes to the learning of a wide range of mathematical outcomes and enables students to explore and create patterns, examine relationships, test conjectures and solve problems.

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Technology can be used to:

- explore and demonstrate mathematical relationships and patterns

- organize and display data

- extrapolate and interpolate

- assist with calculation procedures as part of solving problems

- decrease the time spent on computations when other mathematical learning is the focus

- reinforce the learning of basic facts

- develop personal procedures for mathematical operations

- create geometric patterns

- simulate situations

- develop number sense.

Technology contributes to a learning environment in which the growing curiosity of students can lead to rich mathematical discoveries at all grade levels.

Visualization [V]

Visualization "involves thinking in pictures and images, and the ability to perceive, transform and recreate different aspects of the visual-spatial world" (Armstrong, 1993, p. 10). The use of visualization in the study of mathematics provides students with opportunities to understand mathematical concepts and make connections among them.

Visualization is fostered through the use of concrete materials, technology and a variety of visual representations.

Visual images and visual reasoning are important components of number, spatial and measurement sense. Number visualization occurs when students create mental representations of numbers.

Being able to create, interpret and describe a visual representation is part of spatial sense and spatial reasoning. Spatial visualization and reasoning enable students to describe the relationships among and between 3-D objects and 2-D shapes.

Measurement visualization goes beyond the acquisition of specific measurement skills. Measurement sense includes the ability to determine when to measure, when to estimate and which estimation strategies to use (Shaw and Cliatt, 1989).

Nature of Mathematics

- *Change*
- *Constancy*
- *Number Sense*
- *Patterns*
- *Relationships*
- *Spatial Sense*
- *Uncertainty*

Mathematics is one way of trying to understand, interpret and describe our world. There are a number of components that define the nature of mathematics and these are woven throughout this curiculum guide. The components are change, constancy, number sense, patterns, relationships, spatial sense and uncertainty.

Change

Change is an integral part of mathematics and the learning of mathematics.

It is important for students to understand that mathematics is dynamic and not static. As a result, recognizing change is a key component in understanding and developing mathematics.

Within mathematics, students encounter conditions of change and are required to search for explanations of that change. To make predictions, students need to describe and quantify their observations, look for patterns, and describe those quantities that remain fixed and those that change. For example, the sequence 4, 6, 8, 10, 12, … can be described as:

- the number of a specific colour of beads in each row of a beaded design
- skip counting by 2s, starting from 4
- an arithmetic sequence, with first term 4 and a common difference of 2
- a linear function with a discrete domain

(Steen, 1990, p. 184).

Constancy

Constancy is described by the terms stability, conservation, equilibrium, steady state and symmetry.

Different aspects of constancy are described by the terms stability, conservation, equilibrium, steady state and symmetry (AAAS-Benchmarks, 1993, p.270). Many important properties in mathematics and science relate to properties that do not change when outside conditions change. Examples of constancy include the following:

*   The ratio of the circumference of a teepee to its diameter is the same regardless of the length of the teepee poles.

*   The sum of the interior angles of any triangle is 180°.

*   The theoretical probability of flipping a coin and getting heads is 0.5.

Some problems in mathematics require students to focus on properties that remain constant. The recognition of constancy enables students to solve problems involving constant rates of change, lines with constant slope, direct variation situations or the angle sums of polygons.

Number Sense

An intuition about number is the most important foundation of a numerate child.

Number sense, which can be thought of as intuition about numbers, is the most important foundation of numeracy (British Columbia Ministry of Education, 2000, p.146).

A true sense of number goes well beyond the skills of simply counting, memorizing facts and the situational rote use of algorithms. Mastery of number facts is expected to be attained by students as they develop their number sense. This mastery allows for facility with more complex computations but should not be attained at the expense of an understanding of number.

Number sense develops when students connect numbers to their own real-life experiences and when students use benchmarks and referents. This results in students who are computationally fluent and flexible with numbers and who have intuition about numbers. The evolving number sense typically comes as a by product of learning rather than through direct instruction. It can be developed by providing rich mathematical tasks that allow students to make connections to their own expereinces and their previous learning.

Patterns

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns.

Mathematics is about recognizing, describing and working with numerical and non-numerical patterns. Patterns exist in all strands of mathematics.

Working with patterns enables students to make connections within and beyond mathematics. These skills contribute to students' interaction with, and understanding of, their environment.

Patterns may be represented in concrete, visual or symbolic form. Students should develop fluency in moving from one representation to another.

Students must learn to recognize, extend, create and use mathematical patterns. Patterns allow students to make predictions and justify their reasoning when solving routine and non-routine problems.

Learning to work with patterns in the early grades helps students develop algebraic thinking, which is foundational for working with more abstract mathematics.

Relationships

Mathematics is used to describe and explain relationships.

Mathematics is one way to describe interconnectedness in a holistic worldview. Mathematics is used to describe and explain relationships. As part of the study of mathematics, students look for relationships among numbers, sets, shapes, objects and concepts. The search for possible relationships involves collecting and analyzing data and describing relationships visually, symbolically, orally or in written form.

Spatial Sense

Spatial sense offers a way to interpret and reflect on the physical environment.

Spatial sense involves visualization, mental imagery and spatial reasoning. These skills are central to the understanding of mathematics.

Spatial sense is developed through a variety of experiences and interactions within the environment. The development of spatial sense enables students to solve problems involving 3-D objects and 2-D shapes and to interpret and reflect on the physical environment and its 3-D or 2-D representations.

Some problems involve attaching numerals and appropriate units (measurement) to dimensions of shapes and objects. Spatial sense allows students to make predictions about the results of changing these dimensions; e.g., doubling the length of the side of a square increases the area by a factor of four. Ultimately, spatial sense enables students to communicate about shapes and objects and to create their own representations.

Uncertainty

Uncertainty is an inherent part of making predictions.

In mathematics, interpretations of data and the predictions made from data may lack certainty.

Events and experiments generate statistical data that can be used to make predictions. It is important to recognize that these predictions (interpolations and extrapolations) are based upon patterns that have a degree of uncertainty.

The quality of the interpretation is directly related to the quality of the data. An awareness of uncertainty allows students to assess the reliability of data and data interpretation.

Chance addresses the predictability of the occurrence of an outcome. As students develop their understanding of probability, the language of mathematics becomes more specific and describes the degree of uncertainty more accurately.

Essential Graduation Learnings

Essential graduation learnings are statements describing the knowledge, skills and attitudes expected of all students who graduate from high school. Essential graduation learnings are cross-curricular in nature and comprise different areas of learning: *aesthetic expression, citizenship, communication, personal development, problem solving, technological competence* and *spiritual and moral development.*

Aesthetic Expression

Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.

Citizenship

Graduates will be able to assess social, cultural, economic and environmental interdependence in a local and global context.

Communication

Graduates will be able to use the listening, viewing, speaking, reading and writing modes of language(s) and mathematical and scientific concepts and symbols to think, learn and communicate effectively.

Personal Development

Graduates will be able to continue to learn and to pursue an active, healthy lifestyle.

Problem Solving

Graduates will be able to use the strategies and processes needed to solve a wide variety of problems, including those requiring language and mathematical and scientific concepts.

Technological Competence

Graduates will be able to use a variety of technologies, demonstrate an understanding of technological applications, and apply appropriate technologies for solving problems.

Spiritual and Moral Development

Graduates will be able to demonstrate an understanding and appreciation for the place of belief systems in shaping the development of moral values and ethical conduct.

See *Foundations for the Atlantic Canada Mathematics Curriculum*, pages 4-6.

The mathematics curriculum is designed to make a significant contribution towards students' meeting each of the essential graduation learnings (EGLs), with the communication, problem-solving and technological competence EGLs relating particularly well to the mathematical processes.

Outcomes and Achievement Indicators

The curriculum is stated in terms of general outcomes, specific outcomes and achievement indicators.

General Outcomes

General outcomes are overarching statements about what students are expected to learn in each course.

Specific Outcomes

Specific outcomes are statements that identify the specific skills, understanding and knowledge that students are required to attain by the end of a given course.
In the specific outcomes, the word *including* indicates that any ensuing items must be addressed to fully meet the learning outcome. The phrase *such as* indicates that the ensuing items are provided for illustrative purposes or clarification, and are not requirements that must be addressed to fully meet the learning outcome.

Achievement Indicators

Achievement indicators are samples of how students may demonstrate their achievement of the goals of a specific outcome. The range of samples provided is meant to reflect the scope of the specific outcome.

Specific curriculum outcomes represent the means by which students work toward accomplishing the general curriculum outcomes and ultimately, the essential graduation learnings.

Program Organization

Program Level	Course 1	Course 2	Course 3	Course 4
Advanced	Mathematics	Mathematics 2200	Mathematics 3200	Calculus 3208
Academic	1201	Mathematics 2201	Mathematics 3201	
Applied	Mathematics 1202	Mathematics 2202	Mathematics 3202	

The applied program is designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into the majority of trades and for direct entry into the workforce.

The academic and advanced programs are designed to provide students with the mathematical understandings and critical-thinking skills identified for entry into post-secondary programs. Students who complete the advanced program will be better prepared for programs that require the study of calculus.

The programs aim to prepare students to make connections between mathematics and its applications and to become numerate adults, using mathematics to contribute to society.

Summary

The conceptual framework for Grades 10-12 Mathematics (p. 3) describes the nature of mathematics, mathematical processes and the mathematical concepts to be addressed. The components are not meant to stand alone. Activities that take place in the mathematics classroom should result from a problem-solving approach, be based on mathematical processes and lead students to an understanding of the nature of mathematics through specific knowledge, skills and attitudes among and between topics.

ASSESSMENT AND EVALUATION

Purposes of Assessment

What learning is assessed and evaluated, how it is assessed and evaluated, and how results are communicated send clear messages to students and others about what is really valued.

Assessment techniques are used to gather information for evaluation. Information gathered through assessment helps teachers determine students' strengths and needs in their achievement of mathematics and guides future instructional approaches.

Teachers are encouraged to be flexible in assessing the learning success of all students and to seek diverse ways in which students might demonstrate what they know and are able to do.

Evaluation involves the weighing of the assessment information against a standard in order to make an evaluation or judgment about student achievement.

Assessment has three interrelated purposes:

- assessment *for* learning to guide and inform instruction;

- assessment *as* learning to involve students in self-assessment and setting goals for their own learning; and

- assessment *of* learning to make judgements about student performance in relation to curriculum outcomes.

Assessment *for* Learning

Assessment *for* learning involves frequent, interactive assessments designed to make student understanding visible. This enables teachers to identify learning needs and adjust teaching accordingly. It is an ongoing process of teaching and learning.

Assessment *for* learning:

- requires the collection of data from a range of assessments as investigative tools to find out as mush as possible about what students know

- provides descriptive, specific and instructive feedback to students and parents regarding the next stage of learning

- actively engages students in their own learning as they assess themselves and understand how to improve performance.

Assessment *as* Learning

Assessment *as* learning actively involves students' reflection on their learning and monitoring of their own progress. It focuses on the role of the student as the critical connector between assessment and learning, thereby developing and supporting metacognition in students.

Assessment *as* learning:

- supports students in critically analysing their learning related to learning outcomes
- prompts students to consider how they can continue to improve their learning
- enables students to use information gathered to make adaptations to their learning processes and to develop new understandings.

Assessment *of* Learning

Assessment *of* learning involves strategies to confirm what students know, demonstrate whether or not they have met curriculum outcomes, or to certify proficiency and make decisions about students' future learning needs. Assessment *of* learning occurs at the end of a learning experience that contributes directly to reported results.

Traditionally, teachers relied on this type of assessment to make judgments about student performance by measuring learning after the fact and then reporting it to others. Used in conjunction with the other assessment processes previously outlined, however, assessment *of* learning is strengthened.

Assessment *of* learning:

- provides opportunities to report evidence to date of student achievement in relation to learning outcomes, to parents/guardians and other stakeholders
- confirms what students know and can do
- occurs at the end of a learning experience using a variety of tools.

Because the consequences of assessment *of* learning are often far-reaching, teachers have the responsibility of reporting student learning accurately and fairly, based on evidence obtained from a variety of contexts and applications.

Assessment Strategies

Assessment techniques should match the style of learning and instruction employed. Several options are suggested in this curriculum guide from which teachers may choose, depending on the curriculum outcomes, the class and school/district policies.

Observation (formal or informal)

This technique provides a way of gathering information fairly quickly while a lesson is in progress. When used formally, the student(s) would be aware of the observation and the criteria being assessed. Informally, it could be a frequent, but brief, check on a given criterion. Observation may offer information about the participation level of a student for a given task, use of a concrete model or application of a given process. The results may be recorded in the form of checklists, rating scales or brief written notes. It is important to plan in order that specific criteria are identified, suitable recording forms are ready, and all students are observed within a reasonable period of time.

Performance

This curriculum encourages learning through active participation. Many of the curriculum outcomes promote skills and their applications. In order for students to appreciate the importance of skill development, it is important that assessment provide feedback on the various skills. These may be the correct manner in which to use a manipulative, the ability to interpret and follow instructions, or to research, organize and present information. Assessing performance is most often achieved through observing the process.

Paper and Pencil

These techniques can be formative or summative. Whether as part of learning, or a final statement, students should know the expectations for the exercise and how it will be assessed. Written assignments and tests can be used to assess knowledge, understanding and application of concepts. They are less successful at assessing processes and attitudes. The purpose of the assessment should determine what form of paper and pencil exercise is used.

Journal

Journals provide an opportunity for students to express thoughts and ideas in a reflective way. By recording feelings, perceptions of success, and responses to new concepts, a student may be helped to identify his or her most effective learning style. Knowing how to learn in an effective way is powerful information. Journal entries also give indicators of developing attitudes to mathematical concepts, processes and skills, and how these may be applied in the context of society. Self-assessment, through a journal, permits a student to consider strengths and weaknesses, attitudes, interests and new ideas. Developing patterns may help in career decisions and choices of further study.

Interview

This curriculum promotes understanding and applying mathematics concepts. Interviewing a student allows the teacher to confirm that learning has taken place beyond simple factual recall. Discussion allows a student to display an ability to use information and clarify understanding. Interviews may be a brief discussion between teacher and student or they may be more extensive. Such conferences allow students to be proactive in displaying understanding. It is helpful for students to know which criteria will be used to assess formal interviews. This assessment technique provides an opportunity to students whose verbal presentation skills are stronger than their written skills.

Presentation

The curriculum includes outcomes that require students to analyze and interpret information, to be able to work in teams, and to communicate information. These activities are best displayed and assessed through presentations. These can be given orally, in written/pictorial form, by project summary, or by using electronic systems such as video or computer software. Whatever the level of complexity, or format used, it is important to consider the curriculum outcomes as a guide to assessing the presentation. The outcomes indicate the process, concepts and context for which a presentation is made.

Portfolio

Portfolios offer another option for assessing student progress in meeting curriculum outcomes over a more extended period of time. This form of assessment allows the student to be central to the process. There are decisions about the portfolio, and its contents, which can be made by the student. What is placed in the portfolio, the criteria for selection, how the portfolio is used, how and where it is stored, and how it is evaluated are some of the questions to consider when planning to collect and display student work in this way. The portfolio should provide a long-term record of growth in learning and skills. This record of growth is important for individual reflection and self-assessment, but it is also important to share with others. For all students, it is exciting to review a portfolio and see the record of development over time.

INSTRUCTIONAL FOCUS

Planning for Instruction

Consider the following when planning for instruction:
- Integration of the mathematical processes within each topic is expected.
- By decreasing emphasis on rote calculation, drill and practice, and the size of numbers used in paper and pencil calculations, more time is available for concept development.
- Problem solving, reasoning and connections are vital to increasing mathematical fluency and must be integrated throughout the program.
- There should be a balance among mental mathematics and estimation, paper and pencil exercises, and the use of technology, including calculators and computers. Concepts should be introduced using manipulatives and be developed concretely, pictorially and symbolically.
- Students bring a diversity of learning styles and cultural backgrounds to the classroom. They will be at varying developmental stages.

Teaching Sequence

The curriculum guide for Applied Mathematics 1202 is organized by units. This is only a suggested teaching order for the course. There are a number of combinations of sequences that would be appropriate.

Each two page spread lists the topic, general outcome, and specific outcome.

Instruction Time Per Unit

The suggested number of hours of instruction per unit is listed in the guide at the beginning of each unit. The number of suggested hours includes time for completing assessment activities, reviewing and evaluating. The timelines at the beginning of each unit are provided to assist in planning. The use of these timelines is not mandatory. However, it is mandatory that all outcomes are taught during the school year, so a long term plan is advised. Teaching of the outcomes is ongoing, and may be revisited as necessary.

Resources

The authorized resource for Newfoundland and Labrador students and teachers is *Math at Work 10* (McGraw-Hill Ryerson). Column four of the curriculum guide references *Math at Work 10* for this reason. Teachers may use any other resource, or combination of resources, to meet the required specific outcomes.

GENERAL AND SPECIFIC OUTCOMES

GENERAL AND SPECIFIC OUTCOMES WITH ACHIEVEMENT INDICATORS (pages 19-152)

This section presents general and specific outcomes with corresponding achievement indicators and is organized by unit. The list of indicators contained in this section is not intended to be exhaustive but rather to provide teachers with examples of evidence of understanding that may be used to determine whether or not students have achieved a given specific outcome. Teachers may use any number of these indicators or choose to use other indicators as evidence that the desired learning has been achieved. Achievement indicators should also help teachers form a clear picture of the intent and scope of each specific outcome.

Applied Mathematics 1202 is organized into seven units: *Consumerism and Travel*, *Measuring Length*, *Measuring Area*, *Getting Paid for Your Work*, *All About Angles*, *Pythagorean Relation* and *Trigonometry*.

Consumerism and Travel

Suggested Time: 16 Hours

Unit Overview

Focus and Context

In this unit, students will be introduced to unit pricing and currency exchange. Previously studied mathematics concepts, including fractions, percent, rate and ratio, will be used in a new context. Students will determine which purchase is the best buy, giving consideration to quality and quantity as well as unit price. They will investigate sales promotions and compare their effects, and they will convert Canadian dollars into a foreign currency and foreign currencies into Canadian dollars.

Students will consider both the SI and imperial systems of measurement and the relationship between the two. Although Canada officially adopted the SI system in the early 1970s, there are many aspects of daily life in which the imperial system is still used. In this unit, students will explore temperature, mass and capacity, all of which have measurement scales in both the SI and the imperial system. The ability to work within and convert between the two systems is an important skill.

The geometry outcome, which aims to develop spatial sense through the use of puzzles and games, will be introduced in this unit. This is ongoing throughout the course.

Outcomes Framework

GCO

Develop spatial sense through direct and indirect measurement.

GCO

Develop number sense and critical thinking skills.

SCO N1

Solve problems that involve unit pricing and currency exchange, using proportional reasoning.

SCO M1

Demonstrate an understanding of the Système International (SI) by:
- describing the relationships of the units for length, area, volume, capacity, mass and temperature
- applying startegies to convert SI units to imperial units.

SCO M2

Demonstrate an understanding of the imperial system by:
- describing the relationships of the units for length, area, volume, capacity, mass and temperature
- comparing the American and British imperial units for capacity
- applying startegies to convert imperial units to SI units.

GCO

Develop spatial sense.

SCO G1

Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

SCO Continuum

Mathematics 9	Mathematics 1202	Mathematics 2202
Number		
not addressed	N1 Solve problems that involve unit pricing and currency exchange, using proportional reasoning. [CN, ME, PS, R]	N2 Solve problems that involve pesonal budgets. [CN, PS, R, T] N4 Demonstrate an understanding of financial institution services used to access and manage finances. [C, CN, R, T]
Measurement		
not addressed	M1 Demonstrate an understanding of the Système International (SI) by: • describing the relationships of the units for length, area, volume, capacity, mass and temperature • applying startegies to convert SI units to imperial units. [C, CN, ME, V] M2 Demonstrate an understanding of the imperial system by: • describing the relationships of the units for length, area, volume, capacity, mass and temperature • comparing the American and British imperial units for capacity • applying startegies to convert imperial units to SI units. [C, CN, ME, V]	M2 Solve problems that involve SI and imperial units in volume and capacity measurements. [C, CN, ME, PS, V]
Geometry		
not addressed	G1 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	N1 Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies [C, CN, PS, R]

Mathematical Processes

[C]	Communication	[PS] Problem Solving
[CN]	Connections	[R] Reasoning
[ME]	Mental Mathematics and Estimation	[T] Technology
		[V] Visualization

Number

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to **N1 Solve problems that involve unit pricing and currency exchange, using proportional reasoning.** **[CN, ME, PS, R]**	Decisions about the value of consumer purchases are made frequently. In Grade 7, students worked with decimals and percentages (7N2, 7N3). In Grade 8, they worked with problems involving ratio, rate and proportional reasoning (8N4, 8N5). Students will now apply these mathematical skills as they relate to the following: • determining which product to purchase while considering unit price, quality and quantity • investigating sale promotions • calculating percent increase and percent decrease • converting between foreign and Canadian currencies Students will be exposed to both metric and imperial units as they explore unit pricing. Both measurement systems, however, will be addressed further through the measurement outcomes (M1, M2).
Achievement Indicator:	
N1.1 *Compare the unit price of two or more given items.*	Unit pricing is a useful tool for comparing prices. The unit price is the amount a consumer pays for each unit of the product they are buying. Students will work with units such as ounces, litres, pounds, square feet, or individual pieces in a package. The unit price, for example, will show the cost of each ounce in a can of soup while the package price shows the price of the whole can. For each unit listed, students could brainstorm some typical products that are sold that way. Students will use unit pricing to compare prices of items and to determine which price is the better deal. Consider an example such as the following: *The local grocery store sells 2 L of milk at $3.65 and 1.5 L of milk at $2.80. What is the best buy?* Students should be encouraged to identify the unit, which is the litre, and then calculate the unit price for each item. They should then determine that, since the lowest unit price is the 2 L of milk at $3.65, this is the best buy. In this situation, the brand of milk was not identified. Quality of the item, therefore, was not considered when determining the best buy. In stores that provide unit pricing, students can look for special shelf tags below each product. These labels are often easy to overlook. They may be the most helpful items in the store when it comes to getting the best buy.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies	Resources/Notes
Paper and Pencil	**Authorized Resource**
• Ask students to determine the following: Sean buys a package of 15 pencils for $4.50 at Walmart. Angela buys a box of 50 pencils at Costco for $14.00. Which is the better buy? (N1.1)	*Math at Work 10* 1 Get Ready Student Book (SB): pp. 4-5 Teacher Resource (TR): pp. 9-11 BLM 1-3
Performance	
• Students could work in centers, with each center containing similar items in different sizes with prices given (e.g., soup, pop, dog food, shampoo, etc.). In their journals, students discuss the item they would buy and why they feel it is the better deal. (N1.1)	
• In small groups, ask students to research the cost of lumber at the local hardware store(s) to determine: (i) the store that offers the best buy on one piece of $2' \times 4' \times 8'$. (ii) the best rate per foot of a piece of: (a) $2' \times 4' \times 8'$ (b) $2' \times 4' \times 10'$ (c) $2' \times 4' \times 12'$ (N1.1)	1.1 Unit Pricing SB: pp. 6-17 TR: pp. 12-22 BLM 1-4
• Students could compare a jumbo size and a regular size liquid laundry detergent. Ask them to determine what constitutes one use for each size. To visualize how much more the jumbo size contains, they could measure out each serving. They should then determine the cost per usage. (Empty containers with water could be used). (N1.1)	

Number

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to **N1 Continued...** **Achievement Indicators:** N1.2 *Compare, using examples, different sales promotion techniques; e.g., deli meat at $2 per 100 g seems less expensive than $20 per kilogram.*	Discuss various sales promotion techniques stores offer to help sell items. Stores often sell different quantities of the same product at different prices (e.g., soft drinks sold at 4 for $5 as opposed to 1 for $1.49). Promotions, such as buy one get one free, also encourage consumers to shop in a particular store. Students must realize, however, that to effectively compare the prices of two or more items they must use the same units. For example, deli meat sold at $2 per 100 g may seem less expensive than $20 per kilogram. Using the conversion factor of 1000 g = 1 kg, students should realize that deli meat at $2 per 100 g is equivalent to $20 per kilogram. Although students have explored the relationship between metric units of measurement (3SS3, 3SS4, 5SS2, 5SS4), it may be necessary to revisit the following conversions: • 1000 g = 1 kg • 100 cm = 1 m • 1000 mL = 1 L
N1.3 *Solve problems that involve determining the best buy, and explain the choice in terms of the cost as well as other factors, such as quality and quantity.*	After comparing unit prices, discuss other factors that may influence the choice for a "best buy". Students should be reminded that more is not always better. Engage students in a discussion about this. Buying large quantities of items that have a cheaper unit price is not helpful if you end up wasting some of the product because it was not fully used or has expired. Other factors, such as travelling distance from stores and quality of one product over another, must also be considered. Students should realize that decisions to buy an item should not be based on price alone. Present students with a choice such as the following: Paper towels are sold in a 2-roll package for $2.49 and a 12-roll package for $12.99. • Which package has the lower unit price? • How much would you save by buying a 12-roll package rather than six 2-roll packages? • When deciding which package size is the better buy for you, what should you consider in addition to unit price?

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:

  Seth went to the cafeteria at lunch to purchase milk. The cafeteria had two different sizes available. The 250 mL is $0.45 while the 500 mL costs $0.80.

  (i) Which milk is the best buy?
  (ii) What other factors may influence his decision?

  (N1.1, N1.3)

- Students could do a cost comparison of buying conventional oil versus a more expensive synthetic oil for their ATV or snowmobile. Ask them to discuss the benefits of each type of oil.

  (N1.3)

Portfolio

- Ask students to collect flyers to compare various products. Ask them to create their own problem related to comparing unit price and finding the best buy. After doing the comparison, students should determine other factors that could influence their decision to purchase that item.

  (N1.1, N1.2, N1.3)

Journal

- Gather empty containers for dish detergent, including jumbo size, regular and ultra size. Ask students to discuss factors that contribute to deciding which size to purchase. They should be encouraged to consider environmental concerns, such as packaging, use of water and concentration of chemicals.

  (N1.3)

Presentation

- Ask students to work in teams of of 3 to discuss their cell phone packages. They should determine which plan is best in terms of cost, number of texts they can send, etc. Students could present their findings to the class.

  (N1.1, N1.3)

Resources/Notes

Authorized Resource

Math at Work 10

1.1 Unit Pricing

SB: pp. 6-17

TR: pp. 12-22

BLM 1-4

Number

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to	
N1 Continued...	
Achievement Indicator:	
N1.4 *Determine the percent increase or decrease for a given original and new price.*	In Grade 8, students converted between percents and decimals (8N3). A review of this may be necessary. Students were also exposed to problems involving percent increase and decrease (8N3). Percent change is the change in a value over time. Through discussion about the meaning of percent increase and percent decrease, it should be highlighted that in a percent increase problem, the new number is greater than the original number, and in a percent decrease problem, the new number is less than the original number. This should help overcome any difficulty students may have in determining which number is the original number and which is the new number. Relating percent increase to situations such as salary or population growth should make it more concrete for students. They may associate percent decrease with clothing sales or weight loss.

To find the percent increase or decrease, students should first determine the increase or decrease in value, and then write it as a fraction of the original price. They have the choice of converting the fraction to a decimal or setting up a proportion to find the percent.

Strategy 1: *Using Proportions*

$$\frac{increase\ or\ decrease\ in\ price}{original\ price} = \frac{percent\ increase\ or\ decrease}{100}$$

Strategy 2: *Convert from a fraction to a percent*

$$percent\ increase\ or\ decrease = \frac{increase\ or\ decrease\ in\ price}{original\ price} \times 100$$

A common error occurs when students express the new number as a percent of the original number, rather than calculating the percent change. A pictorial representation could help students understand that when they express one number as a percent of another, it will be 100% more than the percent change, because percent change does not include the original amount.

Diagram 1:　[100%]　　　Diagram 2:　[100%]　[75%]

- The second diagram represents 175% of the original diagram.
- The difference between the diagrams is 75%.

This can then be related to calculating percent increase. When the price of a cup of coffee, for example, increases from \$1.25 to \$1.40, the new price is 112% of the original price, but the percent increase is 12%.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:

  (i)   Chelsea bought stock in the Gap for $25.00. Two weeks later she sold it for $60.00. What was the percent increase?

  (N1.4)

  (ii) Jordan bought an adapter for his computer that cost $29.99. Two weeks later he noticed that the same adapter was priced at $19.99. What was the percent decrease?

  (N1.4)

  (iii) A 4Gb MP3 player cost $89.99 in 2008. In 2010 the same MP3 player cost $19.99. What was the percent decrease?

  (N1.4)

  (iv) In 2004 gas was 67¢ per litre. In 2010 gas was $1.06 per litre. What was the percent increase?

  (N1.4)

  (v)   Frank decided to buy an iPod for $297. He scratched a ticket at the checkout and got $50 off. What is the percent decrease?

  (N1.4)

  (vi) What is the difference between a percent increase and a percent decrease? Include examples in your explanation.

  (N1.4)

  (vii) The original price of a car was $19 295. The new sale price of the car was $17 995. John calculated the percent decrease to be 93%. Describe the error in John's calculation.

  (N1.4)

Presentation

- Ask students to choose their favourite clothing store (or some other interest) and interview the store manager regarding trends in markdowns. They should present a summary orally or in written or pictorial form.

  (N1.4)

Performance

- Students could compare in-store versus online percentage discounts for specific products. Ask them to determine if it is cheaper to buy the product in-store or online.

  Extension: Students could consider percentages of taxes and shipping for in-store and online, and consider this in a decision to buy.

  (N1.4)

Resources/Notes

Authorized Resource

Math at Work 10

1.1 Unit Pricing

SB: pp. 6-17

TR: pp. 12-22

BLM 1-4

Number

Specific Outcomes	Suggestions for Teaching and Learning

Specific Outcomes

Students will be expected to

N1 Continued...

Achievement Indicator:

N1.5 *Explain the difference between the selling rate and purchasing rate for currency exchange.*

Suggestions for Teaching and Learning

Students will explore currencies in countries around the world and should recognize the importance of understanding currency rates, especially when travelling and buying and selling goods in different countries. Discuss with students some of the different systems of currencies in a variety of countries. Some samples are provided below.

Currency by Country	
Canada	dollar C$
United States	dollar US$
Germany	Euro €
England	pound £
Japan	yen ¥
Denmark	Krone kr
Cuba	convertible pesos CUC$
Mexico	Mexican Peso MXN$
Dominican Republic	Dominican Peso RD$

Selling rate and purchasing (buying) rate are terms related to currency exchange. Selling rate is the rate at which a bank, for example, sells money to the consumer. The buying rate is the rate at which a bank buys money from the consumer. It should be noted that the buying and selling rates are not equal. If you buy US$100 from the bank, for example, and sell it back immediately, you may not get the same money back. Processing costs should also be considered. Students should understand that the buying and selling rates can change on a daily basis and should be able to interpret a table such as the following:

Exchange Rates Compared to the Canadian Dollars			
Buying Rate	Country	Currency Units	Bank Selling Rate
1.344831	Belgium	Euro	1.465633
0.173888	Denmark	krone	0.198592
0.00998	Japan	yen	0.010919

The bank's selling rate for the Euro is 1.465633. This means the bank will sell the consumer 1 Euro for $1.465633 Canadian dollars.

The bank's selling rate for the krone is 0.198592. Therefore, the bank will sell the consumer 1 krone for $0.198592 Canadian dollars.

Current buying and selling rates can be found on most banking websites.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies	Resources/Notes
Paper and Pencil	**Authorized Resource**
• Ask students to define the terms selling rate and purchasing rate, and explain the difference between the two using an appropriate example. (N1.5)	*Math at Work 10* 1.2 Currency Exchange SB: pp. 18-28
• Jason is planning a trip to Florida. He uses $300 Canadian to buy US dollars at the bank at the current daily rate of 0.9084. Later that day his trip gets cancelled so he changes his money back to Canadian dollars at the rate of 1.0361. Ask students to determine how much money he lost and explain why would he not get exactly $300 back. (N1.5)	TR: pp. 23-32 BLM 1-5
Portfolio	**Suggested Resource**
• From class discussion, students could explain how fluctuations in the exchange rates of different countries could affect the import and export business. (N1.5)	Resource Link: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit1.html
Performance	• The Royal Bank website lists both the buy and sell rates in the cash rate table
• Ask students to keep a daily record of the Canadian dollar versus foreign currencies to observe the fluctuations. (N1.5)	

Number

Specific Outcomes	Suggestions for Teaching and Learning

Specific Outcomes

Students will be expected to

N1 Continued...

Achievement Indicators:

N1.6 *Convert between Canadian currency and foreign currency using formulas, charts and tables.*

N1.7 *Solve, using proportional reasoning, a contextual problem that involves currency exchange.*

N1.8 *Explain how to estimate the cost of items in Canadian currency while in a foreign country, and explain why this might be important.*

Suggestions for Teaching and Learning

In Grade 8, students solved problems using proportional reasoning (8N5). This will now be applied to problems involving currency exchange.

Consider the following example:

How much money would a student have in Euros if they have C$200?

This can be calculated using proportions.

$$\frac{\text{Euros}}{\text{Canadian}} = \frac{\text{Euros}}{\text{Canadian}}$$

$$\frac{1}{1.41042} = \frac{x}{200}$$

$$x = 141.80$$

This topic provides opportunities for discussion. Often, for example, both Canadian and American prices are listed on books. Discuss whether or not customers would benefit from choosing which price to pay.

Travellers to countries that use a different currency from their home country's currency can exchange their money to make purchases while they are travelling. The exchange rate may determine how much Canadian travellers will buy in a foreign country. Before items are bought in a foreign country, students need to be aware of what the item actually costs in their own currency to ensure they are not paying more than they would at home. Estimation can help students compare foreign prices to Canadian prices. Consider the following example:

When Susan was vacationing in France she wanted to purchase a print of the Eiffel Tower costing 190 Euros. What would be the cost in Canadian dollars if the exchange rate was 1.41042?

Exact Value	Estimated Value
1 Euro = C$1.41042	1 Euro = C$1.4
190 Euros = C$267.98	200 Euros = C$280

An engaging activity for students involves setting up an international store with food items from different countries (e.g., a can of olives from Greece, lettuce from the United States, bananas from Chile). The items should be labelled with the cost in the currency of the source country. Each group chooses from provided recipes and selects the required ingredients. They calculate the cost, in Canadian dollars, of the completed dish. As an extension, they could determine the cost per serving.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies

Performance

- Ask students to use currency exchange rates from the Internet or the newspaper to answer the following:

  Stephanie is travelling to the Philippines on vacation.

  (i) What is the name of the currency used in the Philippines?
  (ii) What is the current value of that currency in Canadian dollars?
  (iii) What is the value in Canadian dollars of 4500 units of Philippine currency?

  Groups of students could also be assigned different countries.
  (N1.6, N1.7)

- Chantelle gets a job in Malaysia where the currency is the Ringitt. She has the option to get paid $60 000 Canadian per year, or 210 000 Ringitts. Ask students to look up today's exchange rates on the Internet to determine the best option for Chantelle.
  (N1.6, N1.7)

Paper and Pencil

- Ask students to answer the following:

  (i) Francine bought two pairs of Italian shoes in Milan at 30 000 lira each. She has been out of Canada for 48 hours and has an import allowance of C$400. Will she have to pay duty on her shoes when she returns to Canada?
  (N1.6, N1.7)

  (ii) While in the United States, you wish to purchase a laptop computer for US$385.00. If the selling rate of the US dollar compared to the Canadian dollar is 1.0375, estimate the cost of the laptop in Canadian funds.
  (N1.8)

  (iii) Mary is in Mexico bargaining with a local cart seller. The cloth she wants to buy costs 85 pesos. If the exact value of 1 Canadian dollar is 12.3 pesos, what is a good estimate (in Canadian dollars) for 85 pesos? Why is estimating quickly useful in a situation like this?
  (N1.8)

  (iv) Compare shopping prices for stores with Canadian and American websites (e.g., Gap, Lululemon, Urban Outfitters, American Apparel). Consider which is more economical based on currency exchange rates. Students might benefit from a video outlining security tips, rules and advice when shopping online.
  Extension: Include shipping and tax rates.
  (N1.6, N1.7)

Resources/Notes

Authorized Resource

Math at Work 10

1.2 Currency Exchange

SB: pp. 18-28

TR: pp. 23-32

BLM 1-5

Suggested Resource

Resource Link: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit1.html

- Weblink to a video discussing security tips, rules and advice when shopping online

Measurement

Specific Outcomes

Students will be expected to

M1 Demonstrate an understanding of the Système International (SI) by:

- **describing the relationships of the units for length, area, volume, capacity, mass and temperature**
- **applying strategies to convert SI units to imperial units.**

[C, CN, ME, V]

M2 Demonstrate an understanding of the imperial system by:

- **describing the relationships of the units for length, area, volume, capacity, mass and temperature**
- **comparing the American and British imperial units for capacity**
- **applying strategies to convert imperial units to SI units.**

[C, CN, ME, V]

Achievement Indicator:

M1.1 *Explain how the SI system was developed, and explain its relationship to base ten.*

Suggestions for Teaching and Learning

Students will explore both the Système International (SI) and imperial systems of measurement. In this unit, they will work with capacity, mass and temperature. While the SI system is the official measurement system in Canada, students will benefit from exposure to the imperial units of measure. In today's global market, goods are manufactured and sold using both SI and imperial measurements. People may find themselves working in different countries around the world and should be knowledgeable in both systems of measurement.

Students were introduced to gram and kilogram as units of measurement in Grade 3 (3SS4). Work with SI units was extended to include millilitre and kilolitre in Grade 5 (5SS4). This measurement will now be developed in greater detail. It is intended that this outcome be limited to the base ten units and the prefixes milli, centi, deci, deca, hecto and kilo. This will be the first introduction to imperial units.

To introduce students to the measurement systems, have them choose an object (e.g., a piece of wood or string) that becomes their unit of measure. They can use this to find various measurements around the school. As students report their measurements, discuss the differences depending on the unit of measure used.

By the eighteenth century, different units of measurement were being used throughout the world. Discuss some challenges that may have been associated with this (e.g., confusion in trades between countries). To promote a more common standard, the metric system was created. The SI system (metric) was developed in France in the late 1700s. Students could research the history of the SI system. All SI units for distance are based on or derived from the metre. The standard unit of length is the metre. The name metre was derived from the Greek word metron, meaning "a measure." Although definitions have evolved over time to become more precise, one metre was originally defined as $\frac{1}{10\,000\,000}$ of the distance between the North Pole and Earth's equator as measured along the meridian passing through Paris. The standard SI unit of mass is the gram. It was initially defined as the mass of one cubic centimetre (a cube that is 0.01 meter on each side) of water at its temperature of maximum density. The standard unit of volume is the litre. It was defined as the volume of a cubic decimetre (a cube 0.1 metre on each side). With the age of rapid technological advances, the metric system is well suited for scientific and engineering work.

Larger and smaller multiples of each unit can be created by multiplying or dividing the basic units by ten and its powers, thus making it a base ten or decimal system. For further background on the origins of the SI System, refer to the website reference in Resources/Notes.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Performance

- Ask students to research the development of the SI and/or imperial systems. Some topics/questions they could focus on are:
    (i) Name two countries that do not use the SI system.
    (ii) What are the origins of some of the units used in the imperial system (e.g., foot, span, etc.)?
    (iii) Give some examples of when one system is used instead of the other.
    (iv) Why was the SI system developed?
    (v) When did Canada adopt the SI system and why?

    (M1.1, M2.1)

- Ask students to bring in objects that depict one or both of the measurement systems. Items may include wrenches, socket sets, feeler gauge for measuring gaps in spark plugs, micrometer, device for measuring crab (used by fishers), measuring cups, etc.

    (M1.1, M2.1)

Presentation

- Working in small teams, students could have an informal debate/discussion about whether Canada should stay with the SI system or switch back to the imperial system.

    (M1.1, M2.1)

Journal

- Ask students to respond to the following:

    What is your favourite sport/activity (e.g., football, quilting, and snowmobiling)? Does it use the SI system, the imperial system, or a combination of both?

    (M1.1, M2.1)

Resources/Notes

Authorized Resource

Math at Work 10

1.3 Measurement Comparisons

SB: pp. 29-47

TR: pp. 33-46

BLM 1-6

Suggested Resource

Resource Link: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit1.html

- Weblink to a site explaining the history of SI units

Measurement

Specific Outcomes	Suggestions for Teaching and Learning

Specific Outcomes

Students will be expected to

M2 Continued...

Achievement Indicators:

M2.1 *Explain how the imperial system was developed.*

M2.2 *Compare the American and British imperial measurement systems; e.g., gallons, bushels, tons.*

Suggestions for Teaching and Learning

Both the imperial system and the USA system of measurement are based on the older English units of measurement. Although measurements in the metric system are derived from scientific principles, the English units of measurements (and subsequent USA and imperial measurements) are based on nature and everyday activities. For example, an inch was first defined as the width of a man's thumb and then later it was defined as the length of 3 grains of barley placed end to end.

In 1824, the English measurement system underwent a review process. The types and sizes of the units were modified and renamed from the English units of measurement to the imperial system of measurement. This new standard was then introduced throughout the UK and its colonies. Since the US was no longer a British colony, they did not adopt these new standards.

The United States is the only country besides Myanmar and Liberia that does not use the metric system as its official system of measurements. Rather, it uses its own system called the U.S. customary system. This system is a derivative of the imperial system, which was used by Britain until the 20th century.

While many of the imperial and US measurements are the same (e.g., yards, pounds, and miles), there are differences. Consider the following:

- A gallon is a measure of volume.

  1.2 US gallons = 1 imperial gallon.

  Students should consider that the miles per gallon (MPG) for a vehicle in Canada, which is based on the imperial system, would be higher than the same vehicle in the US, which is based on the US gallon.

- A ton is a measure of weight.

  US ton = 2000 pounds

  1 imperial ton = 2240 pounds

  1 metric tonne = 1000 kg $\approx$ 2200 pounds

- A bushel is a unit for volumes of dry commodities (not liquids) most often used in agriculture.

  1.03 US bushel = 1 imperial bushel

  (1 imperial bushel = 8 imperial gallons)

Students should be aware that differences exist between the American and British imperial measurement systems. The intent, however, is not to devote a significant amount of time to this.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies	Resources/Notes
Performance	**Authorized Resource**
• Ask students to compare their shoe size in terms of US, UK, and European measurements.	*Math at Work 10*
(M2.2)	1.3 Measurement Comparisons
• Ask students to compare the sizes of various metric wrenches to the closest corresponding imperial wrenches.	SB: pp. 29-47
(M1.1, M2.1, M2.2)	TR: pp. 33-46
	BLM 1-6
Paper and Pencil	
• Bill and Jim are comparing the gas mileage of their snowmobiles. Bill gets 18 MPG (US) and Jim gets 22 MPG (Imp). Ask students to determine which snowmobile gets the best mileage.	
(M2.2)	

Measurement

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to	
M1 and M2 Continued...	
Achievement Indicators:	

M1.2 *Identify contexts that involve the SI system.*

M2.3 *Identify contexts that involve the imperial system.*

In many instances, the SI system is used instead of the imperial system. In Canada, for example, speed limits are posted in kilometres per hour instead of miles per hour. In most science laboratories, researchers use SI units when measuring. Sporting events such as the Olympics use metric measurements (e.g., 100 m sprint versus 100 yard sprint).

There are other instances where the imperial system is used. In the construction industry, for example, lengths are measured in inches (2 in. × 4 in.). Sports, such as football and golf, measure distance using yards.

There are also cases where both systems are used. Wrenches, for example, come in imperial and metric sizes. Snowmobiles made in the US require imperial sized wrenches whereas snowmobiles made elsewhere would use metric sized wrenches. The price of groceries is often given in both imperial (price per pound) and metric (price per 100 g).

M1.3 *Match the prefixes used for SI units of measurement with the powers of ten.*

It is intended that this outcome be limited to the base units and the six common prefixes listed below.

Prefix	Symbol	Power of 10
kilo	k	10^3
hecto	h	10^2
deca	da	10^1
deci	d	10^{-1}
centi	c	10^{-2}
milli	mm	10^{-3}

M1.4 *Explain, using examples, how and why decimals are used in the SI system.*

Since the SI system uses base 10, everything is a multiple of 10. Therefore, conversions are easily done by moving the decimal point left or right or multiplying or dividing by a multiple of 10. For example, 7.500 kL = 7500 L. Base 10 manipulatives can be used to reinforce the powers of 10.

A decimal system is used in the SI System for several reasons:

- the greater of two values can be immediately seen
- it is easier to do calculations involving decimal numbers
- there is less variation in the terminology (e.g., In the SI system, length is always measured using a base unit of metres. In the imperial system, length may be measured using inches, feet, yards, or miles.)

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Performance

- Ask students to make a chart containing some of the units used for one or both of the measurement systems. The chart may include common or uncommon units, conversions, etc. It may also include some locally used units such as fathom, barrel, quintal (of fish), yaffle (armful of dried fish).

  (M1.2, M2.3)

- Ask students to estimate the size of an item using both the SI and imperial systems and record the estimate. Students can then measure the item and record the actual size.

  (M1.2, M2.3)

- Ask students to bring in grocery flyers to examine how both types of measurements are used. Ask them to explain why they think imperial units are usually in the forefront (larger) than the metric units.

  (M1.2, M2.3)

- Students could pin cards with SI prefixes in order on a line (clothesline, string, etc.).

  (M1.3)

Journal

- Ask students to explain why they think certain measurements in Canada still use imperial units instead of metric.

  (M2.3)

Resources/Notes

Authorized Resource

Math at Work 10

1.3 Measurement Comparisons

SB: pp. 29-47

TR: pp. 33-46

BLM 1-6

Measurement

Specific Outcomes	Suggestions for Teaching and Learning

Students will be expected to

M1 and M2 Continued...

Achievement Indicators:

M2.4 *Explain, using examples, how and why fractions are used in the imperial system.*

The imperial system is not based on a base ten system. Most measurements are based on traditional measurements. A foot, for example, is the approximate length of a man's foot. Therefore, in order to break these measurements up into smaller segments, they may need to be divided into fractions. For example, inches on a measuring tape are divided into $\frac{1}{4}$ inch, $\frac{1}{16}$ inch, etc.

M1.5 *Provide an approximate measurement in SI units for a measurement given in imperial units; e.g., 1 inch is approximately 2.5 cm.*

Some common approximations include:

1 inch	≈ 2.5 cm
1 foot	≈ 30 cm
3 feet	≈ 1 m
1 mile	≈ 1.5 km
1 US gallon	≈ 4 L
1 imperial gallon	≈ 4.5 L
2 pounds	≈ 1 kg
1 cup	≈ 250 ml
32°F	≈ 0°C
1 barrell	≈ 35 gallons (imperial) ≈ 160 L

M2.5 *Provide an approximate measure in imperial units for a measurement given in SI units; e.g., 1 litre is approximately $\frac{1}{4}$ US gallon.*

M1.6 *Convert a given measurement from SI to imperial units by using proportional reasoning (including formulas); e.g., Celsius to Fahrenheit, centimetres to inches.*

To compare measurements, one of the measurements must be converted so the units are the same. Proportional reasoning was a focus in Grade 8 (8N5). Students will apply proportional reasoning and formulas to convert measurements from SI to imperial and vice versa.

To convert from Celsius to Fahrenheit, the formula $F = \left(C \times \frac{9}{5}\right) + 32$ can be applied. To convert from Fahrenheit to Celsius, the formula $C = \left(F - 32\right) \times \frac{5}{9}$ is used. Students can use the appropriate formula, for example, to convert 22°C to 71.6°F.

M2.6 *Convert a given measurement from imperial to SI units by using proportional reasoning (including formulas); e.g., Fahrenheit to Celsius, inches to centimetres.*

Students will also convert between pounds and kilograms, ounces and grams and gallons and litres.

A video, "Air Canada Flight 143", can be used to initiate discussion. An airplane that was refueled with pounds of fuel instead of kilograms had to glide to an emergency landing at the former air force base, Gimli. Students could view the condensed re-enactment and segment of the investigation video which explains the miscalculation (See Resources/Notes for web links).

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

Performance

- Students could create a fraction toolkit for reference. Each student needs five coloured strips. They should label one of the strips as "one". They cut a second strip into two pieces and label each piece "one-half". Repeat for one-quarter, one-eighth and one-sixteenth. The strips can be kept in a baggie or envelope for future reference.

  (M2.4)

- Students choose a vacation spot and record the high temperatures for a week. They report the average in both Fahrenheit and Celsius. As an extension, they could create clues so that other students can guess the location.

  (M1.6, M2.6)

Paper and Pencil

- Ask students to bring in a recipe and convert the measurements from imperial to SI or vice versa (including the oven temperature).

  (M1.6, M2.6)

- Ask students to convert:

  (i) $25°C = $ ___ °F
  (ii) 96 lbs = ___ kg
  (iii) 25 L = ___ gallons (US)

  (M1.6, M2.6)

- Lean hamburger meat is on sale at one grocery store for $1.99/lb. The same meat is on sale at a different grocery store for $3.99/kg. Ask students to determine the best deal.

  (M1.6, M2.6)

Portfolio

- Ask students to respond to the following:

  You are planning a trip to Florida and the forecast is predicting temperatures in the low 60s (Fahrenheit). What type of clothes should you pack for the trip? How did you decide?

  (M2.6)

Resources/Notes

Authorized Resource

Math at Work 10

1.3 Measurement Comparisons

SB: pp. 29-47

TR: pp. 33-46

BLM 1-6

Suggested Resources

Resource Links: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit1.html

- A weblink to a video showing a condensed re-enactment of Air Canada Flight 143

- A weblink to a video showing the miscalculation of fuel for the Gimli Glider

Geometry

Specific Outcomes

Students will be expected to

G1 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

[C, CN, PS, R]

Achievement Indicators:

G1.1 *Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g.,*

- *guess and check*
- *look for a pattern*
- *make a systematic list*
- *draw or model*
- *eliminate possibilities*
- *simplify the original problem*
- *work backward*
- *develop alternative approaches.*

G1.2 *Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.*

G1.3 *Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.*

Suggestions for Teaching and Learning

This outcome is intended to be integrated throughout the course by using puzzles and games focused on sliding, rotation, construction and deconstruction. These are intended to help students enhance spatial reasoning and problem-solving strategies.

Puzzles and games used should involve spatial reasoning. Numerical reasoning and logical reasoning will be addressed in 2202 and 3202, respectively. A variety of puzzles and games, such as board games, online puzzles and games, appropriate selections for gaming systems, and pencil and paper games, should be used. The puzzles and games may relate directly to other outcomes in the unit, but this is not a requirement. The main purpose is to develop spatial reasoning.

Games provide opportunities for building self-concept and developing positive attitudes towards mathematics, through reducing the fear of failure and error. In comparison to more formal activities, greater learning can occur through games due to the increased interaction between students, opportunities to test intuitive ideas and problem-solving strategies. Students' thinking often becomes apparent through the actions and decisions they make during a game, so teachers have the opportunity to assess learning in a non-threatening situation.

Problem-solving strategies are crucial to the analysis of strategies and will vary depending on the puzzle or game.

The following are some tips for using games in the mathematics class:

- Use games for specific purposes, not just time-fillers.
- Keep the number of players from two to four, so that turns come around quickly.
- Communicate to students the purpose of the game.
- Invite students to create their own games or variations of known games.
- Engage students in post-game discussions.

Teachers should create opportunities for students to explore mathematical ideas by asking questions that prompt students to reflect about their reasoning and make predictions. As students play games and analyze strategies, they explore mathematical ideas and compare different strategies for efficiency. If relevant to the unit of study, games can be used to help consolidate learning.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Observation

- Playing games creates dialogue and provides a tool for informal assessment. Stations could be set up in the classroom with one or two games at each center. Teachers should circulate among the groups and assess the students' understanding. Puzzles and games involving spatial reasoning could include:

  Board Games:

  (i)   Jenga™
  (ii) Battleship™
  (iii) Checkers™
  (iv) Connect Four™
  (v)   Mancala™

  Online Games:

  (i)   Assemble the Square™
  (ii)   Fifteen Puzzles™
  (iii) Tower of Hanoi™
  (iv)   Rush Hour™
  (v)   Tetris™
  (vi)   Nim™
  (vii) Mahjong Tiles™
  (viii) Tilt™

  Paper and Pencil Games:

  (i)   Sprouts™
  (ii) Pipelayer™
  (iii) Capture™
  (iv) Tic-Tac-Toe™

  Gaming Systems:

  (i) Table Tilt™ - Wii Fit
  (ii) TanZen™

  If students are struggling, teachers could participate with the group and think through the strategies out loud so the group can hear the reasoning for selected moves. Ask the group's opinions about moves in the game and facilitate discussions around each of the other player's moves and strategies.

Discussion

- As students play games or puzzles, ask probing questions and listen to students' responses. Record the different strategies and use these strategies for class discussion. Possible discussion starters include:

  (i)   Thumbs up if you liked the game, thumbs sideways if it was okay, and thumbs down if you didn't like it. What did you like about it? Why?
  (ii) What did you notice while playing the game?
  (iii) Did you make any choices while playing?
  (iv) Did anyone figure out a way to quickly find a solution?

Journal

- Ask students to write about their game strategies in their mathematics journal.

Resources/Notes

Authorized Resource

Math at Work 10

Games and Puzzles

Water Puzzle   SB: p. 53

Canoe Puzzle   SB: p. 53

TR: pp. 54

PL Site: https://www.k12pl.nl.ca/curr/10-12/math/math1202/classroomclips/games-and-puzzles.html

- Classroom clips of students using problem-solving strategies to analyze puzzles and games

Suggested Resources

Resource Links: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit1.html

- Templates for various paper pencil games

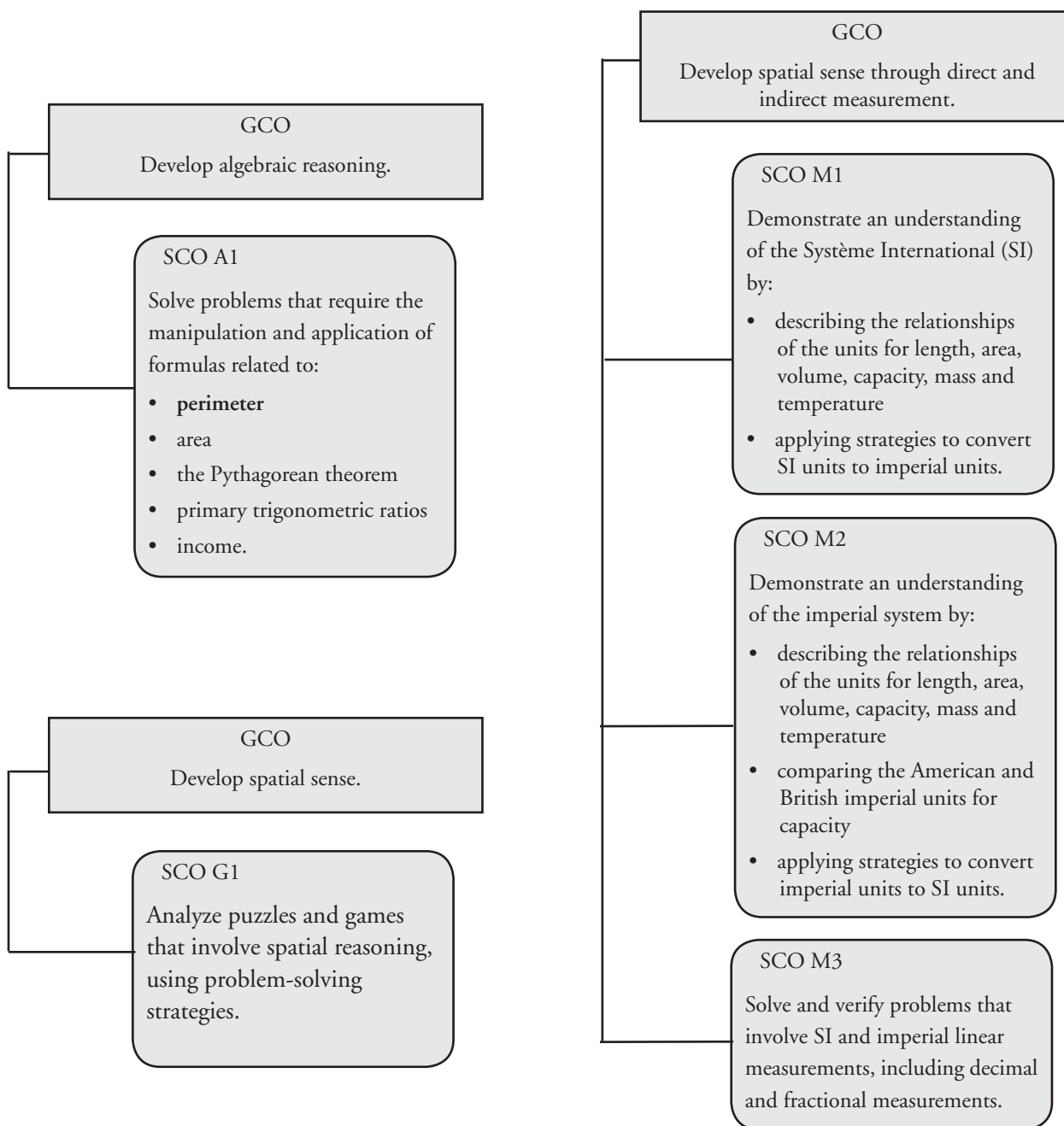- Weblinks to online games

Measuring Length

Suggested Time: 15 Hours

Unit Overview

Focus and Context

Students were previously introduced to the imperial measurement system as they worked with temperature, mass and capacity. In this unit, the focus is on linear measurement. Students will first work with imperial measurements, followed by SI measurements. Then they will convert between both measurement systems in the context of linear measurement problems.

Outcomes Framework

GCO

Develop spatial sense through direct and indirect measurement.

GCO

Develop algebraic reasoning.

SCO M1

Demonstrate an understanding of the Système International (SI) by:

- describing the relationships of the units for length, area, volume, capacity, mass and temperature
- applying strategies to convert SI units to imperial units.

SCO A1

Solve problems that require the manipulation and application of formulas related to:

- **perimeter**
- area
- the Pythagorean theorem
- primary trigonometric ratios
- income.

SCO M2

Demonstrate an understanding of the imperial system by:

- describing the relationships of the units for length, area, volume, capacity, mass and temperature
- comparing the American and British imperial units for capacity
- applying strategies to convert imperial units to SI units.

GCO

Develop spatial sense.

SCO G1

Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

SCO M3

Solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements.

SCO Continuum

Mathematics 9	Mathematics 1202	Mathematics 2202
Measurement		
9SS2 Determine the surface area of composite 3-D objects to solve problems. [C, CN, PS, R, V]	M1 Demonstrate an understanding of the Système International (SI) by: • describing the relationships of the units for length, area, volume, capacity, mass and temperature • applying strategies to convert SI units to imperial units. [C, CN, ME, V] M2 Demonstrate an understanding of the imperial system by: • describing the relationships of the units for length, area, volume, capacity, mass and temperature • comparing the American and British imperial units for capacity • applying strategies to convert imperial units to SI units. [C, CN, ME, V] M3 Solve and verify problems that involve SI and imperial linear msurements, including decimal and fractional measurements. [CN, ME, PS, V]	M1 Solve problems that involve SI and imperial units in surface area measurements and verify the solution. [C, CN, ME, PS, V] M2 Solve problems that involve SI and imperial units in volume and capacity measurements. [C, CN, ME, PS, V]
Algebra		
not addressed	A1 Solve problems that require the manipulation and application of formulas related to: • **perimeter** • area • the Pythagorean theorem • primary trigonometric ratios • income. [C, CN, ME, PS, R]	A1 Solve problems that require the manipulation and application of formulas related to: • volume and capacity • surface area • slope and rate of change • simple interest • finance charges. [CN, PS, R]
Geometry		
not addressed	G1 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	N1 Analyze puzzles and games that involve numerical reasoning, using problem-solving strategies [C, CN, PS, R]

Measurement

Specific Outcomes	Suggestions for Teaching and Learning

Specific Outcomes

Students will be expected to

M2 Demonstrate an understanding of the imperial system by:

- **describing the relationships of the units for length, area, volume, capacity, mass and temperature**
- **comparing the American and British imperial units for capacity**
- **applying strategies to convert imperial units to SI units.**

[C, CN, ME, V]

M3 Solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements.

[CN, ME, PS, V]

Achievement Indicators:

M2.7 *Write a given linear measurement expressed in one imperial unit in another imperial unit.*

M2.4 *Explain, using examples, how and why fractions are used in the imperial system.*

M3.1 *Provide an example of a situation in which a fractional linear measurement would be divided by a fraction.*

Suggestions for Teaching and Learning

Linear measurement is used to express distances. Students are expected to convert from one form of a linear measurement to another. First, they will work with imperial units of length.

In the imperial system, the foot is commonly used to measure length. This is the approximate length of a man's foot. Some common imperial measures for length include inches (in. or ″), feet (ft or ′), yards (yd) and miles (mi). Because the imperial units were developed at different times to meet different needs, each group of units has a particular relationship.

$$1 \text{ foot} = 12 \text{ inches}$$

$$1 \text{ yard} = 3 \text{ feet} = 36 \text{ inches}$$

$$1 \text{ mile} = 1760 \text{ yards} = 5280 \text{ feet}$$

It may be helpful for students to complete an imperial conversion table and have it available for quick reference throughout the unit.

To solve imperial measurement problems, it may be necessary to convert the given measurements into common units. In the context of a problem, for example, students may be required to convert 60 inches to feet. Possible strategies they could use include:

- Proportional Reasoning: $\frac{x}{60''} = \frac{1'}{12''}$

  $\frac{x}{60''}(60) = \frac{1'}{12''}(60)$

  $x = 5'$

- Conversion Factor: A unit conversion factor is a fraction that is equal to 1. Students should be aware that the numerator contains the unit they want to convert to and the denominator contains the original units in which the measurement was taken.

  $60'' \cdot \dfrac{1'}{12''} = 5'$

- Division: $60 \div 12 = 5$

Since imperial measures are based on traditional measurements rather than a base ten system, a portion of an imperial measure of length is usually written in fractional form. Inches on a measuring tape, for example, are divided into $\frac{1}{2}$ inch, $\frac{1}{16}$ inch, etc.

Operations with fractions are an integral part of the Intermediate Mathematics curriculum. Students should be encouraged to use mental math and estimation skills, where appropriate. Technology, however, may be appropriate in situations where the numbers used may increase mental math difficulty.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Paper and Pencil

- Ask students to convert the following:
    (i)   4 ft = _____ in.
    (ii)   3 mi = _____ yd

(M2.7)

- Cory is measuring a fishing "haul-up" line for turbot nets. He uses his two outstretched arms as his fathom referent (1 fathom = 6 feet). If he measures 125 fathoms, how many feet and inches has he measured?

(M2.7)

- Ask students to determine, with the aid of a ruler, the value of $6 \div \frac{1}{4}$. Have them explain their reasoning.

(M3.1)

- Ask students to respond to the following:

When you convert a measurement from a larger unit to a smaller unit, do you expect the number of units to increase or decrease? Why?

(M2.7)

Resources/Notes

Authorized Resource

Math at Work 10

2 Get Ready

Student Book (SB): pp. 56-57

Teacher Resource (TR): pp. 63-64

BLM 2-3

2.1 Imperial Length Measurements

SB: pp. 58-69

TR: pp. 65-78

BLM 2-4, 2-5

Suggested Resource

Resource Link: https://www. k12pl.nl.ca/curr/10-12/math/ math1202/links/unit2.html

- This website explains the history of SI units

Measurement

Specific Outcomes	Suggestions for Teaching and Learning

Specific Outcomes

Students will be expected to

M3 Continued...

Achievement Indicators:

M3.2 *Measure inside diameters, outside diameters, lengths, widths of various given objects, and distances, using various measuring instruments.*

M3.3 *Identify a referent for a given common SI or imperial unit of linear measurement.*

M3.4 *Estimate a linear measurement, using a referent.*

M3.5 *Estimate the dimensions of a given regular 3-D object or 2-D shape, using a referent; e.g., the height of the desk is about three rulers long, so the desk is approximately three feet high.*

Suggestions for Teaching and Learning

Discuss the proper use and reading of a ruler in both metric and imperial measurements. Teachers could choose a measurement on a ruler and discuss the number of halves, quarters or eighths that make up the measurement. In 3 inches, for example, there are 6 half inches.
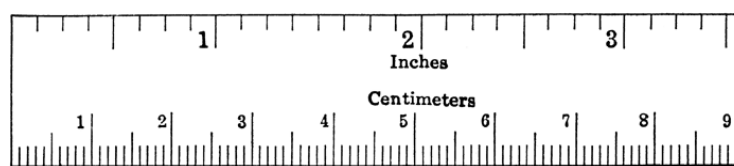
Ensure there are a variety of measuring devices, such as a measuring tape, metre stick and ruler, available. Discuss objects that interest students and ask them to bring the objects to class to be measured using an appropriate tool. Some objects may include a CD, hockey puck, tire, laptop, coins, etc. Students were introduced to diameter and radius in Grade 7 (7SS1). A review of these terms may be necessary.

A referent is an object that can be used to help estimate a measurement. From the earliest introduction to metric units, students have had experience relating non-standard and standard units of measurement. They have used personal referents to estimate the length of an object in centimetres, metres and millimetres. Opportunities should be provided to explore imperial units using personal referents to measure objects. Students should be encouraged to use a referent that represents approximately one unit of measurement.

Some common referents for linear measurement include:

width of a quarter, width of a hockey puck	≈ 1 in.
length of a standard floor tile, length of a sub sandwich	≈ 1 ft
distance from the tip of the nose to the outstretched fingers	≈ 1 yd

Encourage students to select their own personal referents that make sense to them. To approximate 1 mile, for example, students may choose a distance from two well known points in their own communities.

Students can estimate the dimensions of 2-D and 3-D objects within the classroom. Examples include finding the dimension of a classroom window, door or picture frame. Other examples include the dimensions of a dice or box.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies

Performance

- Ask students to measure items in the classroom using traditional imperial units. For example, students could measure their desktops using the length from their knuckle to their thumb to represent an inch or measure the length of the classroom using their foot, as people did in ancient times. Students should then see that the measurements are not consistent and, therefore, why a more accurate and precise system was developed. They can then use an accurate measuring tool to measure these same lengths to compare to their original measurements. They should also recognize that the old measurement techniques can still be used as a useful estimation tool.

    (M3.2, M3.3, M3.4, M3.5)

- Students could measure the outside and the inside diameter to find the wall thickness of a piece of pipe (e.g., ABS or clay).

    (M3.2)

- Students may choose to work in pairs or small groups to measure their total body height and navel height and record the information on a data sheet. Students should then calculate the ratio of total height to navel height and record this as a ratio. Note: It is important to remove shoes.

Student's Name	Body Height	Naval Height	body height ÷ naval height

This is a good opportunity to introduce Phi (The Golden Ratio) and explore its presence in everyday life.

    (M3.2)

- Ask students to measure the height, width and diagonal of their TV screen and determine which measurement represents the advertised size of the TV.

    (M3.2)

Journal

- Ask students to discuss why it is useful to establish referents to measure objects.

    (M3.3)

Paper and Pencil

- Ask students to identity objects within the classroom or school that would be approximately 10 inches long.

    (M3.4)

Resources/Notes

Authorized Resource

Math at Work 10

2.1 Imperial Length Measurements

SB: pp. 58-69

TR: pp. 65-78

BLM 2-4, 2-5

PL Site: https://www.k12pl.nl.ca/curr/10-12/math/math1202/classroomclips/linear-referents.html

- Classroom clip of students using referents to measure the lengths of various objects

Measurement

Specific Outcomes	Suggestions for Teaching and Learning

Specific Outcomes

Students will be expected to

M1 Demonstrate an understanding of the Système International (SI) by:

- **describing the relationships of the units for length, area, volume, capacity, mass and temperature**
- **applying strategies to convert SI units to imperial units.**

[C, CN, ME, V]

M3 Continued...

Achievement Indicators:

M1.7 *Write a given linear measurement expressed in one SI unit in another SI unit.*

M3.3 *(Continued) Identify a referent for a given common SI or imperial unit of linear measurement.*

M3.4 *(Continued) Estimate a linear measurement, using a referent.*

M3.5 *(Continued) Estimate the dimensions of a given regular 3-D object or 2-D shape, using a referent; e.g., the height of the desk is about three rulers long, so the desk is approximately three feet high.*

M3.2 *(Continued) Measure inside diameters, outside diameters, lengths, widths of various given objects, and distances, using various measuring instruments.*

Suggestions for Teaching and Learning

Students will also work with SI linear measurements and, as with imperial units, it will sometimes be necessary to convert from one SI unit to another. It is intended that this outcome be limited to the base units and the prefixes milli, centi, deci, deca, hecto and kilo. For example, students will convert centimetres to millimetres or kilometres to hectometres.

In Grade 5, students worked with linear measurements and explored the relationship between millimetres and centimetres, millimetres and metres, and metres and kilometres (5SS2). These relationships should be revisited.

The base 10 system in SI units makes conversions more straightforward than in the imperial system. To convert from one linear unit to another, students can multiply or divide by a multiple of 10.

The standard unit of length is the metre. Relationships include:

1 m = 10 dm	1 km = 1000 m
1 m = 100 cm	1 hm = 100 m
1 m = 1000 mm	1 dam = 10 m

A portion of a SI measure of length is usually written in decimal form.

Opportunities should be provided to explore SI units using personal referents. Some common referents for SI linear measurements include:

thickness of a dime, thickness of a fingernail	$\approx$ 1 mm
width of a paper clip	$\approx$ 1 cm
distance from a door knob to the floor	$\approx$ 1 m

Students may relate 1 kilometre to a distance between two well known points in their own communities.

Students should estimate the lengths of objects using referents. The length of a standard white board, for example, is about 3 m because it is approximately 3 metre sticks long. They should also use various measuring tools to measure lengths in SI units.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Paper and Pencil

- Ask students to convert the following:

    (i) 25 dam = ____ cm

    (ii) 1.7 kg = ____ dg

    (M1.7)

- Ask students to identify objects within the classroom or school that would be approximately:

    (i) 1 cm long
    (ii) 2 m long

    (M3.4)

- Ask students to identify and use an appropriate referent to estimate the length, height or distance of objects such as the following:

    (i)   classroom wall
    (ii)   distance from one classroom to another
    (iii)   perimeter of the cafeteria
    (iv)   height the clock is off the floor
    (v)   diameter of a basketball net
    (vi)   width of an IPod screen

    (M3.3, M3.4, M3.5)

- Ask students to identify and use an appropriate referent to estimate the dimensions of objects such as the following:

    (i)   teacher's desk
    (ii)   cereal box
    (iii)   milk can
    (iv)   soccer field
    (v)   school

    (M3.3, M3.4, M3.5)

Performance

- Students could estimate the length, width and perimeter of the classroom. Then they measure the length and width and calculate the perimeter. Ask them to compare their estimates and calculations and discuss how they could improve their estimation techniques. Discuss which measurement system they chose to use.

    (M3.2, M3.4)

Resources/Notes

Authorized Resource

Math at Work 10

2.2 SI Length Measurements

SB: pp. 70-81

TR: pp. 79-90

BLM 2-6, 2-7

PL Site: https://www.k12pl.nl.ca/
curr/10-12/math/math1202/
classroomclips/linear-referents.
html

- Classroom clip of students using referents to estimate linear measurements

Measurement

| Specific Outcomes | Suggestions for Teaching and Learning |

Specific Outcomes

Students will be expected to

M1, M2 Continued...

Achievement Indicators:

M1.5 *Provide an approximate measurement in SI units for a measurement given in imperial units; e.g., 1 inch is approximately 2.5 cm.*

M2.5 *Provide an approximate measure in imperial units for a measurement given in SI units; e.g., 1 litre is approximately $\frac{1}{2}$ US gallon.*

M1.6 *Convert a given measurement from SI to imperial units by using proportional reasoning (including formulas); e.g., Celsius to Fahrenheit, centimetres to inches.*

M2.6 *Convert a given measurement from imperial to SI units by using proportional reasoning (including formulas); e.g., Fahrenheit to Celsius, inches to centimetres.*

Suggestions for Teaching and Learning

Along with converting within measurement systems, it is sometimes necessary to convert between the systems. Students will examine the relationships between imperial and SI units of length.

Some common approximations include:

Imperial Measure	SI Approximation
1 inch	≈ 2.5 cm
1 foot	≈ 30 cm
3 feet	≈ 1 m
1 mile	≈ 1.5 km

Students could examine measuring tools containing both imperial and SI units, and create their own conversion table.

SI to Imperial	Imperial to SI
1 mm = 0.0394 in.	1 in. = 25.4 mm
1 cm = 0.3937 in.	1 in. = 2.54 cm
1 m = 3.2808 ft	1 ft = 0.3048 m
1 m = 1.0936 yd	1 yd = 0.9144 m
1 km = 0.6214 mi	1 mi = 1.6093 km

To compare lengths or distances, one of the measurements must be converted so the units are the same. The methods used to convert within the imperial system can be revisited here. Students can apply proportional reasoning or conversion factors to convert measurements from SI to imperial and vice versa.

Proportional reasoning can be used, for example, when the cost of material is in metres but measurements were taken in feet.

$$\frac{1\,m}{3.2808\,ft} = \frac{x\,m}{12\,ft}$$

Alternatively, students could use a conversion factor to determine how many metres are in 12 feet.

$$12\,ft \cdot \frac{1\,m}{3.2808\,ft}$$

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies	**Resources/Notes**

Paper and Pencil

- Students can work in pairs to measure their height in feet and inches. They should convert their measurements to metres.

(M2.8)

- Ask students to convert:
  - (i) 42 cm = ____ inches
  - (ii) 45 mph = ____ km/h
  - (iii) 26.2 miles = ____ km

(M1.8, M2.8)

Authorized Resource

Math at Work 10

2.3 Length Conversions

SB: pp. 82-93

TR: pp. 91-101

BLM 2-8, 2-9

Measurement

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to	
A1 Solve problems that require the manipulation and application of formulas related to:	It is intended that this outcome be integrated throughout the course. In this unit, some of the problems will require the manipulation and application of formulas related to perimeter.
• **perimeter**	Students were introduced to perimeter in Grade 6 (6SS3) and circumference in Grade 7 (7SS1). A review of these terms may be necessary. Formulae for calculating the perimeter of a rectangle and the perimeter of a square, as well as the circumference of a circle, should also be reviewed.
• area	
• the Pythagorean theorem	
• primary trigonometric ratios	
• income.	
[C, CN, ME, PS, R]	

M3 Continued...

Achievement Indicators:

M3.6 *Solve a linear measurement problem including perimeter, circumference, and length + width + height (used in shipping and air travel).*

It is intended that the four arithmetic operations on decimals and fractions will be integrated into problems throughout this unit. As an introduction, it will be necessary to review these operations as well as formula manipulation skills. When discussing perimeter of a rectangle, for example, students should recognize that $P = 2l + 2w$ can be rewritten in terms of length or width. Similarly, the formula for circumference of a circle, $C = 2\pi r$, can be rewritten in terms of r. Students should be exposed to a variety of situations using perimeter and circumference, determining which operations are appropriate.

M3.7 *Determine the operation that should be used to solve a linear measurement problem.*

A1.1 *Create and solve a contextual problem that involves a formula.*

This outcome provides a good opportunity to make a connection between mathematics and the skill trades. For example, a carpenter has to measure the dimensions of a room to install baseboards and a seamstress has to measure the diameter/radius of a circular table to make tablecloths.

A1.2 *Describe, using examples, how a given formula is used in a trade or occupation.*

Various other connections can be made as well. A courier sometimes has to measure packages to determine shipping costs. For example, to ship with the post office, the length + 2(width + height) must be less than 3 m in total. The maximum linear dimensions ($l + w + h$) of checked baggage for some airlines must not exceed a certain limit. When travelling with one airline, the maximum linear dimensions per bag is 158 cm (62 in.). Students could investigate the baggage policies (checked and carry on) for various airlines.

A1.3 *Solve a contextual problem that involves the application of a formula that does not require manipulation.*

A1.4 *Solve a contextual problem that involves the application of a formula that requires manipulation.*

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following questions:

  (i) A tire has radius 12 inches. What is its circumference?

  (M3.6)

  (ii) The circumference of a CD is 28.26 cm. What is its diameter?

  (M3.6)

  (iii) A rectangular window frame measures 24 inches by 36 inches. If trim for the window costs $ 2.75 per linear foot, how much will it cost to put trim around the window?

  (M3.6, M3.7)

  (iv) A box has a height of 30 cm, a width of 20 cm, and a length of 60 cm. Determine if the box can be shipped via Canada Post. Students could use the Canada Post calculator to determine the cost of shipping the package from Newfoundland to Calgary, Alberta if its weight is 4 lbs.

  (M3.6, M3.7)

  (v) Compare and contrast the terms perimeter and circumference. When would it be appropriate to use each?

  (M3.7)

  (vi) A rectangular room has a width 12 feet. If the floor of the room is to be covered with boards of width $3\frac{1}{4}$ inches, how many rows of boards will be needed?

  (M3.1, M3.6, M3.7)

Journal

- Ask students to respond to the following:

  In everyday life, when do you think you will be working with perimeter? Do you think you will need to estimate or calculate more frequently?

  (M3.6)

Performance

- Bring in a variety of boxes. Students can work together to measure the dimensions and determine the length + girth of each one.

  (M3.6)

Resources/Notes

Authorized Resource

Math at Work 10

2.4 Working with Length

SB: pp. 94-105

TR: pp. 102-113

BLM 2-10

PL Site: https://www.k12pl.nl.ca/ curr/10-12/math/math1202/ classroomclips/skilled-trades.html

- Classroom clip of Skilled Trades students applying mathematics

Suggested Resource

Resource Link: https://www. k12pl.nl.ca/curr/10-12/math/ math1202/links/unit2.html

- Weblink to Canada Post calculator for parcels

Measurement

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to	
M3, A1 Continued...	
Achievement Indicators:	
M3.8 *Determine, using a variety of strategies, the midpoint of a linear measurement such as length, width, height, depth, diagonal and diameter of a 3-D object, and explain the strategies.*	Some strategies for determining the midpoint of a linear measurement include:
	• dividing the total length by 2
	• using a string to cover the full distance, fold it in half and use this distance to locate the middle
	• constructing diagonals, where the point of intersection is the middle
M3.9 *Determine if a solution to a problem that involves linear measurement is reasonable.*	Discuss instances of when it would be necessary to find the midpoint and which strategies would be appropriate. Hanging pictures sometimes requires finding the horizontal middle (midpoint) of a wall. Constructing diagonals could be beneficial in situations such as the following:
	• finding the centre of a ceiling to install a light fixture
A1.5 *Identify and correct errors in a solution to a problem that involves a formula.*	• finding the centre of a floor to install ceramic tile
	• finding the centre of a sheet of plywood to create a circular table
	It would be appropriate to find the midpoint of a diameter when trying to determine the centre of a circular table.
	Encourage students to contribute suggestions of other such instances when it is necessary to determine the midpoint of a linear measurement.
	Students should always be encouraged to use referents and estimation to determine if linear measurements are reasonable.
G1 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.	Continue to provide opportunities for students to solve puzzles and play games focused on spatial reasoning. They should explain the problem-solving strategies used to solve a puzzle or win a game. Students should also be given opportunities to create a variation on a puzzle or a game.
[C, CN, PS, R]	

General Outcome: Develop spatial sense through direct and indirect measurement.

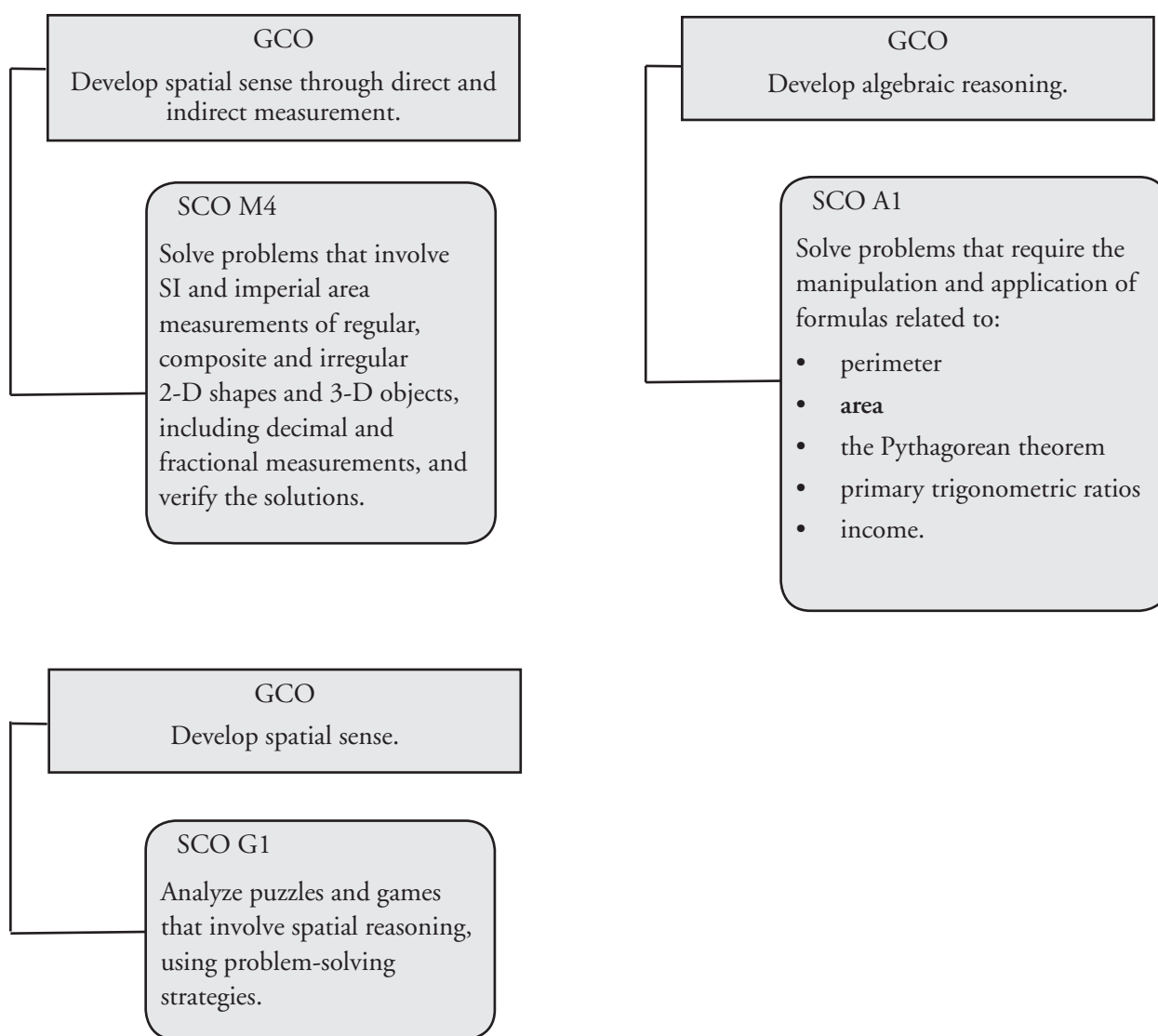| Suggested Assessment Strategies	Resources/Notes
	Authorized Resource
Performance	*Math at Work 10*
• Ask students to determine the midpoint of the whiteboard, or some other object in the classroom, using two or more methods. They should discuss the advantages, disadvantages and accuracy of each method.	2.4 Working with Length
	SB: pp. 94-105
(M3.8)	TR: pp. 102-113
	BLM 2-10
Paper and Pencil	
• An interior decorator wants to install vertical blinds and requires one support bracket to be centred above the window. Ask students to explain how to find the location of this bracket.	Games and Puzzles
	Win 10
(M3.8)	SB: p. 111
• A family wishes to install lighting around their circular pool. The diameter of the pool is 6 m and the lights are to be placed approximately 1.5 m apart. Lights are purchased in packages of 10. The contractor charges for 4 packages of lights. Ask students to determine if this is reasonable.	TR: p. 120
(M3.5, M3.7, M3.9)	

Measuring Area

Suggested Time: 16 Hours

Unit Overview

Focus and Context

In this unit, students will calculate area and surface area using both the imperial and SI measurement systems. Work in this unit will build on the concepts from the previous units. Students will work with regular, irregular and composite 2-D shapes and 3-D objects.

Outcomes Framework

GCO
Develop spatial sense through direct and indirect measurement.

SCO M4

Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions.

GCO
Develop algebraic reasoning.

SCO A1

Solve problems that require the manipulation and application of formulas related to:

- perimeter
- **area**
- the Pythagorean theorem
- primary trigonometric ratios
- income.

GCO
Develop spatial sense.

SCO G1

Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

SCO Continuum

Mathematics 9	Mathematics 1202	Mathematics 2202
Measurement		
M2 Determine the surface area of composite 3-D objects to solve problems. [C, CN, PS, R, V]	M4 Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions. [ME, PS, R, V]	M1 Solve problems that involve SI and imperial units in surface area measurements and verify the solutions. [C, CN, ME, PS, V] M2 Solve problems that involve SI and imperial units in volume and capacity measurements. [C, CN, ME, PS, V]
Algebra		
not addressed	A1 Solve problems that require the manipulation and application of formulas related to: • perimeter • **area** • the Pythagorean theorem • primary trigonometric ratios • income. [C, CN, ME, PS, R]	A1 Solve problems that require the manipulation and application of formulas related to: • volume and capacity • surface area • slope and rate of change • simple interest • financial charges. [CN, PS, R]
Geometry		
not addressed	G1 Analyze puzzles and games that involve spatial reasoning, using problem-solving stragies [C, CN, PS, R]	N1 Analyze puzzles and games that involve numerical resoning, using problem-solving strategies. [C, CN, PS, R]

Mathematical Processes

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[ME]	Mental Mathematics and Estimation	[T]	Technology
		[V]	Visualization

Measurement

Specific Outcomes	Suggestions for Teaching and Learning

Specific Outcomes

Students will be expected to

M4 Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions.

[ME, PS, R, V]

Achievement Indicators:

M4.1 *Identify and compare referents for area measurements in SI and imperial units.*

M4.2 *Estimate an area measurement, using a referent.*

M4.3 *Estimate the area of a given regular, composite or irregular 2-D shape, using an SI square grid and an imperial square grid.*

M4.4 *Determine if a solution to a problem that involves an area measurement is reasonable.*

Suggestions for Teaching and Learning

In Grade 9, students determined the surface area of composite 3-D objects, limited to right cylinders, right rectangular prisms and right triangular prisms. In this unit, SI and imperial area measurements are used to estimate and calculate area and surface area. Students will use both systems of measurement as they explore the area of regular, composite and irregular 2-D shapes and 3-D objects.

It is intended that the four arithmetic operations on decimals and fractions be integrated into the problems.

Students will first estimate and calculate area in imperial units. Following this, they will work with area in SI units.

Students will use estimation by referring to benchmarks or referents to determine approximate values and to determine the reasonableness of calculated values.

Some common referents for area measurement include:

$1 \text{ ft}^2 \approx 900 \text{ cm}^2$	≈ 1 floor tile
1 cm^2	$\approx$ area of a fingernail
1 in^2	$\approx$ area of a postage stamp
$93.5 \text{ in}^2 \approx 600 \text{ cm}^2$	$\approx$ area of an exercise notebook
2 m^2	$\approx$ area of an exterior house door
$1500 \text{ m}^2 \approx 17\,000 \text{ ft}^2$	$\approx$ area of an ice rink surface
8100 ft^2	$\approx$ area of a baseball diamond (infield)
$32 \text{ ft}^2 \approx 3 \text{ m}^2$	$\approx$ area of a sheet of gyproc or plywood

Using inch grids and cm grids (possibly on transparencies), students can place the various 2-D shapes on each of the grids and count the squares that are covered and partially covered to estimate the total area of the shape.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies	Resources/Notes
Performance	**Authorized Resource**
• Ask students to use SI and imperial referents to measure the area of the classroom, gymnasium, whiteboard, etc.	*Math at Work 10*
(M4.1, M4.2)	3 Get Ready
	Student Book (SB): pp. 114-115
Paper and Pencil	Teacher Resource (TR): pp. 129-130
• Provide students with different shapes and ask them to estimate the area of the shapes using square centimetres and square inches. Students can then overlay the shapes on the cm and inch grid papers to determine the area.	BLM 3-3
(M4.3)	
• Ask students to respond to the following:	3.1 Imperial Area Measurements
Sally estimates that the top of her school desk is 60 cm². Does this estimate seem reasonable? Explain.	3.2 SI Area Measurements
(M4.4)	SB: pp. 116-127, 128-139
	TR: pp. 131-142, 143-153
	BLM 3-4
	Suggested Resource
	Resource Link: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit3.html
	• Centimetre grid paper

Measurement

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to **M4 Continued...** **Achievement Indicators:** M4.5 *Identify a situation where a given SI or imperial area unit would be used.*	Many flooring materials are priced in square feet or square yards. Hardwood, ceramic tiles and laminate are priced per square foot while carpet and linoleum are priced per square yard. As well, dimensions for new houses, including blueprints, are usually given in square feet. Paving a driveway is also usually based on square feet. Other situations more often use the SI measurement system. Surveyors usually use square metres when giving the area of a piece of land, fabric is sold using square metres, and population density is given as the number of people per square kilometre.
M4.6 *Write a given area measurement expressed in one SI unit squared in another SI unit squared.* M4.7 *Write a given area measurement expressed in one imperial unit squared in another imperial unit squared.*	The focus should be on converting square units that are used in everyday events. One would not typically convert, for example, square inches to square miles or square centimetres to square hectares. Students could convert between square millimetres and square centimetres, square centimetres and square metres, square inches and square feet, and square feet and square yards.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Paper and Pencil

- Johnny is carpeting a bedroom that measures 10 feet by 12 feet. He performed the following calculation:

  10 ft × 12 ft = 120 ft^2

  Since there are 3 feet in a yard, I need to order 40 yd^2 of carpet

  120 ft^2 ÷ 3 = 40 yd^2

  When he is done he finds he has over 25 yd^2 of carpet left over.

  Ask students to identify the error in Johnny's thinking.

  (A1.6, M4.7)

- Ask students to convert the following:

  (i)   20 cm^2 = ____ mm^2
  (ii) 2 yd^2 = ____ in.^{2}

  (M4.6, M4.7)

- Ask students to give three examples of situations which would require calculating area using imperial measurements and three which would require using SI measurements.

  (M4.5)

Resources/Notes

Authorized Resource

Math at Work 10

3.1 Imperial Area Measurements

3.2 SI Area Measurements

SB: pp. 116-127, 128-139

TR: pp. 131-142, 143-153

BLM 3-4, 3-5

Algebra

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to	

A1 Solve problems that require the manipulation and application of formulas related to

- perimeter
- **area**
- the Pythagorean theorem
- primary trigonometric ratios
- income.

[C, CN, ME, PS, R]

M4 Continued...

Achievement Indicators:

M4.8 *Solve a contextual problem that involves the area of a regular, a composite or an irregular 2-D shape.*

M4.9 *Solve a problem, using formulas for determining the areas of regular, composite and irregular 2-D shapes, including circles.*

A1.3 *Solve a contextual problem that involves the application of a formula that does not require manipulation.*

A1.4 *Solve a contextual problem that involves the application of a formula that requires manipulation.*

A1.1 *Create and solve a contextual problem that involves a formula.*

A1.5 *Identify and correct errors in a solution to a problem that involves a formula.*

It is intended that this outcome be integrated throughout the course. Students have already been exposed to problems related to perimeter (M3). In this unit, some of the problems will require the manipulation and application of formulas related to area.

When calculating area, focus should first be on regular and irregular 2-D shapes. A regular shape is defined as having all sides congruent and all angles equal (e.g., equilateral triangle, square). An irregular shape is one in which all sides and/or angles are not equal (e.g., rectangle, rhombus, scalene triangle, etc.). Discussion of regular and irregular shapes is appropriate. It is more important, however, that students be given the opportunity to work with area of these various figures rather than being able to distinguish between the two types.

In Grade 7, formulae for the area of various quadrilaterals, triangles, and circles were developed and applied (7SS2). Students will now use these formulae to find the area of regular and irregular shapes. Required formulae include:

- $A = b \times h$ (parallelogram, rectangle, rhombus, square)
- $A = \frac{1}{2}b \times h$ (triangle)
- $A = \pi r^2$ (circle)

In some situations, the area will be calculated when all measurements are given, requiring no formula manipulation. Given the area and one dimension, students should also determine the missing measurement, such as in the following problem:

What is the radius of a circle having an area of 46 cm²?

Students are not expected to rearrange formulas at this level. They should first substitute the given information into the formula and then solve for the missing dimension.

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Paper and Pencil

- Given the area of the top of an engine piston is 25.43 cm², ask students to determine the diameter of the piston.

(A1.4)

Project

- Ask students to assume the role of owner of a contracting company that installs flooring. They should develop a proposal for a bid on a new house that is being built.
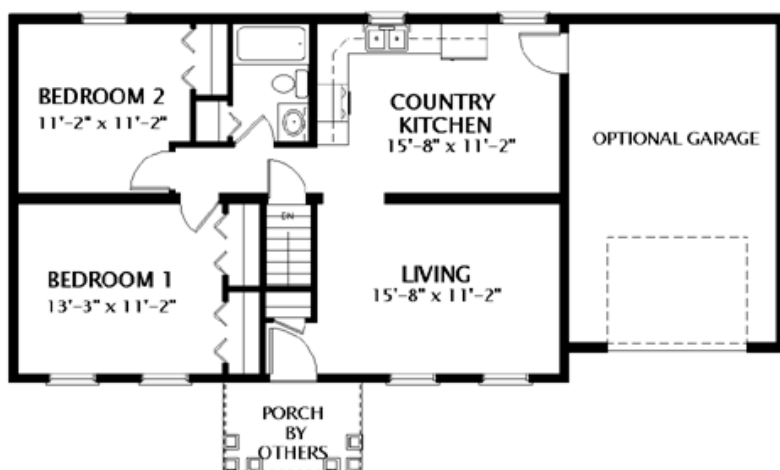
Using the measurements in the floor plans, students should determine the amount of the bid (including profit) if:

(i)   each bedroom is to have carpet ($9.99 per square yard)
(ii)   the living room has hardwood flooring ($6.99 per square foot)
(iii)   the kitchen/dining room has ceramic tile ($3.99 per square foot)
* above prices include installation costs   (M4.8, M4.9, A1.3)

Alternatively, students could be asked to determine the cost of flooring for the living room and bedrooms based on the following floor plan.

Resources/Notes

Authorized Resource

Math at Work 10

3.1 Imperial Area Measurements

3.2 SI Area Measurements

SB: pp. 116-127, 128-139

TR: pp. 131-142, 143-153

BLM 3-4, 3-5

Note

Solving contextual problems involving the manipulation of area formulas are not addressed in *Math at Work 10.*

Measurement

Specific Outcomes	Suggestions for Teaching and Learning

Specific Outcomes

Students will be expected to

M4, A1 Continued...

Achievement Indicators:

M4.8 *(Continued) Solve a contextual problem that involves the area of a regular, a composite or an irregular 2-D shape.*

M4.9 *(Continued) Solve a problem, using formulas for determining the areas of regular, composite and irregular 2-D shapes, including circles.*

A1.3 *(Continued) Solve a contextual problem that involves the application of a formula that does not require manipulation.*

M4.10 *Explain, using examples, the effect of changing the measurement of one or more dimensions on area and perimeter of rectangles.*

Suggestions for Teaching and Learning

Area formulae can be used to determine such things as the amount of:

- plywood or gyproc needed for a new house
- flooring needed
- paint needed to cover walls
- material needed to make a new dress
- materials being used so there is no wastage.

Once students have had exposure to regular and irregular 2-D shapes, they should work with more complex area problems, such as asking them to calculate the area of a wall with a window removed. They should also work with composite shapes (e.g., a semicircle on top of a triangle).

There are often several ways to decompose a composite shape. The way in which a shape is decomposed may affect the dimensions used, but not its area. When decomposing a composite shape, encourage students to look for figures they have previously worked with (i.e., rectangles, triangles and circles).

Students should explore changing the dimension(s) of one or two sides of a rectangle to see what effect it will have on the area and perimeter of the rectangle. Consider the following rectangle:

2 cm

6 cm
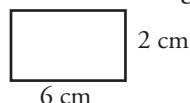
Ask students:

- How would the area change if the width was doubled?
- How would the area change if the length was tripled?
- How would the perimeter change if the width was doubled?
- How would the perimeter change if the length was tripled?

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies	Resources/Notes

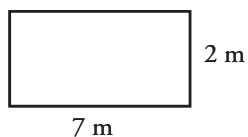### Suggested Assessment Strategies

Paper and Pencil

- The diagram below shows the front of a shed. Ask students to determine the amount of siding needed (in square feet) to cover the front of the shed.
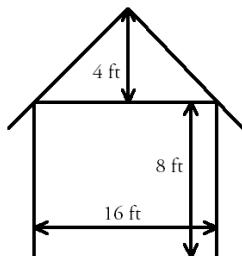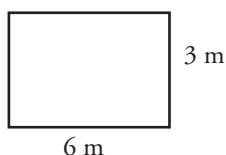

(M4.8, M4.9)

Journal

- Susie says that if the length of the rectangle shown below is doubled then the area will double.

  Ask students whether they agree or disagree. They should explain their thinking.

- Ask students how they could change the dimensions of the rectangle shown to triple its area.

(M4.10)

Resources/Notes

Authorized Resource

Math at Work 10

3.3 Working With Area

SB: pp. 140-151

TR: pp. 154-164

BLM 3-6

3.4 Surface Area of Three-Dimensional Objects

SB: pp. 152-163

TR: pp. 165-175

BLM 3-7, 3-8, 3-9, 3-10, 3-11, 3-12

Measurement

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to	
M4 Continued...	
Achievement Indicator:	

M4.11 *Solve a problem that involves determining the surface area of 3-D objects, including right cylinders and cones.*

Many area problems involve finding surface areas of three-dimensional figures. In this unit, the focus is on the surface area of rectangular prisms and right cylinders. The surface area of cones will also be explored.

To find these surface areas, nets should be used. Students should have prior experience with nets and their application to surface area of 3-D objects. To calculate surface area, students must identify the faces or surfaces, determine the dimensions of each face, and apply appropriate formulae to calculate area. Use of concrete models allows students to visualize the figures and encourages them to use reasoning rather than merely follow procedure. Prior to generalizing formulas and using symbolic representations to calculate surface area, they should use nets of 3-D objects. Working from the net also allows for easy identification of congruent faces, which sometimes avoids the necessity of having to find the areas of each face individually. Students may conclude that the surface area of a rectangular prism can be calculated using the formula SA = 2*lw* + 2*lh* + 2*wh*. To ensure they have gained the conceptual understanding of surface area, however, concrete models and nets should be explored before introducing the formula. Some students may never use this formula. Students should always be encouraged to include the units as part of the solution.

Next, consider the right cylinder. The net of a solid cylinder consists of two circles and one rectangle. The curved surface opens up to form a rectangle. A good way to demonstrate this is to unpeel the label on a can to show that it is a rectangle.
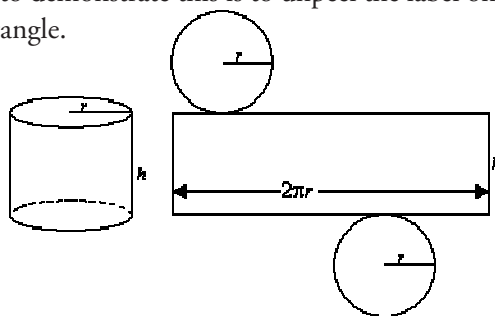
Surface Area = 2 × (area of circle) + area of rectangle

SA = $(2 \times \pi r^2) + (2\pi r \times h)$

SA = $2\pi r^2 + 2\pi rh$

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies	**Resources/Notes**

Paper and Pencil

- Chloe takes an empty tin can (with the lid removed) and plans to cover it with craft paper (sides and bottom). Ask students to determine how much paper she will need if the height of the can is 12 cm and the radius is 3 cm.

(M4.11)

Authorized Resource

Math at Work 10

3.4 Surface Area of Three-Dimensional Objects

SB: pp. 152-163

TR: pp. 165-175

BLM 3-7, 3-8, 3-9, 3-10, 3-11, 3-12

Measurement

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to	

M4 Continued...

Achievement Indicator:

M4.11 *(Continued) Solve a problem that involves determining the surface area of 3-D objects, including right cylinders and cones.*

Another example of a cylinder is a shipping tube for holding blueprints.

Students should also be exposed to situations where calculating the surface area involves only one circle (e.g., a can with the lid removed).

Finally, students should solve problems involving the surface area of a cone. This is students' first exposure to the surface area of a cone: $A = \pi r^2 + \pi rs$, where r is the radius of the base and s is the slant height of the cone.
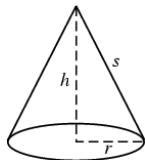
If the cone has no top (like a drinking cup), the formula is simply $A = \pi rs$. Discuss examples such as an ice cream cone, the tip of a pen and a traffic pylon.

General Outcome: Develop spatial sense through direct and indirect measurement.

Suggested Assessment Strategies	**Resources/Notes**

Paper and Pencil

- Ask students to answer the following:

  Jill is planning to make a witch's hat for Halloween. She makes the hat using a big sheet of Bristol board. The hat is 36 cm high, has a slant height of 39 cm, and a radius of 15 cm. What is the surface area of the hat?
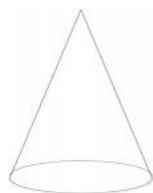
  (Note: the area would not include the bottom)

  (M4.11)

Authorized Resource

Math at Work 10

3.4 Surface Area of Three-Dimensional Objects

SB: pp. 152-163

TR: pp. 165-175

BLM 3-7, 3-8, 3-9, 3-10, 3-11, 3-12

Note

The surface area of a cone is not addressed in *Math at Work 10*.

Algebra

Specific Outcomes	Suggestions for Teaching and Learning

Students will be expected to

G1 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

[C, CN, PS, R]

In this unit students should experience puzzles and games with a focus on sliding, rotation, construction and deconstruction. These are intended to help them enhance spatial reasoning and problem-solving strategies.

Achievement Indicators:

G1.1 *Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g.,*

- *guess and check*
- *look for a pattern*
- *make a systematic list*
- *draw or model*
- *eliminate possibilities*
- *simplify the original problem*
- *work backward*
- *develop alternative approaches.*

G1.2 *Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.*

G1.3 *Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.*

Teachers could set up learning centres using a collection of physical puzzles and puzzle books. Consider the following tips when creating puzzle centres:

- Create several stations, each with several related puzzles (e.g., sequential movement puzzles, packing puzzles).

- Divide students into small groups. At regular intervals, have groups rotate to the next centre.

- As a supplemental activity, students could work at the puzzle centres if they finish other work.

Encourage students to work through the first puzzle in the learning centre to learn the rules. Then, they can work through various puzzles. If students find the puzzle too challenging, encourage them to try an easier one. If they are working on a sequential movement puzzle, such as Rush Hour, they could remove some of the pieces and solve just part of the problem. As they work through different challenge levels, they will begin to develop effective strategies for solving the puzzle.

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies

Observation

- Sequential movement puzzles could include:
    (i)   Rush Hour™
    (ii)   Unblock Me™
    (iii) Lunar Lockout™
    (iv)   Flipit™
    (v)   Mancala™
    (vi)   Labyrinth™

- Packing puzzles could include:
    (i)   Shape by Shape™
    (ii)   Brick by Brick™
    (iii) TanZen™ - iPod Touch Application

Students can work through the puzzles on their own either individually or with a partner. They could record their progress on a sheet such as the one shown below.

Puzzle	Solved?	Comments, Hints
1		
2		
3		
4		
5		

Journal

- Ask students to write about their thought processes. Which puzzles did they find interesting and why?

Resources/Notes

Authorized Resource

Math at Work 10

Games and Puzzles

Design a Puzzle SB: p. 169

TR: p. 182

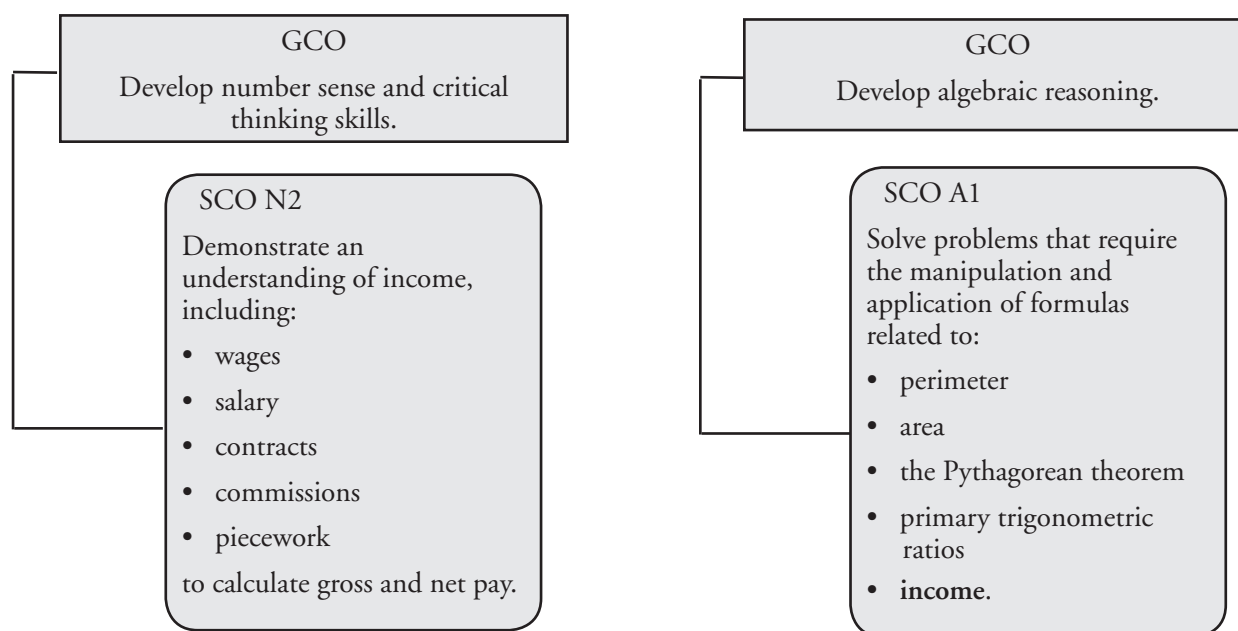# Getting Paid for Your Work

Suggested Time: 15 Hours

Unit Overview

Focus and Context

In this unit, various methods of earning income, such as piecework and commission, as well as hourly, monthly and annual rates of pay, are introduced. Students will also explore income from overtime shifts, shift premiums, tips and bonuses for achieving results. They will calculate the basic deductions that most working Canadians pay including government-mandated deductions such as Canada Pension Plan, Employment Insurance and income tax, as well as benefit plans such as pension plans, dental benefits and paid vacation time. They will calculate gross and net income.

Some students will have familiarity with wages and associated deductions from part-time work or casual jobs.

Outcomes Framework

GCO
Develop number sense and critical thinking skills.

SCO N2
Demonstrate an understanding of income, including: • wages • salary • contracts • commissions • piecework to calculate gross and net pay.

GCO
Develop algebraic reasoning.

SCO A1
Solve problems that require the manipulation and application of formulas related to: • perimeter • area • the Pythagorean theorem • primary trigonometric ratios • **income**.

SCO Continuum

Mathematics 9	Mathematics 1202	Mathematics 2202
Number		
not addressed	N2 Demonstrate an understanding of income, including: • wages • salary • contracts • commissions • piecework to calculate gross and net pay. [C, CN, R, T]	N2 Solve problems that involve personal budgets. [C, CN, PS,R] N3 Demonstrate an understanding of compound interest. [CN, ME, PS, T] N4 Demonstrate an understanding of financial institution services used to access and manage finances. [C, CN, R, T] N5 Demonstrate an understanding of credit options, including: • credit cards • loans. [CN, ME, PS, R]
Algebra		
not addressed	A1 Solve problems that require the manipulation and application of formulas related to: • perimeter • area • the Pythagorean theorem • primary trigonometric ratios • **income**. [C, CN, ME, PS, R]	A1 Solve problems that require the manipulation and application of formulas related to: • volume and capacity • surface area • slope and rate of change • simple interest • financial charges. [CN, PS, R]

Mathematical Processes

[C]	Communication	[PS] Problem Solving
[CN]	Connections	[R] Reasoning
[ME]	Mental Mathematics and Estimation	[T] Technology
		[V] Visualization

Number

Specific Outcomes

Students will be expected to

N2 Demonstrate an understanding of income, including:

- **wages**
- **salary**
- **contracts**
- **commissions**
- **piecework**

to calculate gross and net pay.

**[C, CN, R, T]**

Achievement Indicators:

N2.1 *Describe, using examples, various methods of earning income.*

N2.2 *Identify and list jobs that commonly use different methods of earning income; e.g., hourly wage, wage and tips, salary, commission, contract, bonus, shift premiums.*

Suggestions for Teaching and Learning

More than ever students are working in part time jobs. In addition to providing an income, work experience enhance resumes, college applications and future job applications. In this unit, students will be introduced to the various methods of income payment, deductions and calculations involving gross and net pay. They will also be presented with flawed solutions involving gross or net pay that require them to identify and correct mistakes.

There are many methods of earning income.
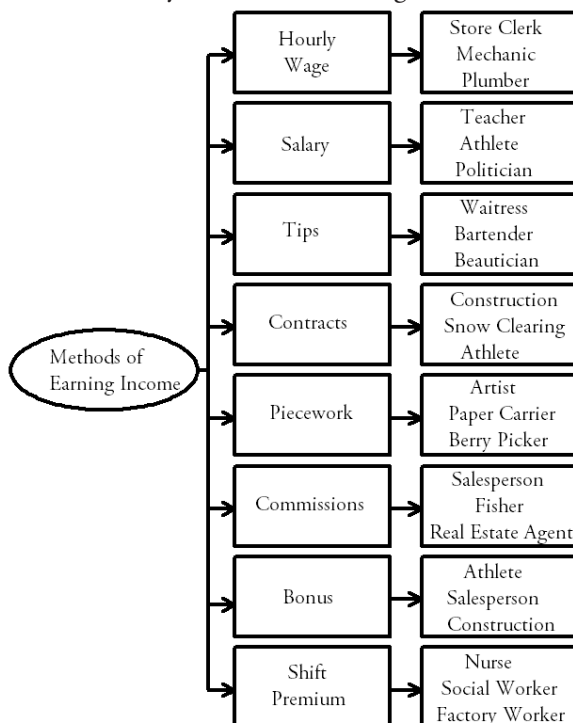
Various combinations of these methods of earning income, such as hourly wage and tips, are also common.

Students will begin work with jobs that earn an hourly wage, wage and tips, or a salary. Some jobs may offer the employee an opportunity to earn a bonus. An athlete, for example, may receive a performance bonus. Construction companies sometimes offer a bonus when contract deadlines are met. Salespersons who achieve sales targets sometimes earn bonuses.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies	**Resources/Notes**

Performance

- Ask students to determine how three of the following occupations are paid:

  (i)   Nurse

  (ii) Doctor

  (iii) Electrician

  (iv) PartyLite® salesperson

  (v)   Hairdresser

(N2.2)

Authorized Resource

Math at Work 10

4 Get Ready

Student Book (SB): pp. 172-173

Teacher Resource (TR): pp. 191-193

BLM 4-3, 4-4

4.1 Wages and Salary

SB: pp. 174-185

TR: pp. 194-206

BLM 4-5

Suggested Resources

Resource Links: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit4.html

- Webink to the XE Currency Encyclopedia, which provides rates and information for every world currency.

- Weblink to The Money Belt, which provides interactive tools, quizzes, polls and resources to interest students in financial matters.

Number

Specific Outcomes	Suggestions for Teaching and Learning

Specific Outcomes

Students will be expected to
N2 Continued ...

Achievement Indicator:

N2.3 *Determine in decimal form, from a time schedule, the total time worked in hours and minutes, including time and a half and/or double time.*

Suggestions for Teaching and Learning

Employees sometimes work extra hours in addition to their regular hours. Overtime usually begins when they work beyond 40 hours in a work week. Overtime pay must be received for those additional hours. Overtime pay is typically $1\frac{1}{2}$ times the employee's regular rate of pay. Students may be familiar with this as time and a half. If regular pay is $12 an hour, for example, then the overtime rate is $18 an hour $\left(\$12 \times 1\frac{1}{2}\right)$ for every hour worked beyond 40 hours in each week.

Other employees, in professions such as nursing, receive a shift premium. In this case, they receive an extra amount of money per hour because they work non-standard hours. Students should be provided with a time schedule and asked to compute regular, overtime and shift premium hours. They should be exposed to situations where an employee works a fraction of an hour (e.g., 15 minutes, 30 minutes or 45 minutes). In such cases, they should convert minutes to hours in decimal form. They should recognize, for example, that 15 minutes is 0.25 of an hour.

When considering an example such as the following, students must be careful not to include overtime hours twice.

Alyson works at a local donut shop. Her regular pay is $10.00 per hour and she earns a shift premium of $1.00 per hour for hours worked between 12 a.m. and 8 a.m.. Alyson gets 1.5 times the regular rate of pay for overtime hours worked above 40 hours per week. Alyson's time schedule is shown below:

Monday	8 a.m. - 4 p.m.
Tuesday	8 a.m. - 8 p.m.
Wednesday	12 a.m. - 8 a.m.
Thursday	12 a.m. - 5:30 a.m.
Friday	6 a.m. - 12:15 p.m.
Saturday	Holiday
Sunday	8 a.m. - 4 p.m.

Ask students to:

(i)   Calculate Alyson's regular hours. *(24.5)*

(ii) Calculate Alyson's premium hours. *(15.5)*

(iii) Calculate Alyson's overtime hours. *(7.75)*

An extension of this activity would be to ask students to calculate the gross pay *($531.75)*. It may be a good idea to encourage students to compute overtime hours first. Otherwise, because overtime hours can occur during the regular work day, the tendency could be to include the overtime hours with the regular hours as well.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies	**Resources/Notes**

Paper and Pencil

- Crystal is working at a fish plant for the summer. She makes $12 per hour and she earns a shift premium of $2.00 per hour for hours worked between 12 a.m. and 8 a.m. She gets 1.5 times the regular pay for overtime hours worked above 40 hours per week. Her weekly time schedule is shown below:

Monday	12 p.m. - 6 p.m.
Tuesday	8 a.m. - 4:30 p.m.
Wednesday	6 a.m. - 2 p.m.
Thursday	12 a.m. - 10 a.m.
Friday	Holiday
Saturday	10 p.m. - 4 a.m.
Sunday	4 p.m. - 11 p.m.

Ask students to:

(i) Calculate Crystal's regular hours.
(ii) Calculate Crystal's premium hours.
(iii) Calculate Crystal's overtime hours.
As an extension, ask students to calculate the gross pay for the week.

(N2.3)

- Ask students to design their own time schedule and create a problem for other students to answer.

(N2.3)

Resources/Notes

Authorized Resource

Math at Work 10

4.1 Wages and Salary

SB: pp. 174-185

TR: pp. 194-206

BLM 4-5

Number

Specific Outcomes	Suggestions for Teaching and Learning

Students will be expected to

A1 Solve problems that require the manipulation and application of formulas related to:

- perimeter
- area
- the Pythagorean theorem
- primary trigonometric ratios
- **income.**

[C, CN, ME, PS, R]

N2 Continued ...

Achievement Indicators:

N2.4 *Determine gross pay from given or calculated hours worked when given:*

- *the base hourly wage, with and without tips*
- *the base hourly wage, plus overtime (time and a half, double time).*

A1.3 *Solve a contextual problem that requires the application of a formula that does not require manipulation.*

It is intended that this outcome be integrated throughout the course. Students have already been exposed to problems related to perimeter (M3) and area (M4). In this unit, the problems require the **application** of formulas related to income.

A discussion of terminology related to income calculations is necessary before doing the calculations. A pre-assessment of what students know about gross pay, pay periods, deductions, and the types of deductions may be beneficial here. Students could be given an admit card with a job and three options of how to be paid. They choose the best method and justify their choice.

Gross pay is the total amount earned before any deductions are taken out. There are three types of fixed gross earnings:

- wages   (rate × hours worked)
- salaries (set amount)
- bonuses (discretionary)

Students should be able to calculate weekly (52 pay periods), bi-weekly (26 pay periods), or monthly (12 pay periods) gross wages given the hourly rate of pay or the annual income. The gross pay calculation is straightforward when the number of hours worked in the pay period and the hourly pay rate are known. Gross pay calculations for salaried employees may require more focus. The total annual pay is divided by the number of pay periods per year.

Students should recognize the difference between hourly gross pay and salaried gross pay through exploration of situations such as the following:

Hourly Gross Pay	Salaried Gross Pay
An employee works 20 hours a week at $12/hour. If he or she is paid weekly, the gross pay for each of the 52 pay periods is $240.	If an employee's annual salary is $30 000 and he or she is paid biweekly, the gross pay for each of the 26 pay periods is $1 153.85.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Journal

- Ask students to respond to the following:

  You have a summer job at the local restaurant as a waiter/waitress. The owner presents three choices for your income:

  (i) $14 per hour (no tips)
  (ii) $10 per hour (plus tips)
  (iii) salary of $320 per week

  Which option would you choose and why? What aspects of the job should you consider before choosing an option?

  (N2.4)

Performance

- Ask students to choose a job that suits their skills and interests. They can research the rate of pay in NL and post on cards on a job wall.

  (N2.4)

Paper and Pencil

- Ask students to answer questions such as the following:

  (i) Seth is paid $12.50 per hour for 40 hours a week. If he works more than 40 hours per week, he makes time and a half. If Seth worked 52 hours this week, what would his gross earnings be?
  (N2.4, A1.1)

  (ii) Hannah is a hairstylist who works for a base hourly wage of $12. She works 35 hours per week and receives $160.50 in tips for the week. Calculate her gross pay.
  (N2.4, A1.1)

  (iii) Joshua is a waiter at the local restaurant and is paid an hourly rate of $10 plus tips. Joshua earns 6% of the tips received in a shift. During his shift on Tuesday, $1500 in total was received in tips. Calculate Joshua's gross pay for Tuesday.
  (N2.4)

Resources/Notes

Authorized Resource

Math at Work 10

4.1 Wages and Salary

SB: pp. 174-185

TR: pp. 194-206

BLM 4-5

Suggested Resource

Resource Link: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit4.html

- Weblink identifying hourly wages and gross annual income for jobs in various Canadian cities.

Number

Specific Outcomes	Suggestions for Teaching and Learning

Students will be expected to

N2 Continued ...

Achievement Indicators:

N2.5 *Investigate, with technology, "what if …" questions related to changes in income; e.g., "What if there is a change in the rate of pay?"*

Technology (e.g., calculator, online payroll calculator, tax programs, etc.) should be used to compare various income rates. Students should examine how changes in rate of pay, number of hours worked, increases in income or decreases in deductions affect net income.

N2.6 *Identify and correct errors in a solution to a problem that involves gross or net pay.*

Engaging students in error analysis heightens awareness of common errors. Along with providing the correct solutions, they should be able to identify incorrect solutions, including why errors might have occurred and how they can be corrected. Questions requiring error analysis can be effective tools to assess students' understanding of gross and net pay calculations because it requires a deeper understanding than simply "doing the problem." Analyzing errors is a good way to focus discussion on "How did you get that?" rather than being limited to "Is my answer right?" This reinforces the idea that the process is as important as the solution.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer questions such as the following:

  John works 48 hours per week at an hourly rate of $16 per hour. After 40 hours he receives time and a half. John calculated his income using the following method.

  Step 1: Regular pay = 40 hrs x $16 = $640.00
  Step 2: Overtime hours = 48 - 40 = 8hrs
  Step 3: Overtime pay = 8hrs x $16 = $128.00
  Step 4: Gross pay = regular pay + overtime pay
  Step 5: Gross pay = $640.00 + $128.00 = $768.00

  Is John's gross pay correct? If not, identify the step in which the error occurred and determine John's correct gross pay.

  (N2.4, N2.6)

Resources/Notes

Authorized Resource

Math at Work 10

4.1 Wages and Salary

SB: pp. 174-185

TR: pp. 194-206

BLM 4-5

Number

Specific Outcomes

Students will be expected to

N2, A1 Continued ...

Achievement Indicators:

N2.7 *Explain why gross pay and net pay are not the same.*

N2.8 *Determine the Canadian Pension Plan (CPP), Employment Insurance (EI) and income tax deductions for a given gross pay.*

N2.9 *Determine net pay when given deductions; e.g., health plans, uniforms, union dues, charitable donations, payroll tax.*

A1.3 *(Continued) Solve a contextual problem that requires the application of a formula that does not require manipulation.*

Suggestions for Teaching and Learning

Gross pay is the total of all earnings including regular pay and overtime pay. Net pay is the gross pay minus any deductions. Students may be familiar with this as "take home" pay. Ask them if anyone has had the experience of being surprised when they received their first paycheque and realized how much money had been deducted. Students need to be aware of the deductions their employers take from their cheques and by how much this will reduce their gross income. Some typical deductions are listed below:
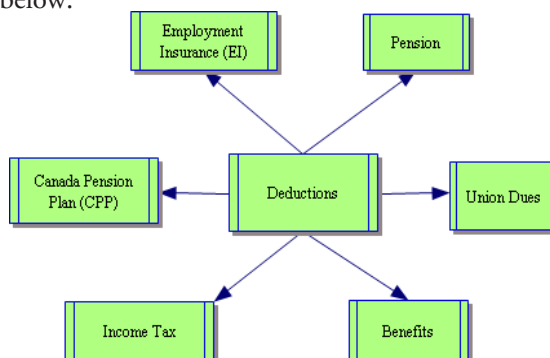
Three mandatory government deductions are CPP, EI and income tax. The employee's share will be deducted from his/her paycheque and the employer's share will be a cost to the company. In Grade 7, students solved problems involving determining the percent of a number. They will use that knowledge to determine deductions. Students will calculate these deductions given the appropriate percentages rates found on the Canada Revenue Agency website.

Employment Insurance (EI) offers financial assistance for some people who lose their jobs through no fault of their own. The number of hours or weeks you need to qualify for EI are based on where you live and the unemployment rate in your economic region at the time you file your claim. In Newfoundland and Labrador, most people will need between 420 and 700 insurable hours of work in the last 52 weeks. EI is a fund into which employees and employers pay. Employers pay 1.4 times the employee's rate. The greater a worker's earnings, the greater the deductions and EI payments, if collected. EI contributions on all eligible earnings will continue throughout the year until the maximum contribution levels are reached. How fast you reach that figure, or if you reach it at all, depends on how much you earn.

For 2015, the EI premium rate was 1.88% of gross earnings and the maximum annual employee premium was $930.60. Illustrate with a cheque stub an example of EI being deducted, and one where it is paid up.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer questions such as the following:

  (i) Jeff's bi-weekly gross salary is $2800.00. He has to pay the following deductions:
  - EI – 1.73%
  - CPP – 4.95%
  - Income Tax – 25%

  (a) Calculate each deduction.
  (b) Determine Jeff's net pay.

  (N2.8, N2.9, A1.1)

  (ii) Lesley earns $11.50 per hour. She works 35 hours a week. Her weekly deductions are:
  - EI - $9.06
  - CPP - $14.41
  - Income Tax - $49.10
  - Company Pension Plan - $10.77
  - Health Plan - $4.85

  Determine her: (a) gross pay
  (b) total deductions
  (c) net pay

  (N2.9, A1.1)

Journal

- Ask students to respond to the following:

  (i) Describe, in your own words, how a higher gross income affects deductions.
  (ii) Sam wants to move in to an apartment and is wondering how much he can afford to pay for rent. Offer him advice on whether he should consider his gross income or his net income. Explain.

  (N2.7, N2.8, N2.9)

Performance

- In the role of a business owner, ask students to use the Payroll Deductions Online Calculator (see resources) to determine the CPP, EI and tax deductions for an employee.

  (N2.8)

Resources/Notes

Authorized Resource

Math at Work 10

4.2 Net Pay

SB: pp. 186-197

TR: pp. 207-216

BLM 4-6, 4-7

Suggested Resources

Resource Links: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit4.html

- Weblink to the Canada Revenue Agency

- Weblink to a Payroll Deductions Online Calculator which calculates federal and provincial payroll deductions for provinces and territories.

- Pay stub template

Number

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to **N2, A1 Continued ...** **Achievement Indicators:** N2.7 *(Continued) Explain why gross pay and net pay are not the same.* N2.8 *(Continued) Determine the Canadian Pension Plan (CPP), Employment Insurance (EI) and income tax deductions for a given gross pay.* N2.9 *(Continued) Determine net pay when given deductions; e.g., health plans, uniforms, union dues, charitable donations, payroll tax.* N2.6 *(Continued) Identify and correct errors in a solution to a problem that involves gross or net pay.* A1.3 *(Continued) Solve a contextual problem that requires the application of a formula that does not require manipulation.*	Canada Pension Plan (CPP) protects families against income loss due to retirement, disability, or death. Both employees and employers contribute a portion to the Canada Pension Plan. The employer matches the contributions made by the employee. For CPP, there are yearly maximum contribution amounts (e.g., \$2 479.95 for 2015) and once these are reached during the calendar year the contributions will cease. In 2015, the CPP deduction rate was 4.95% of any gross earnings above \$3 500.00 and the maximum rate for pensionable earnings was \$53 600. CPP = (Earnings − \$3500) × 0.0495 Students should be exposed to situations where income is below the minimum contribution level. For example, Kyle earned \$3280 through a summer job, \$220 less than the minimum contribution level. As a result, he did not have to contribute to the CPP in 2015. If Kyle's employer deducted any CPP contributions, they would have been refunded to him when he filed his tax return. They should also be given examples of income between the minimum and maximum contribution rates, and of income higher than the maximum contribution level. • Mark is employed as a photographer. His annual salary is \$45 000. This figure is between the minimum contribution level of \$3500 and the 2015 maximum rate of \$53 600. His maximum contribution for 2015 is \$2 054.25 • Cindy is employed as a dental hygienist. Her annual salary is \$68 700. Her CPP contribution is given by (\$68 700 - \$3 500) × 0.0495 = 3 227.40. To determine Cindy's bi-weekly payment, students could divide this value by 26, resulting in \$124.13. Once her contribution reaches the maximum annual contribution is \$2479.95 she will see an increase in her net pay since there will no longer be CPP deductions. CPP contributions will begin the following year. Students could use Cindy's bi-weekly payment to determine how many pay periods it will take for her to make her full CPP contribution: 2479.95 ÷ 124.13 = 19.98 (approximately 20 pay periods or 40 weeks).

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Paper and Pencil

- Assign a job to each student or pair of students. Ask them to complete the following for the assigned job:

  (i)   Calculate gross annual salary.
  (ii) Determine which federal tax bracket it fits and calculate the federal income tax deduction.
  (iii) Repeat (ii) for NL tax.
  (iv) Calculate EI and CPP deductions.
  (v)   Complete a blank T4 form.
  (vi) Students draw five cards from a deck of cards containing other considerations, such as:

childcare	charitable donations
rental	RRSP
tuition amounts	tips
student loan	moving expenses
transportation	dependents

  They can complete an income tax return form for their assigned jobs.

  (N2.7, N2.8, N2.9, A1.1)

- Ask students to answer the following:

  Dawn's bi-weekly gross salary is $2 400.00. She has to pay the following deductions:

  - EI – 1.73%
  - CPP – 4.95% (up to a yearly maximum of $2 479.95)
  - Income Tax – 25%

  Dawn used the following steps to calculate her net income for one pay period.

  Step 1: EI = $2 400 × 0.0173 = $41.52
  Step 2: CPP = $2 400 × 0.0495 = $118.80
  Step 3: Tax = $2 400 × 0.25 = $600.00
  Step 4: Total deductions = $760.32
  Step 5: Net income = $2 400.00 - $760.32
  $\qquad\qquad\qquad$ = $1 639.68

  Is Dawn's net pay correct? If not, identify the step in which the error occurred and calculate Dawn's correct net pay.

  (N2.6, N2.8 and N2.9, A1.5)

Resources/Notes

Authorized Resource

Math at Work 10

4.2 Net Pay

SB: pp. 186-197

TR: pp. 207-216

BLM 4-6, 4-7

Number

Specific Outcomes

Students will be expected to

N2, A1 Continued ...

Achievement Indicators:

N2.7 *(Continued) Explain why gross pay and net pay are not the same.*

N2.8 *(Continued) Determine the Canadian Pension Plan (CPP), Employment Insurance (EI) and income tax deductions for a given gross pay.*

N2.9 *(Continued) Determine net pay when given deductions; e.g., health plans, uniforms, union dues, charitable donations, payroll tax.*

N2.6 *(Continued) Identify and correct errors in a solution to a problem that involves gross or net pay.*

A1.3 *(Continued) Solve a contextual problem that requires the application of a formula that does not require manipulation.*

Suggestions for Teaching and Learning

Income Tax is a type of deduction used to help pay for everything from building roads and maintaining law and order to funding our health service. Students should examine federal and provincial tax deduction tables and discuss the differences in provincial and federal amounts. Most students will be interested in actual deductions for different ranges of income. Federal and provincial/territorial tax rates vary depending on the employee's taxable income.

Students should also discuss optional deductions, such as company health and pension plans. Canada, for example, is recognized for its effective health care system. Employees still opt, however, to buy extended coverage from their place of work to cover unforeseen expenses such as vision care, dental care, prescription drugs and accidental death. Once all the deductions have been totaled, the net pay will then be calculated. This can be demonstrated using a sample pay stub.

Teachers should also discuss vacation pay with students. In NL, vacation pay is 4% of gross wages for employees who have worked less than 15 continuous years with the same employer and 6% for employees who have worked 15 continuous years or more with the same employer. Employees may be given their vacation pay before going on vacation or they may be paid each pay period. Students should work with problems that include vacation pay.

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Performance

- Provide students with a problem that involves calculating net pay. Call on one student to calculate the first deduction. The student should explain how to complete this step. The student then calls on another student to calculate the next deduction and "passes the pen". This continues until the problem is finished. When a question arises, the student holding the pen must answer the question, call on another student to help, or "pass the pen" to a different student. This activity could also be completed in small groups.

(N2.6, N2.9)

Paper and Pencil

- John works 40 hours a week at Costco, where he earns $15.00/hr plus 4% vacation pay. Ask students to complete the following:
  (i) Determine his vacation pay.
  (ii) Determine his EI contribution - 1.88%.
  (iii) Determine his CPP contribution - 4.95%.
  (iv) Determine the amount of income tax that is deducted - 22%
  (v) Determine his net pay.

(N2.4, N2.6, N2.8)

Resources/Notes

Authorized Resource

Math at Work 10

4.2 Net Pay

SB: pp. 186-197

TR: pp. 207-216

BLM 4-6, 4-7

4.3 Other Forms of Income

SB: pp. 198-209

TR: pp. 217-225

BLM 4-8

Number

Specific Outcomes

Students will be expected to

N2 Continued ...

Achievement Indicators:

N2.10 *Determine gross pay for earnings acquired by:*

- *base wage, plus commission*

- *single commission rate.*

N2.11 *Describe the advantages and disadvantages for a given method of earning income; e.g., hourly wage, tips, piecework, salary, commission, contract work.*

Suggestions for Teaching and Learning

Students will now investigate commission earnings and earnings for piecework and contract work. They should explore various methods of calculating regular pay, including commission only, salary plus commission and tips.

Students should brainstorm, research and discuss possible advantages and disadvantages of various income earning methods. Some suggestions follow. However, these are not intended to be all-inclusive.

Method	Advantage	Disadvantage
Hourly Wage	guaranteed income for hours worked	reduced hours during slow perioeds
Tips	additonal income beyond regular salary	job may be low paying
Piecework	more money if you work faster	may ignore safety standards to work faster
Salary	income continues during slow sales periods	work overtime without extra income
Commission	increased income during good sales periods	decreased income during slow sales periods

General Outcome: Develop number sense and critical thinking skills.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer questions such as the following:

    (i) Stephen works at Sears selling appliances. His base salary is $300/week and he makes 5% on his sales. During the month of August, he sold appliances worth $120 000. What is his gross pay for that particular month?

    (N2.10)

    (ii) Jordan fishes with his granddad in the summertime. He is paid 6% commission on the amount of catch landed. If $7500 worth of fish is landed, what is his gross pay?

    (N2.10)

- Ask students to research the business and classified ads section of the newspaper and clip out a job that pays by:

    (i) a salary
    (ii) an hourly wage
    (iii) a straight commission
    (iv) a salary plus commission
    (v) piecework

    They can share the clippings with other classmates and discuss which jobs are of interest to them.

    (N2.2, N2.11)

Portfolio

- Students could identify jobs that commonly use different methods of earning income and describe some advantages and disadvantages of each method.

    (N2.2, N2.11)

Resources/Notes

Authorized Resource

Math at Work 10

4.3 Other Forms of Income

SB: pp. 198-209

TR: pp. 217-225

BLM 4-8

All About Angles

Suggested Time: 18 Hours

Unit Overview

Focus and Context

Visualizing angles is important for estimating, describing, measuring and creating angles in workplace and everyday environments. In this unit, students will work with acute, right, obtuse, straight and reflex angles. They will draw, replicate and construct, and bisect the various types of angles, as well as use them in problem-solving situations.

Students will also explore the relationships of angles that are formed by parallel lines and a transversal. They will work with corresponding angles, vertically opposite angles, alternate interior and exterior angles, and angles on the same side of the transversal.

Outcomes Framework

GCO
Develop spatial sense.

SCO G1

Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

SCO G5

Solve problems that involve parallel, perpendicular and transversal lines, and pairs of angles formed between them.

SCO G6

Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by:

- drawing
- replicating and constructing
- bisecting
- solving problems.

SCO Continuum

Mathematics 9	Mathematics 1202	Mathematics 2202
Geometry		
not addressed	G1 Analyze puzzles and games that involve spatial reasoning, using problem-solving stragies [C, CN, PS, R] G5 Solve problems that involve parallel, perpendicular and transversal lines, and pairs of angles formed between them [C, CN, PS, V] G6 Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by: • drawing • replicating and construing • bisecting • solving problems. [C, CN, ME, PS, R]	N1 Analyze puzzles and game that involve numerical resoning, using problem-solving strategies. [C, CN, PS, R] G3 Model and draw 3-D objects and their views. [CN, R, V]

Mathematical Processes

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[ME]	Mental Mathematics and Estimation	[T]	Technology
		[V]	Visualization

Geometry

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to	

G6 Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by:

- **drawing**
- **replicating and constructing**
- **bisecting**
- **solving problems.**

[C, ME, PS, T, V]

In this unit, students will explore a variety of ways to identify, estimate, measure and draw angles. They will work with various types of angles and will be expected to classify them according to their measure. Students were previously introduced to angles in Grade 6 (6SS1). In Grade 7, they performed various geometric constructions (7SS3). Although students are familiar with the terminology acute, obtuse, right, and straight, and with bisecting angles using a variety of techniques, review may be necessary.

Achievement Indicators:

G6.1 *Measure, using a protractor, angles in various orientations.*

In addition to standard dimensions in length, the angle is one of the most common home construction calculations. Framing, roofing and basic woodworking all depend on exact angle measurements and cuts. The angle at which a ball is kicked is an important factor in whether or not a goal results. The required angle measurement of a wheelchair ramp is also an important calculation. As an introduction to this unit, students should be given the opportunity to measure angles using a protractor. Students could be given various angles and asked to measure them accurately or they can measure the angles of objects within the classroom. Students will categorize angles according to their measure. The terms acute, right, straight, obtuse and reflex should be defined.

G6.2 *Draw and describe angles with various measures, including acute, right, straight, obtuse and reflex angles.*

Students should then independently draw a variety of angles and measure them. Encourage students to exchange their angles with other students so they can compare their answers and discuss how close the angle measurements actually were.

Students will also construct angles accurately. This can be done using a variety of techniques such as a compass and straightedge, a protractor, a set square or a rafter angle square. Teachers could illustrate how to construct a 30° angle on a piece of wood before it is to be cut with the carpenter's tool, a rafter angle square.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Performance

- Set up stations with the required protractors and worksheets. Students will move around the stations and measure the different types of angles on the worksheets. If a Smartboard is available, students can use the hands-on protractor tool. The last station can be a writing piece where students summarize their understanding of the different types of angles in their own words.

(G6.1, G6.2)

- Ask students to play Angle Tic-Tac-Toe.

(G6.2)

Paper and Pencil

- Ask students to answer the following:

    The floor plan represents a kitchen and living room. Jamie is going to re-tile the kitchen floor. Determine the measure of the angle indicated below.
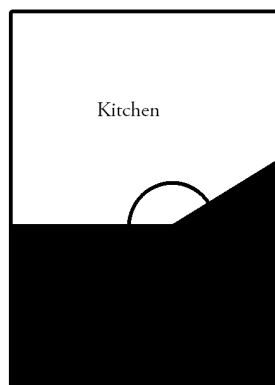
(G6.1)

Resources/Notes

Authorized Resource

Math at Work 10

5 Get Ready

Student Book (SB): pp. 220-221

Teacher Resource (TR): pp. 241-242

BLM 5-3, 5-4

5.1 Estimating and Measuring Angles

SB: pp. 222-233

TR: pp. 243-251

BLM 5-5, 5-6, 5-7, 5-8

Suggested Resources

Resource Links: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit5.html

- Video illustrating how to use a rafter-angle square for measuring and drawing angles

- An applet providing interactive lessons on angle measurement and estimation (up to 180°) and on the proper use of protractors

- Weblink for exploring angle measurement

- Carnival game for approximaing angles, including reflex angles

- Interative protractor to measure given angles

- Angle Tic-Tac-Toe

Geometry

Specific Outcomes	Suggestions for Teaching and Learning

Students will be expected to

G6 Continued ...

Achievement Indicators:

G6.3 *Identify referents for angles.*

G6.4 *Sketch a given angle.*

G6.5 *Estimate the measure of a given angle, using 22.5°, 30°, 45°, 60°, 90° and 180° as referent angles.*

G6.6 *Solve a contextual problem that involves angles.*

Students worked with angles and referents in Grade 6 (6SS1). Referent angles are commonly used angle measurements such as 22.5°, 30°, 45°, 60°, 90° and 180°. Students should use these angle measures as a point of reference to estimate or "eyeball" unknown angle measurements. Students should be able to estimate, for example, an angle that is approximately 100° or estimate the angle they hold their pencil when used for writing.

The values of these referent angles should be compared to real-life situations. Most corners on cabinets, for example, are 90°. Cabinets on an angle in the corner are typically set at 45°. Discuss with students that each corner has two angles. A 90° angle is made up of two 45° angles, and a 45° angle is made up of two 22.5° angles. When installing crown mouldings, 22.5° angles are often used. When installing baseboards, 45° angles are common.

Students should be able to sketch the approximate measure of various angles using their knowledge of referent angles. For example, they should be able to sketch a 50° angle because they can estimate it is between 45° and 60°.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Performance

- Ask students to answer the following:

  Estimate the measure of the angle below using referent angles as a guide.

  (G6.2, G6.4)
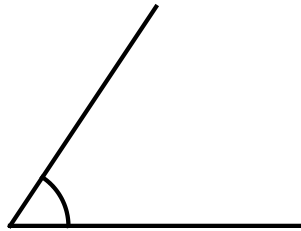
Observation

- Ask students to find examples of the 6 referent angles in the classroom/school.

  (G6.2, G6.4)

Journal

- Ask students to sketch a 30° angle and explain how they estimated the size.

  (G6.4)

Resources/Notes

Authorized Resource

Math at Work 10

5.1 Estimating and Measuring Angles

SB: pp. 222-233

TR: pp. 243-251

BLM 5-5, 5-6, 5-7, 5-8

Geometry

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to **G6 Continued ...** **Achievement Indicators:** G6.7 *Explain and illustrate how angles can be replicated in a variety of ways; e.g., Mira, protractor, compass and straightedge, carpenter's square, dynamic geometry software.* G6.8 *Replicate angles in a variety of ways, with and without technology.* G6.6 (*Continued*) *Solve a contextual problem that involves angles.* G6.9 *Bisect an angle, using a variety of methods.*	Students may use various methods to make an exact copy of, or replicate, an angle. A Mira can be used, for example, to copy the reflection of an angle. Another tool is a t-bevel (see diagram) which is an adjustable carpenter's tool used to copy angles. Angles can also be bisected using various methods. Students can use a protractor, a compass, a Mira, paper folding, or a carpenter's square to bisect the angle, or divide it into two equal parts.
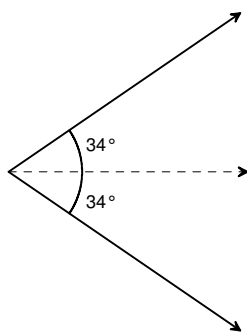
Discuss with students examples of real-life and workplace situations where it is necessary to bisect angles. One example occurs when a carpenter installs mouldings in a corner (not always restricted to a right angle). The mouldings must be cut at an angle so that the two pieces fit together tightly. The carpenter is creating an angle bisector of the corner angle.

The compass tool in Notebook software for Smartboards is useful for bisecting and replicating angles.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer the following:

  (i) Sally is building a patio and one of the corners is going to look like the diagram below. She will need to bisect the angle to make the correct cut. At what angle should she cut the boards?
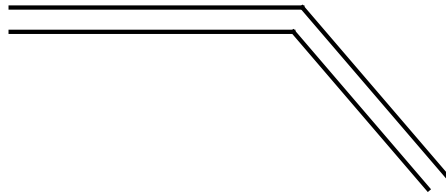
  (G6.1, G6.5, G6.9)

  (ii) Sally needs to copy the angle of this corner to use on the next corner. Explain and demonstrate how she can replicate the angle.
  (G6.6, G6.8, G6.9)

Performance

- Ask students to answer the following:

  A carpenter is building an octagon-shaped deck.

  (i)   Determine the angles at which the boards must be cut.
  (ii) Bisect one of the angles and check to see if it matches.

  (G6.6, G6.9)

- Ask students to complete the following paper folding activity:

  (i)   Construct a 60° angle on a loose leaf sheet of paper and label the rays as A and B.
  (ii) Fold the paper from the vertex so that ray A is folded exactly onto ray B.
  (iii) Draw the angle bisector by tracing the paper along the fold.
  (iv) Measure each angle to verify they are equal.

  (G6.1, G6.2, G6.9)

Resources/Notes

Authorized Resource

Math at Work 10

5.2 Angle Constructions

SB: pp. 234-245

TR: pp. 252-260

BLM 5-9, 5-10

Suggested Resources

Resource Link: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit5.html

- Weblink explaining how to use a Mira™

- Weblink explaining how to use a framing square

Geometry

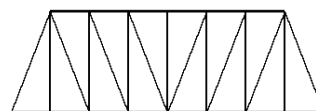| Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to **G5 Solve problems that involve parallel, perpendicular and transversal lines, and pairs of angles formed between them.** [C, CN, PS, V] **Achievement Indicator:** G5.1 *Sort a set of lines as perpendicular, parallel or neither, and justify this sorting.*	Students worked with perpendicular and parallel lines in Grade 7 (7SS3). When lines are parallel, they do not intersect and the perpendicular distances between the lines at different points are equal. Perpendicular lines intersect at 90°. In this unit, the focus will be on angle relationships that occur with perpendicular lines and with parallel lines cut by a transversal. Students will distinguish between parallel and non-parallel lines or perpendicular and non-perpendicular lines by inspection. Diagrams and pictures can help promote discussion. The miniature truss bridge that follows is one such example. When exploring parallel and perpendicular lines, various items in a classroom can be identified (e.g., whiteboard, tiles, windows, etc.). The design on a quilt, rails of an escalator, road map, or even construction scaffolding can also be used as illustrative examples. Students should then define the terms parallel and perpendicular based on their observations. Students are not required to formally prove that lines are perpendicular or parallel.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Observation

- Ask students to identify each set of lines as perpendicular, parallel, or neither.
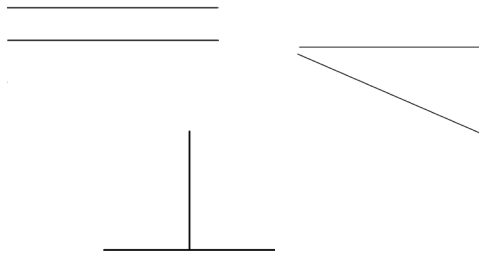
(G5.1)

Performance

- Ask students to make their own drawing or bring in pictures from magazines or newspapers of examples of parallel and perpendicular lines in the world. One example of parallel and perpendicular lines can be seen on a basketball backboard.

(G5.1)

- Students could design and build a popsicle stick bridge. To sort the lines, they could colour parallel sticks blue, colour sticks that are perpendicular to each other red, and colour all others with random colours.

(G5.1)

Resources/Notes

Authorized Resource

Math at Work 10

5.3 Lines and Angles

5.4 Angles in Our World

SB: pp. 246-261, 262-273

TR: pp. 261-274, 275-284

BLM 5-11, 5-12, 5-13, 5-14

Geometry

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to	
G5 Continued ...	
Achievement Indicators:	

G5.2 *Illustrate and describe complementary and supplementary angles.*

G5.3 *Identify, in a set of angles, adjacent angles that are not complementary or supplementary.*
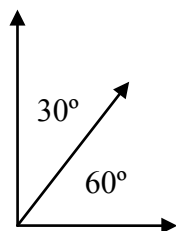
Adjacent angles are any two angles that share a common vertex and a common ray separating the two angles.
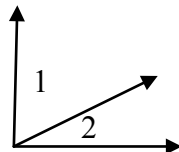
Complementary and supplementary angles are new terms for students. Two angles are complementary if the sum of their angles equals 90°, whereas two angles are supplementary if the sum of their angles equals 180°.
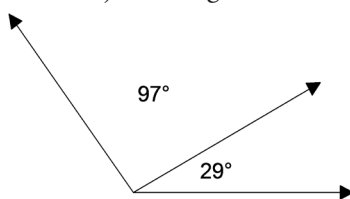
Complementary   Supplementary
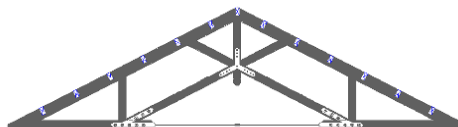
Adjacent and supplementary
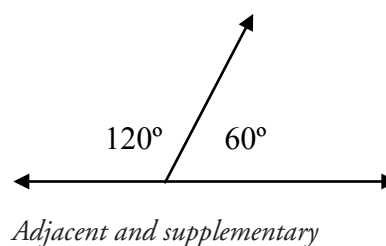
Adjacent and complementary

Some adjacent angles are neither complementary nor supplementary.
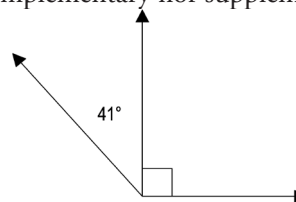
Looking at the roof truss, students can identify which angles are adjacent, complementary or supplementary.

Students should also be aware that there are cases where complementary or supplementary angles are not adjacent angles.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to determine the missing angles in diagrams like those below.
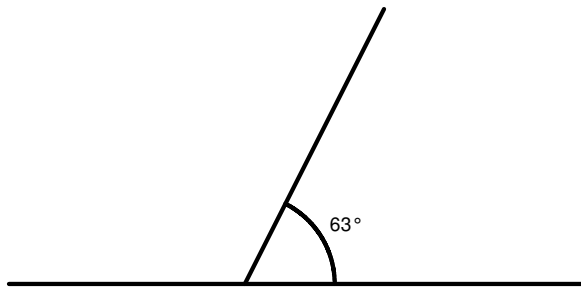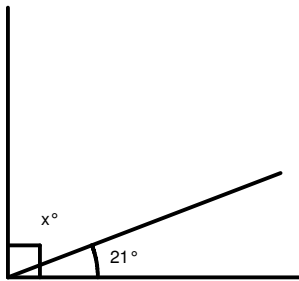
(G5.2)

- Sue is using a compound mitre saw to cut boards for a tree house. The largest angle indicated on the saw is 45°. Ask students to explain how she could cut a 60° angle.

(G5.3)

Performance

- Refer to the popsicle stick bridge from page 107. Ask students to identify any angles that are complementary or supplementary.

(G5.2)

Resources/Notes

Authorized Resource

Math at Work 10

5.3 Lines and Angles

5.4 Angles in Our World

SB: pp. 246-261, 262-273

TR: pp. 261-274, 275-284

BLM 5-11, 5-12, 5-13, 5-14

Geometry

Specific Outcomes

Students will be expected to

G5 Continued ...

Achievement Indicators:

G5.4 *Identify and name pairs of angles formed by parallel lines and a transversal, including corresponding angles, vertically opposite angles, alternate interior angles, alternate exterior angles, interior angles on same side of transversal, and exterior angles on same side of transversal.*
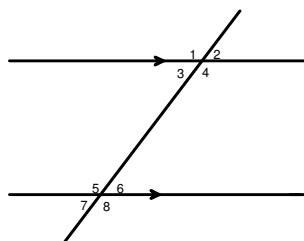
G5.5 *Explain and illustrate the relationships of angles formed by parallel lines and a transversal.*

G5.6 *Determine the measures of angles involving parallel lines and a transversal, using angle relationships.*

Suggestions for Teaching and Learning

Using a diagram, students should be able to identify the different types of angles that are formed when two parallel lines are cut by a transversal. Students should measure all the angles and determine which sets of angles are equal and which sets are supplementary. From this they should be able to recognize the various relationships that exist when a set of parallel lines is intersected by a transversal. In the diagram below, exterior angles are labelled 1, 2, 7, 8. Interior angles are labelled 3, 4, 5 and 6.

(i)   corresponding (e.g., 2 and 6)
(ii) vertically opposite (e.g., 1 and 4)
(iii) alternate interior (e.g., 4 and 5)
(iv) alternate exterior (e.g., 2 and 7)
(v)   interior angles on the same side of the transversal (e.g., 3 and 5)
(vi) exterior angles on the same side of the transversal (e.g., 1 and 7)


Students could investigate these angles using a parking lot. When workers paint lines for a parking lot, they aim to paint lines that are parallel to each other. The lines in a parking lot, therefore, provide an ideal illustration of the relationships between angles created by parallel lines and a transversal. Using chalk, students can discuss and mark the different types of angles in the school's parking lot. They can then measure the angles to determine which angles are equal and which are supplementary.
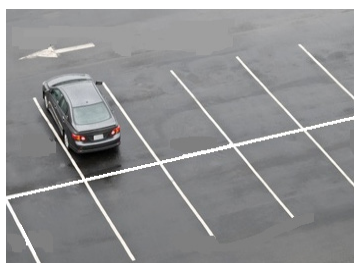
General Outcome: Develop spatial sense.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Paper and Pencil

- Ask students to determine the measures of the missing angles.
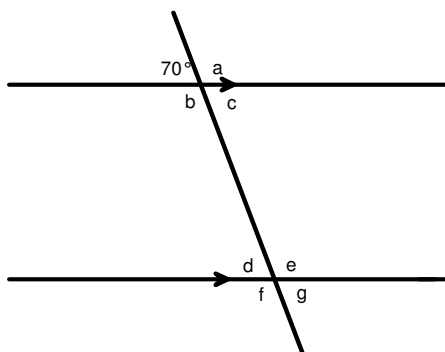
(G5.5, G5.6)

Journal

- Ask students to respond to the following:

  Why do you think lines on a parking lot are usually painted parallel to each other?

  (G5.5)

Performance

- Use tape to create two parallel lines with a transversal on the floor. Students should work in pairs. Student A stands on an angle. Student B picks a card to direct Student A to move to an adjacent angle, vertically opposite angle, etc. Students switch roles after 10 moves.

  Alternatively, this activity could be done with the lines drawn on paper using coloured bingo chips to represent each type of angle.

  (G5.4)

Resources/Notes

Authorized Resource

Math at Work 10

5.3 Lines and Angles

5.4 Angles in Our World

SB: pp. 246-261, 262-273

TR: pp. 261-274, 275-284

BLM 5-11, 5-12, 5-13, 5-14

Suggested Resources

Resource Links: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit5.html

- Interactive website that allow students to work with angles formed by parallel lines and a transversal

- Weblink providing interactive questions on determining missing measures

- Weblink to a Jeopardy Game covering classifying angles, measuring angles and angles formed by parallel lines

Geometry

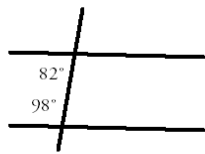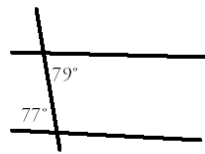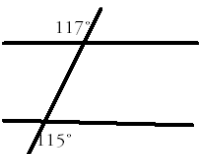| Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to	
G5 Continued ...	
Achievement Indicators:	
G5.7 *Explain, using examples, why the angle relationships do not apply when the lines are not parallel.*	Students should also be given a set of non-parallel lines intersected by a transversal and be asked to measure each angle. They should discover that the same angle relationships (corresponding, alternate interior, alternate exterior, interior angles on the same side of the transversal, and exterior angles on the same side of the transversal) do not exist.
G5.8 *Determine if lines or planes are perpendicular or parallel, e.g., wall perpendicular to floor, and describe the strategy used.*	Students should determine, with justification, whether a set of lines are perpendicular or parallel. To determine if lines are perpendicular, students could use a protractor, a carpenter's square, or Pythagorean Triples. For example, if a flooring installer wants to ensure that the corner of a room is square, a 3-4-5 Pythagorean Triple could be used.
G5.9 *Solve a contextual problem that involves angles formed by parallel lines and a transversal (including perpendicular transversals).*	To identify parallel lines, students could draw a transversal, and use the angle properties to confirm the lines are parallel.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Provide students with 3 different diagrams and ask them to determine which diagram illustrates parallel lines.

Diagram 1	Diagram 2	Diagram 3
82° 98°	79° 77°	117° 115°

(M5.8)

- Explain to students that the diagram below is an architect's drawing of an ice surface. Ask students to determine if the blue lines are parallel.

Blue Line   Blue Line

(G5.8)

Resources/Notes

Authorized Resource

Math at Work 10

5.3 Lines and Angles

5.4 Angles in Our World

SB: pp. 246-261, 262-273

TR: pp. 261-274, 275-284

BLM 5-11, 5-12, 5-13, 5-14

Geometry

Specific Outcomes	**Suggestions for Teaching and Learning**

Students will be expected to

G1 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

[C, CN, PS, R]

This outcome is intended to be integrated throughout the course by using puzzles and games focused on sliding, rotation, construction and deconstruction. These are intended to help students enhance spatial reasoning and problem-solving strategies.

Achievement Indicators:

G1.1 *Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g.,*

- *guess and check*
- *look for a pattern*
- *make a systematic list*
- *draw or model*
- *eliminate possibilities*
- *simplify the original problem*
- *work backward*
- *develop alternative approaches.*

G1.2 *Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.*

G1.3 *Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.*

Students will benefit more from solving puzzles and playing games if they take time to reflect on their experiences.

Ask them to choose one of the puzzles or games they have worked on and write about the following:

- Explain the rules of the game in your own words. Show your rules to another student. Do they agree with your explanation? Can other people find loopholes in your rules?

- What did you do when you got stuck? Explain through words or diagrams the strategies you tried in solving the puzzle or playing the game.

- What general advice would you give to other students trying to solve the puzzle or play the game?

Students may find it easier to record their thoughts if they talk about what they are thinking as they work through a puzzle, and have their partner take notes.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Journal

- Using a puzzle or game of their choice, ask students to write about the problem solving strategies they tried. Which worked well and which did not?

- Students could write hints for puzzles that they found interesting and then try their hints on another student to determine if they are helpful.

Resources/Notes

Authorized Resource

Math at Work 10

Games and Puzzles

Within 10   SB: p. 279

By the Letter   SB: p. 279

TR: p. 290

BLM 5-18, 5-19

Suggested Resource

Resource Link: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit5.html

- Interactive games involving angle measures

Pythagorean Relation

Suggested Time: 12 Hours
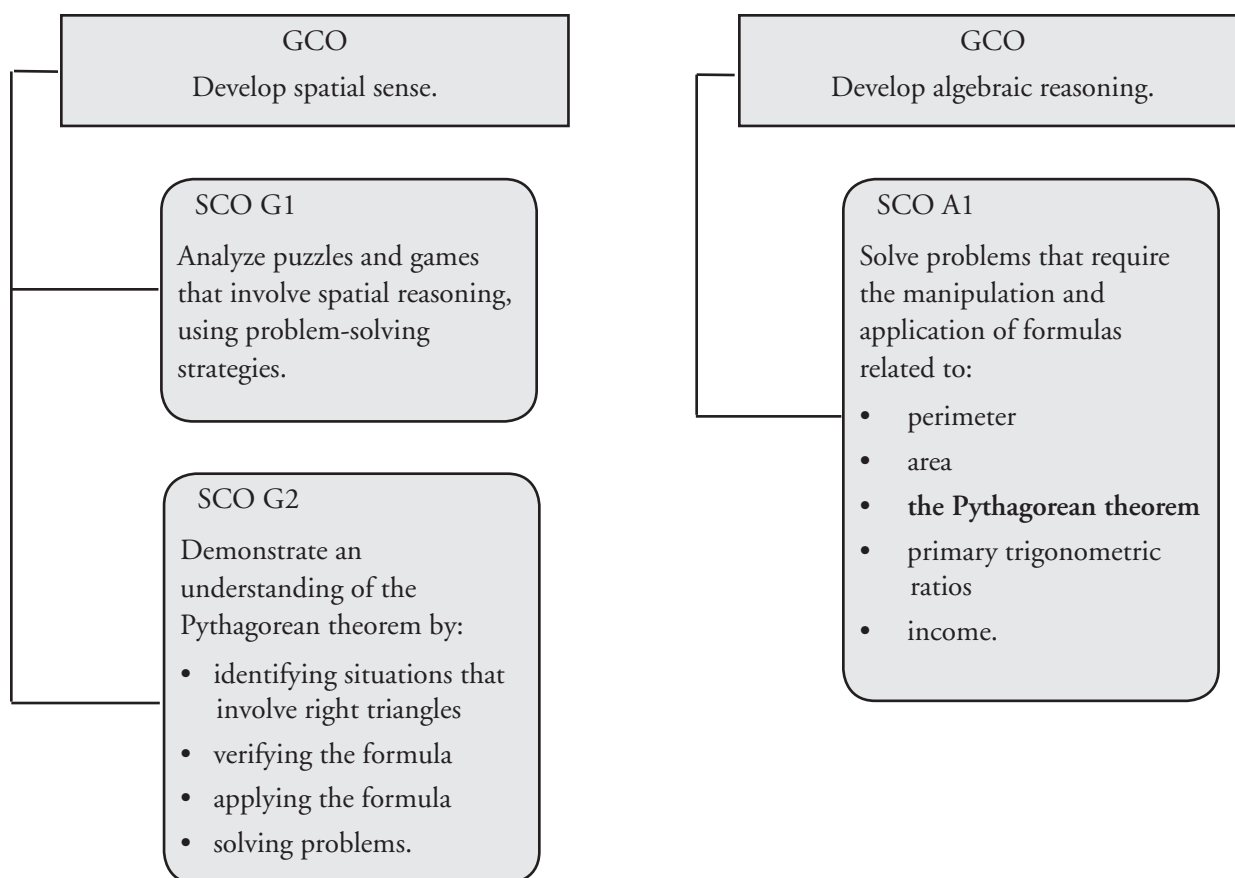
Unit Overview

Focus and Context

In this unit, students will verify the Pythagorean theorem and apply it to solve problems. The Pythagorean theorem is used by people from many walks of life on a daily basis. Examples related to trades and occupations will be used to develop students' understanding of this theorem.

It can be argued that the Pythagorean theorem is the most powerful mathematical equation used in the construction industry. It can be used to enlarge drawings, lay foundations and create perfect right angles. Carpenters can use it to keep their work square. Draftsmen use it to make sure their architectural drawings are accurate.

The Pythagorean theorem can be used to calculate inaccessible distances like the height of a mountain, the width of a river, the distance to Mars, or the diameter of solar systems. Its use is pervasive and powerful.

Outcomes Framework

GCO
Develop spatial sense.

SCO G1

Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

SCO G2

Demonstrate an understanding of the Pythagorean theorem by:

- identifying situations that involve right triangles
- verifying the formula
- applying the formula
- solving problems.

GCO
Develop algebraic reasoning.

SCO A1

Solve problems that require the manipulation and application of formulas related to:

- perimeter
- area
- **the Pythagorean theorem**
- primary trigonometric ratios
- income.

SCO Continuum

Mathematics 9	Mathematics 1202	Mathematics 2202
Geometry		
N5 Determine the square root of positive rational numbers that are perfect squares. [C, CN, PS, R, T]	G1 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	N1 Analyze puzzles and games that involve numerical reasoning, using prolem-solving strategies. [C, CN, PS, R]
N6 Determine an approximate square root of positive rational numbers that are non-perfect squares. [C, CN, PS, R, V]	G2 Demonstrate an understanding of the Pythagorean theorem by: • identifying situations that involve right triangles • verifying the formula • applying the formula • solving problems. [C, CN, PS, V]	G1 Solve problems that involve two and three right triangles [CN, PS, T, V]

Mathematics 9	Mathematics 1202	Mathematics 2202
Algebra		
not addressed	A1 Solve problems that require the manipulation and application of formulas related to: • perimeter • area • **the Pythagorean theorem** • primary trigonometric ratios • income. [C, CN, ME, PS, R]	A1 Solve problems that require the manipulation and application of formulas related to: • volume and capacity • surface area • slope and rate of change • simple interest • finance charges. [CN, PS, R]

Mathematical Processes

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[ME]	Mental Mathematics and Estimation	[T]	Technology
		[V]	Visualization

Geometry

Specific Outcomes

Students will be expected to

G2 Demonstrate an understanding of the Pythagorean theorem by:

- **identifying situations that involve right triangles**
- **verifying the formula**
- **applying the formula**
- **solving problems.**

[C, CN, PS, V]

Achievement Indicator:

G2.1 *Describe historical and contemporary applications of the Pythagorean theorem.*

Suggestions for Teaching and Learning

In this unit, real life applications will be used to promote the importance and relevance of the Pythagorean theorem. Students were introduced to the Pythagorean theorem in Grade 8 (8SS1). In any right triangle, the square of the length of the hypotenuse is the same as the sum of the squares of the lengths of the other two sides. Students should understand how to represent this relationship symbolically: (hypotenuse)^2 = (leg)^2 + (leg)^2. A review of the parts of a right triangle and how they relate to the Pythagorean theorem may be necessary. As well, a basic knowledge of determining the square and square root of a number is required. Students would have been exposed to this in Grade 9 (9N5, 9N6), but a review may be necessary.

Often the sides of a right triangle are labelled *a*, *b*, and *c*, where *c* is the hypotenuse and *a* and *b* are legs. Such a triangle results in the relationship $a^2 + b^2 = c^2$. Students should not become too dependent on this particular labelling, as the variables representing side lengths are not restricted to *a*, *b*, and *c*.

An area interpretation states that if a square is made on each side of a right triangle, then the sum of the areas of the two smaller squares will equal the area of the square on the longest side.

Pythagoras was born in the late 6th century BC on the island of Samos in Ancient Greece. He was a Greek philosopher and religious leader responsible for important developments in mathematics, astronomy and the theory of music. It is believed that Egyptian surveyors, called "rope stretchers", used the converse of the Pythagorean theorem to help build perfect corners for their buildings and pyramids. They had a rope with 12 evenly spaced knots.
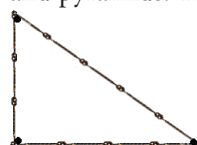
If the rope was pegged to the ground in the dimensions 3-4-5, a right triangle would result. It is believed that they only knew about the 3-4-5 triangle. It was Pythagoras who generalized this relationship to all right triangles and is credited with its first geometrical demonstration.

There are many opportunities to use the Pythagorean relation. Typical applications involve determining how high a ladder will reach up a wall or finding the length of the diagonal of a square or rectangle. It is used to determine indirect measurements (e.g., the width of a river, the height of a tree) and is the basis for computing the distance between two points. In problems requiring the application of the Pythagorean theorem, objects such as buildings, poles and towers are often used. Students should realize that, unless otherwise stated, these objects are assumed to be at right angles to the ground.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Project

- Students could research Pythagoras and his contribution to modern day mathematics.

(G2.1)

Journal

- Ask students to design a question that involves the application of the Pythagorean theorem.

(G2.1)

Performance

- Provide students with a piece of string and a marker. They should make 11 equally spaced marks that separate the string into 12 equal lengths. Teachers could explain that the ancient Egyptians would have used a similar cord to ensure they had right angles when they laid out the boundaries of their fields.

  (i)   Ask students how the ancient Egyptians would have used this cord to ensure they had a right angle.
  (ii) Students can determine what the lengths of the sides of the triangles would have been.

(G2.1)

- Ask students which of the following objects require right angles. This can be done using Thumbs Up/Down, Yes/No cards or interactive response systems.

  (i)   walls
  (ii)   TVs
  (iii) picture frames
  (iv)   sheds
  (v)   tire alignment
  (vi)   swimming pools
  (vii) wall junctions
  (viii) columns

(G2.1)

Resources/Notes

Authorized Resource

Math at Work 10

6 Get Ready

Student Book (SB): pp. 282-283

Teacher Resource (TR): pp. 297-298

BLM 6-3

6.1 Right Triangles

6.2 The Pythagorean Relation

SB: pp. 284-291, 292-302

TR: pp. 299-304, 305-314

BLM 6-4, 6-5

Suggested Resource

Resource Link: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit6.html

- Video showing the history of the Pythagorean theorem

Geometry

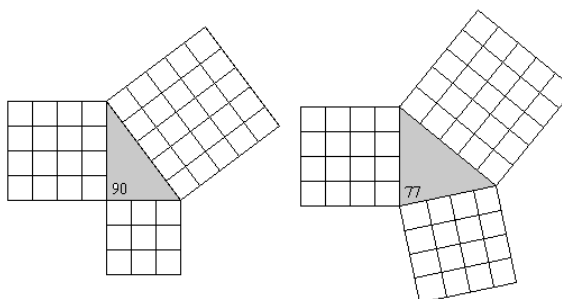| Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to **G2 Continued ...** **Achievement Indicators:** G2.2 *Explain, using illustrations, why the Pythagorean theorem only applies to right triangles.*	Students should use grids that contain right and non-right triangles to verify that the Pythagorean theorem works only for right triangles. Squares are drawn on each side of the triangles shown. Students could compare the area of the square on the hypotenuse with the areas of the squares on the other two sides. To illustrate the Pythagorean relationship give groups of students a variety of right triangles (or ask students to draw such triangles) which have whole number sides, such as the 3 cm–4 cm–5 cm triangle, the 6 cm–8 cm–10 cm triangle, or the 5 cm–12 cm–13 cm triangle. Have students cut out squares from centimetre grid paper so the sides of each square are the same as the side lengths for each triangle. They place the squares on the sides of the triangle, as shown, and find the area of each square. Ask students what they notice. An alternative to cutting out the squares could be to count the blocks to see if any patterns are noticed. Technology, such as the Smartboard or an overhead Pythagorean manipulative set, may also be useful.
G2.3 *Verify the Pythagorean theorem, using examples and counterexamples, including drawings, concrete materials and technology.*	Students should use the Pythagorean theorem to determine whether or not three given side lengths form a right triangle. The focus here is not on finding the length of the sides of a triangle. Rather, it is on showing equality; that is $c^2 = a^2 + b^2$. There are online interactive websites that can be used, as well as the Pythagorean theorem application found in the gallery of Notebook.

To illustrate the Pythagorean relationship give groups of students a variety of right triangles (or ask students to draw such triangles) which have whole number sides, such as the 3 cm–4 cm–5 cm triangle, the 6 cm–8 cm–10 cm triangle, or the 5 cm–12 cm–13 cm triangle. Have students cut out squares from centimetre grid paper so the sides of each square are the same as the side lengths for each triangle. They place the squares on the sides of the triangle, as shown, and find the area of each square. Ask students what they notice. An alternative to cutting out the squares could be to count the blocks to see if any patterns are noticed. Technology, such as the Smartboard or an overhead Pythagorean manipulative set, may also be useful.

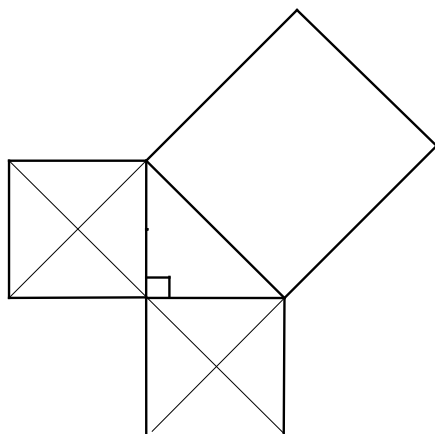Students could explore the Pythagorean theorem with an activity such as the following:

- Use two pieces of string – one 3 feet long and the other 4 feet long.
- Lay the pieces on the floor. Pull them out straight. Hold one end of each piece together. Move one piece until the angle created where the ends meet is a right angle.
- Discuss how to check that it is a right angle.
- Measure the distance between the free end of one piece and the free end of the other.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Performance

- The Pythagorean theorem can be verified through the following activity. Students can trace the diagram shown onto another sheet of paper, cut out the four triangles in each of the two small equal squares and arrange them to exactly cover the large square. Teachers could ask students to describe their findings in their own words.

(G2.3)

- Ask students to build triangles using 12 toothpicks, 24 toothpicks and 30 toothpicks. They should then use the Pythagorean theorem to verify that they are right triangles.

(G2.3)

- Provide students with square shaped food (e.g., Cinnamon Toast Crunch™, Life™ cereal, Golden Grahams™). Give students the dimensions of 2 triangles (e.g., 3-4-5, 2-3-4) and ask them to determine which is a right triangle.

(G2.3)

Journal

- Ask students to respond to the following prompt:

  Write to other types of triangles bragging about your superiority as a right triangle. Be sure to highlight your "special" qualities.

(G2.2)

Resources/Notes

Authorized Resource

Math at Work 10

6.1 Right Triangles

6.2 The Pythagorean Relation

SB: pp. 284-291, 292-302

TR: pp. 299-304, 305-314

BLM 6-4, 6-5, 6-9, 6-10

PL Site: https://www.k12pl.nl.ca/curr/10-12/math/math1202/classroomclips/pythagorean-relation.html

- Classroom clip of students working with the Pythagorean theorem

Suggested Resource

Resource Link: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit6.html

- Link to *Build On Your Talents Using Trades Math,* Construction Sector Council - this resource provides real life applications of the Pythagorean theorem, as well as measurement, area, and trigonometry

Geometry

Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to	
G2 Continued ...	
Achievement Indicators:	
G2.4 *Determine if a given triangle is a right triangle, using the Pythagorean theorem.*	The converse of the theorem states that if the sides of a triangle have lengths *a*, *b* and *c* such that $a^2 + b^2 = c^2$, then the triangle is a right triangle. Once students have an understanding that the Pythagorean theorem is applicable only to right triangles, they can use it to determine if a triangle is right-angled. Examples of both right and non-right triangles should be used. For example, students should be able to determine that a right triangle results from side lengths of 6 cm, 8 cm and 10 cm, while a triangle with sides lengths of 6 cm, 7 cm and 9 cm is not a right triangle.
G2.5 *Explain why a triangle with the side length ratio of 3:4:5 is a right triangle.*	A Pythagorean triple consists of three positive integers *a*, *b*, and *c*, such that $c^2 = a^2 + b^2$. Such a triple is commonly written (*a*, *b*, *c*), and a well-known example is (3, 4, 5). If (*a*, *b*, *c*) is a Pythagorean triple, then so is (k*a*, k*b*, k*c*) for any positive integer k. For example, since (3, 4, 5) is a triple then so is (6, 8, 10). The carpenter square is a good example of this ratio.
G2.6 *Explain how the ratio of 3:4:5 can be used to determine if a corner of a given 3-D object is square (90 degrees) or if a given parallelogram is a rectangle.*	To classify a parallelogram as a rectangle it is sufficient to identify one right angle. Students should be able to use the 3:4:5 ratio to show one of the triangles in a parallelogram is a right triangle, thereby proving the parallelogram is a rectangle.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Paper and Pencil

- Ask students to determine whether each triangle with sides of given lengths is a right triangle. (Units such as feet or inches could be used as well.)

  (i) 9 cm, 12 cm, 15 cm
  (ii) 16 mm, 29 mm, 18 mm
  (iii) 9 m, 7 m, 13 m

  (G2.4, G2.5)

- Keith is building a garage on a floor that measures 18 feet by 24 feet.

  (i) Ask students to calculate the length of the diagonal of the rectangular floor.
  (ii) Keith measures the length of the diagonal to be 29.5 feet. Are the angles at the corners of the garage right angles? Explain.

  (G2.4, G2.7, A1.1)

- Carpenters often use a 3-4-5 triangle to determine if corners are square (90°). Ask students to explain why this works.

  (G2.6)

- Ask students to draw triangles other than right triangles. Have them measure the side lengths and check to see if the Pythagorean theorem works for these non-right triangles.

  (G2.4)

Resources/Notes

Authorized Resource

Math at Work 10

6.1 Right Triangles

6.2 The Pythagorean Relation

SB: pp. 284-291, 292-302

TR: pp. 299-304, 305-314

BLM 6-4, 6-5

Suggested Resource

Resource Links: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit6.html

- An interative website where students can demonstrate the Pythagorean theorem by moving two small squares to cover the area of the large square

- An interactive site where students can solve a variety of problems involving the Pythagorean theorem

Algebra

Specific Outcomes	Suggestions for Teaching and Learning

Specific Outcomes

Students will be expected to

A1 Solve problems that require the manipulation and application of formulas related to:

- perimeter

- area

- **the Pythagorean theorem**

- primary trigonometric ratios

- income.

[C, CN, ME, PS, R]

Achievement Indicator:

A1.2 *Describe, using examples, how a given formula is used in a trade or an occupation.*

Suggestions for Teaching and Learning

It is intended that this outcome be integrated throughout the course. Students have already been exposed to problems related to perimeter (M3), area (M4) and income (N2). In this unit, the problems require the manipulation and application of formulas related to the Pythagorean theorem.

The Pythagrean theorem is useful in many trades. In the construction industry, the 3-4-5 method is used to lay out the perimeter of buildings, to ensure corners are square, and to calculate the length of rafters and the length of stringers on stairs.
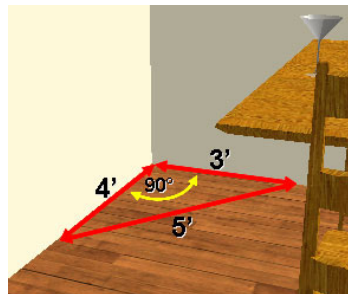
Plumbers use this method to calculate lengths of pipes in piping systems. Crane operators and riggers use it to calculate the length of the boom and the length of slings used to lift loads. Draftsmen use it to ensure architectural drawings are accurate.

General Outcome: Develop algebraic reasoning.

Suggested Assessment Strategies	Resources/Notes
Interview	**Authorized Resource**
• Ask students how they could ensure a wall is perpendicular (square) to the floor if the only tool available is a measuring tape.	*Math at Work 10*
(A1.2)	6.1 Right Triangles
	6.2 The Pythagorean Relation
Paper and Pencil	SB: pp. 284-291, 292-302
• Craig is laying out forms for the cement footings of a house. The house is to be 36 feet by 48 feet. Ask students to determine how long the diagonal of the rectangle should be to ensure there will be right angles at the corners of the house.	TR: pp. 299-304, 305-314
	BLM 6-4, 6-5
(A1.2)	
• Ask students to verify that a quilting square is actually square.	
(A1.2)	

Geometry

Specific Outcomes

Students will be expected to

G2, A1 Continued ...

Achievement Indicators:

G2.7 *Solve a problem, using the Pythagorean theorem.*

A1.3 *Solve a contextual problem that involves the application of a formula that does not require manipulation.*

A1.4 *Solve a contextual problem that involves the application of a formula that requires manipulation.*

A1.1 *Create and solve a contextual problem that involves a formula.*

A1.6 *Explain and verify why different forms of the same formula are equivalent.*

Suggestions for Teaching and Learning

Since the Pythagorean theorem describes the relationship between the three sides of a right triangle, applications involve determining a missing side. Students should be provided with examples that involve finding the length of the hypotenuse given two sides in addition to finding the length of a side given the hypotenuse and another side. For example, students may recognize in baseball that the distance from home plate to 2nd base is the hypotenuse of a right triangle and can determine that distance on a field knowing the length of a baseline. They can also determine the length of a baseline given the hypotenuse.

Problems showing the practical application of the Pythagorean theorem should be explored. Students could discuss, for example, the length the elbow pipe shown in the diagram should be cut.
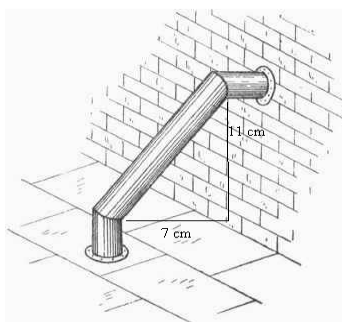
To find a missing leg, *a* or *b*, students can rearrange the formula before substituting values. Alternatively, students may substitute first and then rearrange the formula. The formula can be expressed using symbols or words.

$$\left(leg\right)^2 + \left(leg\right)^2 = \left(hyp\right)^2$$
$$\left(leg\right)^2 + \left(leg\right)^2 - \left(leg\right)^2 = \left(hyp\right)^2 - \left(leg\right)^2$$
$$\left(leg\right)^2 = \left(hyp\right)^2 - \left(leg\right)^2$$

$$a^2 + b^2 = c^2$$
$$a^2 + b^2 - b^2 = c^2 - b^2$$
$$a^2 = c^2 - b^2$$

It is important to present diagrams of right triangles in various orientations. Students should recognize the hypotenuse as being the side opposite the right angle, regardless of the orientation of the figure. They should also recognize that the hypotenuse is the longest side of the triangle. While the use of technology is permissible, students should be encouraged to attempt to find an unknown side without the use of calculators. This will help develop mental math skills and number sense.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to answer questions such as the following:

  (i) Ross has a rectangular garden in his backyard. He measures one side of the garden as 7 m and the diagonal as 11 m. What is the length of the other side of his garden? (Hint: draw a diagram.)
  (G2.7, A1.3)

  (ii) The dimensions of a rectangular frame are 30 cm by 50 cm. A carpenter wants to put a diagonal brace between two opposite corners of the frame. How long should the brace be?
  (G2.7, A1)

  (iii) Rebecca and Julia have to run one lap around the soccer field during practice.
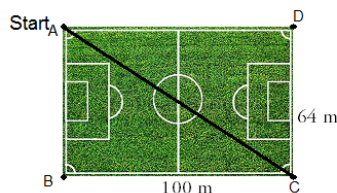
They begin at the corner of the field, as shown in the above diagram. They both run from A to C. However, while running Rebecca gets tired and decides to cut across the soccer field (from C to A), while Julia completes her lap around the field. Ask students to determine how much further Julia runs.
  (G2.7, A1)

  (iv) Joan is giving her boyfriend a hockey stick for Christmas. She wants to wrap it in a box so that he can't guess what it is. The hockey stick is 63 inches long. She goes to the furniture store. They have two boxes:
  • a TV box that is 52 × 10

  • a coffee table box that is 48 × 48

  Which one should she pick?
  (G2.7, A1)

  (v) A baseball diamond is a square having a perimeter of approximately 360 ft. Ask students to determine the direct distance from home plate to second base.
  (G2.7, A1)

Resources/Notes

Authorized Resource

Math at Work 10

6.3 Using the Pythagorean Relation

SB: pp. 303-313

TR: pp. 315-321

BLM 6-6

Geometry

Specific Outcomes

Students will be expected to

G1 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

[C, CN, PS, R]

Achievement Indicators:

G1.1 *Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g.,*

- *guess and check*
- *look for a pattern*
- *make a systematic list*
- *draw or model*
- *eliminate possibilities*
- *simplify the original problem*
- *work backward*
- *develop alternative approaches.*

G1.2 *Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.*

G1.3 *Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.*

Suggestions for Teaching and Learning

This outcome is intended to be integrated throughout the course by using puzzles and games focused on sliding, rotation, construction and deconstruction. These are intended to help students enhance spatial reasoning and problem-solving strategies.

Problem posing is an important part of problem solving. As such, students should be given an opportunity to create their own puzzles or games. They could create one based on an existing puzzle they have worked with, or they could develop a unique puzzle or game.

Following are some ideas that may help students get started:

- use as many or few pieces as possible
- figure out an unlikely move, then build a puzzle that requires it
- form an attractive pattern and try to solve it
- design a challenging puzzle and then create easier puzzles that lead up to it

If students are inventing a game based on an existing one, prompt them to consider how they could change the board, the piece shapes, the way pieces move, or the goal of the game.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Journal

- Students describe how they constructed their puzzles or games and the criteria used to judge whether the puzzle was good.

Performance

- Students should give their created puzzles to at least one other student and watch them try to solve it. Have them discuss whether it was easier or more challenging than they expected. They should reflect on if and where the other student got stuck and whether they found a better solution. Based on this, ask students to find a way to improve or change the puzzle.

- Students could create a sequence of three puzzles: easy, medium and hard. Encourage them to link the three puzzles, if possible. For example, they could all use similar pieces or be based on the same strategy. Students should test their puzzles on another student to see if they agree with the assessment of which puzzle is easy, medium and hard.

- Ask students to play the Pythagorean Game.

Resources/Notes

Authorized Resource

Math at Work 10

Games and Puzzles

Pythagorean Puzzles

SB: p. 319

TR: p. 326

BLM 6-9, 6-10

Suggested Resources

Resource Links: https://www. k12pl.nl.ca/curr/10-12/math/ math1202/links/unit6.html

- The Pythagorean Game - game instructions and masters for the game board and game cards

- Tarsia- can be used to create a variety of puzzles

- Link to the Tangram Channel- templates for various tangram puzzles
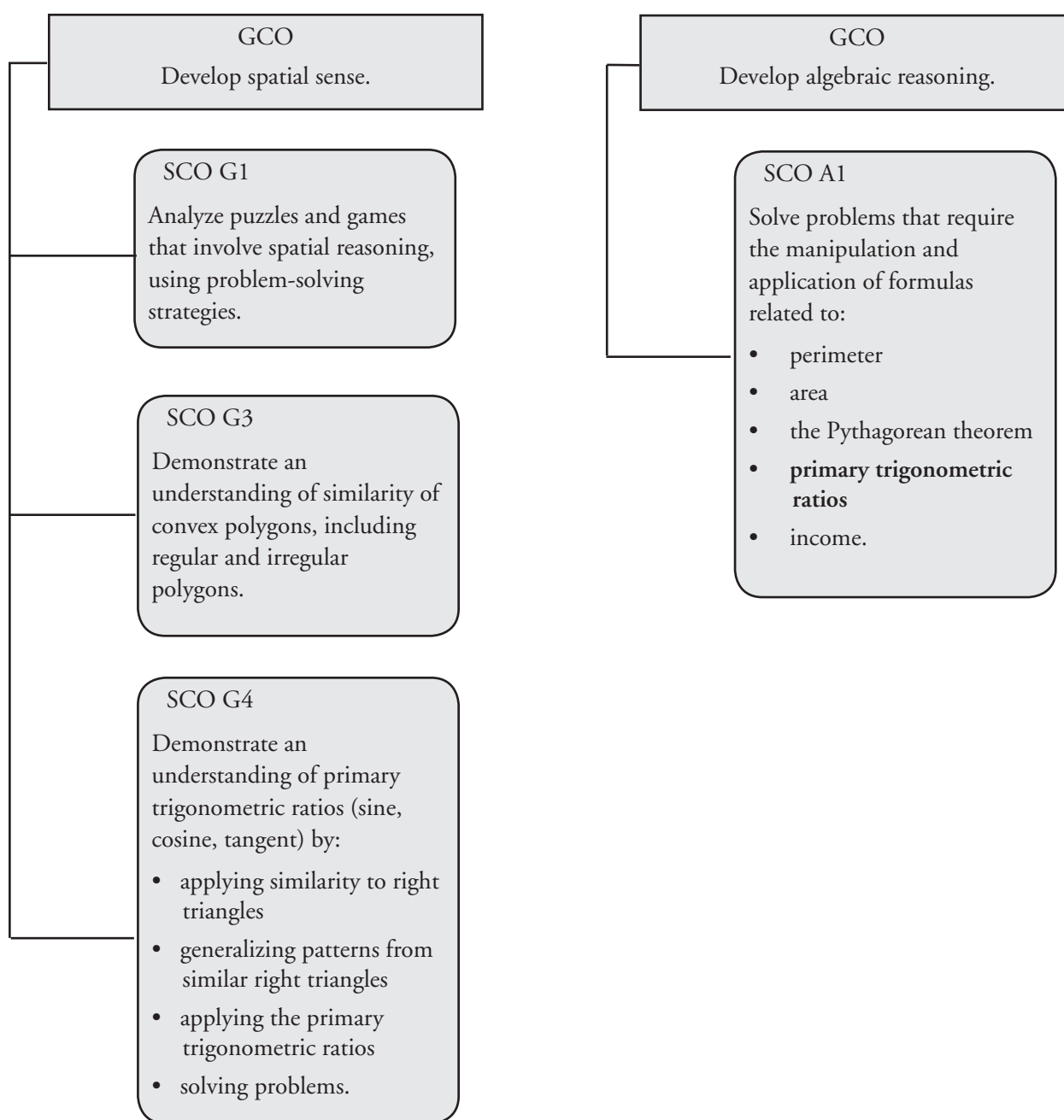
Trigonometry

Suggested Time: 18 Hours

Unit Overview

Focus and Context

In this unit, students will explore the similarity of polygons and generalize patterns from similar right triangles to develop the primary trigonometric ratios. The sine, cosine and tangent ratios will be used to determine side lengths and acute angle measures in right triangles.

Outcomes Framework

GCO
Develop spatial sense.

SCO G1

Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

SCO G3

Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons.

SCO G4

Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by:

- applying similarity to right triangles
- generalizing patterns from similar right triangles
- applying the primary trigonometric ratios
- solving problems.

GCO
Develop algebraic reasoning.

SCO A1

Solve problems that require the manipulation and application of formulas related to:

- perimeter
- area
- the Pythagorean theorem
- **primary trigonometric ratios**
- income.

SCO Continuum

Mathematics 9	Mathematics 1202	Mathematics 2202
Geometry		
SS3 Demonstrate an understanding of similarity of polygons. [C, CN, PS, R, V]	G1 Analyze puzzles and games that involve spatial reasoning, using problem-solving stragies [C, CN, PS, R] G3 Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons. [C, CN, PS, V] G4 Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: • applying similarity to right triangles • generalizing patterns from similar right triangles • applying the primary trigonometric ratios • solving problems. [CN, PS, R, T, V]	N1 Analyze puzzles and game that involve numerical reasoning, using prolem-solving strategies. [C, CN, PS, R] G1 Solve problems that involved two and three right triangles [CN, PS, T, V] G2 Solve problems that involve scale. [PS, R, T, V]
Algebra		
not addressed	A1 Solve problems that require the manipulation and application of formulas related to: • perimeter • area • the Pythagorean theorem • **primary trigonometric ratios** • income. [C, CN, ME, PS, R]	A1 Solve problems that require the manipulation and application of formulas related to: • volume and capacity • surface area • slope and rate of change • simple interest • finance charges. [CN, PS, R]

Mathematical Processes

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[ME]	Mental Mathematics and Estimation	[T]	Technology
		[V]	Visualization

Geometry

| Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to **G3 Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons.** [C, CN, PS, V]	In this unit, students will explore similar polygons. They were previously introduced to the concept of similarity in Grade 9 (9SS3). Students sometimes confuse similarity with equality. Through investigation, students should recognize that similar polygons are polygons whose angles are congruent and whose corresponding side lengths are proportional. Students will work with convex polygons, whose interior angles are less than 180°. They will explore similarity in both regular and irregular polygons.
Achievement Indicators:	
G3.1 *Determine if two or more regular or irregular polygons are similar.*	To test for similarity, students should verify that the corresponding angles are of equal measure and the ratios of the corresponding sides are equal. Regular polygons with the same number of sides are always similar, regardless of size. Students could explore this using regular polygons such as squares or equilateral triangles. Since all the sides of a regular polygon are the same length, corresponding sides of the polygons must always be in the same proportions.
G3.2 *Explain why two or more right triangles with a shared acute angle are similar.*	Students are also expected to determine if various irregular polygons are similar. When working with triangles, there are minimum conditions that prove the figures are similar. Proving two corresponding angles equal would be sufficient information to prove similarity, using angle-angle similarity. In the diagram below, the figure can be separated into two triangles, namely △ABC and △DEC. Students should then notice there are two sets of corresponding equal angles (∠B = ∠E and ∠C = ∠C). Since the sum of the angles in a triangle is equal to 180°, the third set of corresponding angles (∠A and ∠D) must also be equal to each other. Therefore, the two triangles are similar using angle-angle similarity.
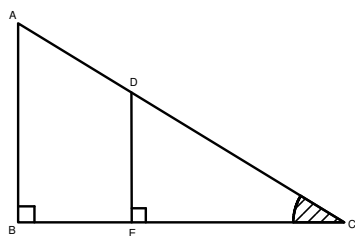
General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Ask students to determine if triangle B or C is similar to triangle A.
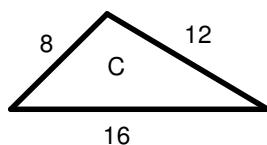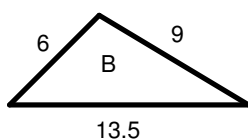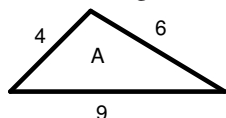
(G3.1, G3.4)

- Ask students to answer the questions that follow based on the diagram given.
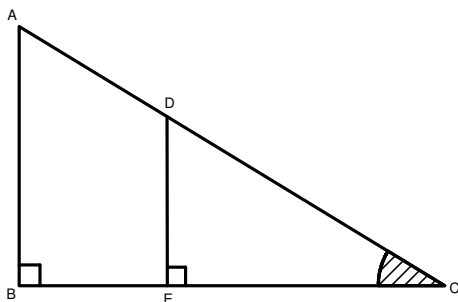
(i)   Which triangles are similar?

(ii) Measure the sides and determine the ratios of:

$$\frac{AB}{DE}, \frac{AC}{DC}$$

$$\frac{AB}{BC}, \frac{DE}{EC}$$

$$\frac{BC}{EC}, \frac{AC}{DC}$$

What do you notice about the values?

(iii) If AB = 9 cm, DE = 6 cm, and EC = 8, what is the length of BC?

(G3.1, G3.2, G3.3)

Resources/Notes

Authorized Resource

Math at Work 10

7 Get Ready

Student Book (SB): pp. 322-323

Teacher Resource (TR): pp. 335-336

BLM 7-3

7.1 Similarity

SB: pp. 324-335

TR: pp. 337-346

BLM 7-4, 7-5

Geometry

| Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to	
G3 Continued ...	

Achievement Indicators:

G3.3 *Explain the relationships between the corresponding sides of two polygons that have corresponding angles of equal measure.*

If two polygons have corresponding angles of equal measure, they are similar. Students could consider similar triangles such as those shown here:

G3.4 *Explain why two given polygons are not similar.*
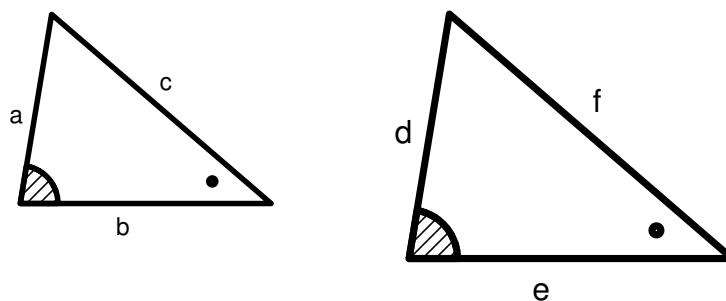
If polygons are similar, corresponding side lengths are all enlarged or reduced by the same factor. In Grade 9, students would have expressed this as "the ratios of their corresponding sides are equal".

Therefore, in the triangles above, $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$.

An alternative test for similarity involves comparing side lengths within each shape. That is, if the ratio between two side lengths on one shape is the same as the ratio between the two corresponding side lengths on the other shape, the figures are similar.

Therefore, $\frac{a}{b} = \frac{d}{e}$, $\frac{c}{b} = \frac{f}{e}$, $\frac{a}{c} = \frac{d}{f}$.

G3.5 *Solve a contextual problem that involves similarity of polygons.*

Indirect measurement is one example of the applications of similar triangles. The concept of similarity is very useful in measuring the heights of inaccessible objects such as buildings, trees and mountains.

The shadowing technique works very well outdoors on sunny days. This "hands-on" project gives students an opportunity to go outside the classroom and take measurements. Students use an object perpendicular to the ground, a metre stick and their shadows to determine the height of the object. Using the fact that the sun's rays are parallel, students can set up a proportion with similar triangles.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Journal

- Ask students to respond to the following:

  Two triangles are similar. The side lengths of the smaller triangle are 3 cm, 4 cm, and 5 cm respectively. Describe how you can determine possible side lengths of the larger triangle.

  (G3.3, G3.5)

Paper and Pencil

- Ask students to answer the following:

  (i) The following quadrilaterals are similar. Determine the value of *x*.
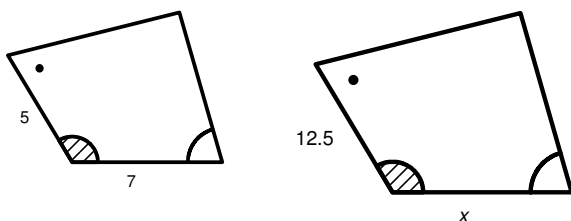
  (G3.3)

  (ii) John wants to build a roof truss that is "4 on 12" (see diagram below). If the roof truss height changes to 6 feet, how wide will the new roof truss be?
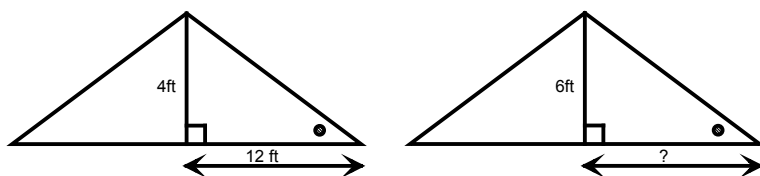
  (G3.5)

Performance

- Each student is given a triangle. Cooperative learning strategies, such as appointment cards, can be used to have students meet in pairs to compare their triangles. Ask them to determine and justify why the triangles are or are not similar. This could also be extended to include other polygons.

  (G3.1, G3.4)

Resources/Notes

Authorized Resource

Math at Work 10

7.1 Similarity

SB: pp. 324-335

TR: pp. 337-346

BLM 7-4, 7-5

Geometry

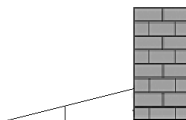| Specific Outcomes	Suggestions for Teaching and Learning
Students will be expected to **G3 Continued ...** **Achievement Indicators:** G3.5 *(Continued) Solve a contextual problem that involves similarity of polygons.*	As an alternative, the mirror technique can be used since this method works both indoors and outdoors. When you look into a mirror, you see light that has bounced off at the same angle that it hit the mirror, so your line of sight creates similar triangles. The goal is to create student interaction and promote mathematical understanding. A suggested classroom activity is to have students guess each other's height using this technique. This investigation can then lead to another group activity where students go to the gymnasium, for example, and measure the height of the basketball net. Students will make a sketch, show the similar triangles formed, take measurements, and find the required height of the object. Engage students in a discussion about using similar triangles to find indirect measures for a variety of real-world applications. Students will then use similar triangles to solve for missing lengths in real world applications. Consider situations such as the following: Many public buildings in the past were built before wheelchair-access ramps became widespread. When it became time to design the ramps, unfortunately the doors of the buildings were already in place. Discussion should be centered around designing a ramp supported by a vertical beam.
G3.6 *Draw a polygon that is similar to a given polygon.*	Reiterate for students that, when creating a similar figure, there are two important features to remember. The measures of their corresponding angles are equal and the measures of their corresponding sides are proportional because they have increased or decreased by the same scale factor. Rulers and protractors can be used to construct similar polygons. Students can use dynamic geometry software to construct a shape and then reduce or enlarge it. As well, students can draw a shape on one square grid and then redraw it on a grid with a different scale (smaller or larger squares).

General Outcome: Develop spatial sense.

Suggested Assessment Strategies

Paper and Pencil

- Students have been asked to create a poster to advertise a field trip to see the lighthouse in Codroy Valley. They want to enlarge a photo of the lighthouse that is 3.5 inches wide and 5 inches long. Ask students to determine how long it will be if the enlargement is to be 16 inches wide. They should draw a sketch of the lighthouse image using the new dimensions.
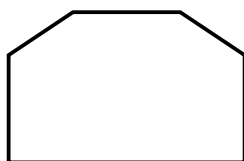
(G3.5)

Performance

- The diagram represents the top view of a patio. Students could choose one of the following parallel tasks.

    Either:

    (i) Measure the angles and sides and reproduce the drawing using a scale factor of 2.

    Or:

    (ii) Given the same diagram with the angle measures, reproduce the drawing using a scale factor of 2.
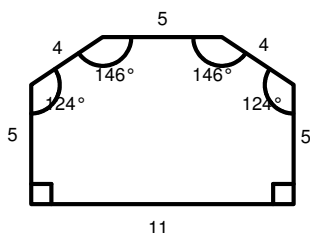
(G3.5, G3.6)

- In small groups, students can create sets of similar triangles (one set per student, with sides and angles labelled). Combine all class triangles and ask students to sort by similarity.

(G3.1, G3.6)

Resources/Notes

Authorized Resource

Math at Work 10

7.1 Similarity

SB: pp. 324-335

TR: pp. 337-346

BLM 7-4, 7-5

Suggested Resources

Resource Links: https://www.k12pl.nl.ca/curr/10-12/math/math1202/links/unit7.html

- Video demonstrating the mirror trick and how similarity of triangles can be used to estimate height

- Video illustrating how similar triangles are used to calculate the position of radiation treatment for cancer patients

- Weblink to dynamic geometry software that can be used to perform dilatations

Geometry

Specific Outcomes	Suggestions for Teaching and Learning

Students will be expected to

G4 Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by:

- **applying similarity to right triangles**
- **generalizing patterns from similar right triangles**
- **applying the primary trigonometric ratios**
- **solving problems.**

[CN, PS, R, T, V]

Students will be introduced to the three primary trigonometric ratios by applying similarity and generalizing patterns from similar right triangles. Since this is the first exposure to trigonometry, students may be interested in some brief history of this particular branch of mathematics.

The term trigonometry comes from two Greek words, trigon and metron, meaning "triangle measurement". A trigonometric ratio is a ratio of the lengths of two sides of a right triangle.

One of the difficulties students sometimes have when working with trigonometric ratios is correctly identifying the opposite and adjacent sides in relation to the reference angle. They should be exposed to right triangles with the reference angle in various locations, so that they recognize that the opposite and adjacent sides are relevant to the reference angle. Angles are often labeled using greek letters, such as theta (θ).

Students should realize that there are three possible pairs of sides with respect to the reference angle, θ: opposite and hypotenuse, opposite and adjacent, and adjacent and hypotenuse.

A1 Solve problems that require the manipulation and application of formulas related to:

- perimeter
- area
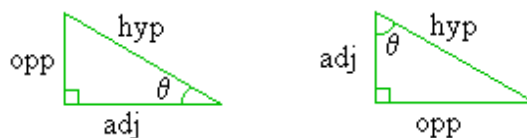- the Pythagorean theorem
- **primary trigonometric ratios**
- income.

[C, CN, ME, PS, R]

It is intended that this outcome be integrated throughout the course. Students have already been exposed to problems related to perimeter (M3), area (M4), the Pythagorean theorem (A2) and income (N2). In this unit, the problems will require the manipulation and application of formulas related to the primary trigonometric ratios.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Paper and Pencil

- Given a right triangle with the right angle and reference angle labelled, ask students to label the sides as opposite, adjacent and hypotenuse.

(G4)

Performance

- Ask students to draw a right triangle, mark the right angle and place a bingo chip on one of the acute angles. Students should then challenge their partner to place sticky notes which read opposite, adjacent and hypotneuse in the proper place.

(G4)

Resources/Notes

Authorized Resource

Math at Work 10

7.2 The Tangent Ratio

7.3 The Sine and Cosine Ratios

SB: pp. 336-347, 348-363

TR: pp. 347-355, 356-367

BLM 7-6, 7-7, 7-8, 7-9, 7-10, 7-11

Geometry

Specific Outcomes	Suggestions for Teaching and Learning

Students will be expected to

G4 Continued ...

Achievement Indicators:

G4.1 *Show, for a specified acute angle in a set of right triangles, that the ratios of length of the side opposite to the length of the side adjacent are equal, and generalize a formula for the tangent ratio.*

G4.2 *Show, for a specified acute angle in a set of right triangles, that the ratios of length of the side opposite to the length of the hypotenuse are equal, and generalize a formula for the sine ratio.*

G4.3 *Show, for a specified acute angle in a set of right triangles, that the ratios of length of the side adjacent to the length of the hypotenuse are equal, and generalize a formula for the cosine ratio.*

Students should investigate the three ratios between the lengths of pairs of sides in right-angled triangles. They could begin this process by constructing several similar right triangles, each with an angle of 30˚.
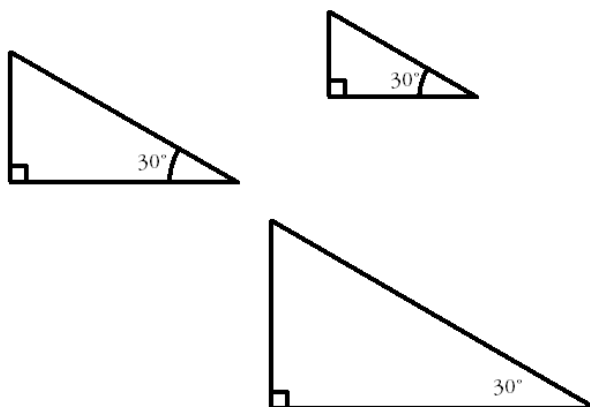
Using 30˚ as the reference angle, students could then measure the opposite and adjacent sides and the hypotenuse for each triangle, and determine the appropriate ratios (opposite to adjacent, opposite to hypotenuse, and adjacent to hypotenuse) for each. Students should notice that the value of any given ratio stays constant, regardless of the size of the triangle. They should then repeat this process for right triangles containing angles of other sizes. As the reference angle changes, the values of the three ratios change. Students should conclude that in any right triangle, the ratios of the different pairs of sides remain constant for a given acute angle, regardless of the size of that angle or the size of the triangle.

Once students have a clear understanding of the concept of these ratios being fixed for any given angle, the formal mathematical names can be introduced.

$$\sin \theta = \frac{opposite}{hypotenuse}$$

$$\cos \theta = \frac{adjacent}{hypotenuse}$$

$$\tan \theta = \frac{opposite}{adjacent}$$

General Outcome: Develop spatial sense.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Portfolio

• Ask students to draw and label a right triangle and state the three primary trigonometric ratios.

(A1.1, G4.1, G4.2, G4.3)

Paper and Pencil

• Ask students to create their own trigonometry problem and distribute to other students to complete a solution to the problem.

(A1.2)

Performance

• Ask students to create their own mnemonic for the primary trigonometric ratios. Students can share their mnemonics and the class can vote for the preferred one.

(G4.1, G4.2, G4.3)

• Ask students to create a foldable outlining the primary trigonometric ratios.

(G4.1, G4.2, G4.3)

Resources/Notes

Authorized Resource

Math at Work 10

7.2 The Tangent Ratio

7.3 The Sine and Cosine Ratios

SB: pp. 336-347, 348-363

TR: pp. 347-355, 356-367

BLM 7-6, 7-7, 7-8, 7-9, 7-10, 7-11

Geometry

Specific Outcomes	Suggestions for Teaching and Learning

Specific Outcomes

Students will be expected to

G4, A1 Continued ...

Achievement Indicators:

G4.4 *Identify situations where the trigonometric ratios are used for indirect measurement of angles and lengths.*

G4.5 *Solve a contextual problem that involves right triangles, using the primary trigonometric ratios.*

A1.3 *Solve a contextual problem that involves the application of a formula that does not require manipulation.*

A1.4 *Solve a contextual problem that involves the application of a formula that requires manipulation.*

A1.6 *Explain and verify why different forms of the same formula are equivalent.*

Suggestions for Teaching and Learning

Once students have a clear picture of the connection between the trigonometric ratios and the reference angle, they can use them to determine missing side lengths and angle measures. Students have used the Pythagorean theorem to find the length of the third side in a right triangle when given the lengths of the other two sides. They can now use the appropriate trigonometric ratio to determine a side length when given an angle measure and another side length. In Grade 9, students solved equations of the form $a = \frac{b}{c}$ (9PR3). A review of this may be necessary prior to solving equations such as $\tan 30° = \frac{x}{10}$ or $\tan 30° = \frac{5}{x}$.

Students will also use the sine, cosine or tangent ratios to determine the measure of a missing acute angle in a right triangle. This will require the use of the inverse of the ratio.

It is expected that students will use a calculator to find trigonometric ratios and angle measurements for given trigonometric ratios. Practice using the calculator is essential and students should be aware of the need to work in degree mode.

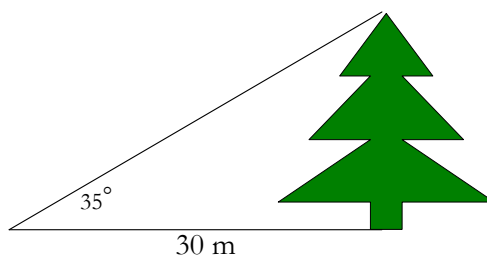Using trigonometric ratios to solve problems, such as the following, will help clarify their value:

- The escalator in a shopping mall rises 76 feet at an angle of 30˚.

To find the distance a person travels on the escalator stairs, the sine ratio would be used.

- Determine the height of the tree.

35°
30 m

Students should also apply the appropriate trigonometric ratios to solve triangles.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies	Resources/Notes

### Suggested Assessment Strategies

Journal

- Ask students to describe how they would determine the height of a flagpole which they cannot climb in each of the following cases:

  (i) on a very sunny day, you and your friend have a tape measure

  (ii) it is a very cloudy, overcast day and you and your friend have a metre stick and a clinometer (a device to measure angle of elevation)

  (G4.4, G4.5)

Performance

- Students could make a graffiti wall. Each student draws a right triangle on a sticky note, labels the right angle, and marks one acute angle with a star. They also write a length value for any two sides. Remind students that the hypotenuse must be the longest. Have them post their notes on the wall. Students then choose a sticky other than their own and determine which ratio can be identified. They then place it in the appropriate section of the board under sine, cosine or tangent.

  (G4.5, A1.3)

- Students can play online games, to practice solving problems that require the use of sine, cosine and tangent (see Resources/Notes).

  (G4.1, G4.2, G4.3)

Paper and Pencil

- Ask students to answer the following:

  (i) A guy wire 6 m long is holding up a telephone pole. The guy wire makes an angle of 70° with the ground. At what height on the telephone pole is the guy wire attached?

  (A1.4, G4.5)

  (ii) An airplane is approaching the St. John's airport as represented by the diagram. Find the line of sight distance from the airplane to the terminal.

250 m

x
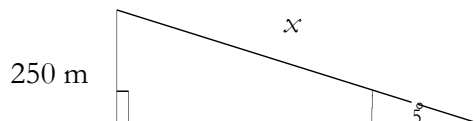
  (A1.4, G4.5)

  (iii) A ramp 4 m long is being built to reach a loading dock that is 1.5 m in height. What is the measure of the angle between the ramp and the ground?

  (A1.2, A1.4, A1.5, G4.5)

Resources/Notes

Authorized Resource

Math at Work 10

7.2 The Tangent Ratio

7.3 The Sine and Cosine Ratios

7.4 Finding Unknown Angles

SB: pp. 336-347, 348-363, 364-375

TR: pp. 347-355, 356-367

BLM 7-6, 7-7, 7-8, 7-9, 7-10, 7-11, 7-12, 7-13

PL Site: https://www.k12pl.nl.ca/curr/10-12/math/math1202/classroomclips/trigonometric-ratios.html

- Classroom clip of the graffiti wall performance strategy being used

PL Site: https://www.k12pl.nl.ca/curr/10-12/math/math1202/classroomclips/clinometer.html

- Classroom clip of students using a clinometer to determine angle measures

Suggested Resource

Resource Link: https://www/k12.pl.nl.ca/curr/10-12/math/math1202/links/unit7.html

- Online trigonometry game

Geometry

Specific Outcomes	Suggestions for Teaching and Learning

Specific Outcomes

Students will be expected to

G4, A1 Continued ...

Achievement Indicators:

A1.2 *Describe, using examples, how a given formula is used in a trade or an occupation.*

A1.1 *Create and solve a contextual problem that involves a formula.*

Suggestions for Teaching and Learning

Trigonometric ratios are used in many different trades and occupations. Carpenters, pipefitters and tailors use them on a regular basis. Trigonometry is used, for example, in surveying to determine height and distances, in navigation to determine location, and in construction to determine the roof pitch. A surveyor could use the sine ratio, for example, to determine the length of a pond if AB is 15 km of a section of highway and ∠A is 50°.
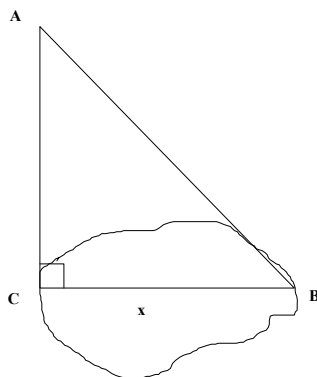
Often, there are terms that are specific to an occupation, but knowledge of mathematics can be used to help solve problems. A pipefitter, for example, would use the terms *offset*, *run* and *travel*, but these are actually the opposite and adjacent sides and the hypotenuse of a right triangle.

A1.5 *Identify and correct errors in a solution to a problem that involves a formula.*

G4.6 *Determine if a solution to a problem that involves primary trigonometric ratios is reasonable.*

Engaging students in error analysis heightens awareness of common errors. Along with providing the correct solutions, they should be able to identify incorrect solutions, including why errors might have occurred and how they can be corrected. This reinforces the importance of recording solution steps rather than only giving a final answer.

To determine if a solution is reasonable, students should ask themselves questions such as:

- Is the hypotenuse the longest side in the triangle?
- Is the smallest angle opposite the shortest side?
- Is the angle measure too big for the triangle whose angles have a sum of 180°?

General Outcome: Develop spatial sense.

Suggested Assessment Strategies	Resources/Notes

Suggested Assessment Strategies

Paper and Pencil

- Ask students to identify and correct any errors in the following solution:
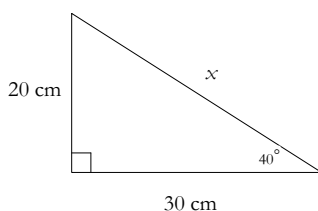
$$\sin 40° = \frac{30}{x}$$

$$x \sin 40° = 30$$

$$x = \frac{30}{\sin 40°}$$

$$x \doteq 46.7$$

(A1.5, G4.6)

Presentation

- Students could research how trigonometric ratios can be used in a trade or an occupation such as:

  (i)   surveying
  (ii)   plumbing/pipefitting
  (iii) construction
  (iv)   warehouse job
  (v)   lighting technician
  (vi)   any other occupation of interest

(A1.2)

Interview

- Ask students to design a question that involves the application of the trigonometric ratios as it applies to a trade or occupation.

  Teachers could invite tradespeople to visit the class. Students could ask their questions to begin discussion of how trigonometry is used in the various trades.

(A1.2, G4.4, G4.5)

Resources/Notes

Authorized Resource

Math at Work 10

7.2 The Tangent Ratio

7.3 The Sine and Cosine Ratios

7.4 Finding Unknown Angles

SB: pp. 336-347, 348-363, 364-375

TR: pp. 347-355, 356-367, 368-376

BLM 7-6, 7-7, 7-8, 7-9, 7-10, 7-11, 7-12, 7-13

Geometry

Specific Outcomes	Suggestions for Teaching and Learning

Students will be expected to

G1 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies.

[C, CN, PS, R]

Achievement Indicators:

G1.1 *Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g.,*

- *guess and check*
- *look for a pattern*
- *make a systematic list*
- *draw or model*
- *eliminate possibilities*
- *simplify the original problem*
- *work backward*
- *develop alternative approaches.*

G1.2 *Identify and correct errors in a solution to a puzzle or in a strategy for winning a game.*

G1.3 *Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.*

This outcome is intended to be integrated throughout the course by using puzzles and games focused on sliding, rotation, construction and deconstruction. These are intended to help students enhance spatial reasoning and problem-solving strategies.

Students internalize what they have learned when they apply their learning to other situations. Considering two puzzles or games they have worked on, discuss what is different and what is the same. Ask them how well strategies for one game would apply to the other. Is it possible to invent a new game that blends rules from both games?

Extend the discussion to other situations. Discuss ways that solving puzzles is similar to solving problems in mathematics. Consider common events, such as planning a trip. Point out to students that when you plan a trip you often make a list to ensure everything gets done before you leave. Also, when you lose an object, you often have to work backwards (re-trace your steps) to try and find the object. Ask them to consider how other strategies used to solve puzzles or play games can be applied to everyday planning.

General Outcome: Develop spatial sense.

Suggested Assessment Strategies	**Resources/Notes**

Paper and Pencil

- Puzzles can be classified using a Venn diagram. Ask students to list 10 puzzles and decide on 3 or 4 attributes for classifying the puzzles. They can sort the puzzles into categories. For categories with no members, they could find existing types of puzzles or invent new ones.

Authorized Resource

Math at Work 10

Games and Puzzles

Trig Solitaire   SB: p. 381

Trig's a Snap   SB: p. 381

TR: p. 382

BLM 7-16

Appendix:

Outcomes with Achievement Indicators
Organized by Topic

(With Curriculum Guide Page References)

Topic: Number	**General Outcome:** Develop number sense and critical thinking skills.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
N1 Solve problems that involve unit pricing and currency exchange, using proportional reasoning. [CN, ME, PS, R]	N1.1 Compare the unit price of two or more given items.	p. 22
	N1.2 Compare, using examples, different sales promotion techniques; e.g., deli meat at $2 per 100 g seems less expensive than $20 per kilogram.	p. 24
	N1.3 Solve problems that involve determining the best buy, and explain the choice in terms of the cost as well as other factors, such as quality and quantity.	p. 24
	N1.4 Determine the percent increase or decrease for a given original and new price.	p. 26
	N1.5 Explain the difference between the selling rate and purchasing rate for currency exchange.	p. 28
	N1.6 Convert between Canadian currency and foreign currency using formulas, charts and tables.	pp. 30, 52
	N1.7 Solve, using proportional reasoning, a contextual problem that involves currency exchange.	pp. 30, 50
	N1.8 Explain how to estimate the cost of items in Canadian currency while in a foreign country, and explain why this might be important.	p. 30

Topic: Number	**General Outcome:** Develop number sense and critical thinking skills.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
N2 Demonstrate an understanding of income, including: • wages • salary • contracts • commissions • piecework to calculate gross and net pay. [C, CN, R, T]	N2.1 Describe, using examples, various methods of earning income.	p. 80
	N2.2 Identify and list jobs that commonly use different methods of earning income; e.g., hourly wage, wage and tips, salary, commission, contract, bonus, shift premiums.	p. 80
	N2.3 Determine in decimal form, from a time schedule, the total time worked in hours and minutes, including time and a half and/or double time.	p. 82
	N2.4 Determine gross pay from given or calculated hours worked when given: • the base hourly wage, with and without tips. • the base hourly wage, plus overtime (time and a half, double time).	p. 84
	N2.5 Investigate, with technology, "what if..." questions related to changes in income; e.g., "What if there is a change in the rate of pay?"	p. 86
	N2.6 Identify and correct errors in a solution to a problem that involves gross or net pay.	pp. 86, 90
	N2.7 Explain why gross pay and net pay are not the same.	pp. 88-93
	N2.8 Determine the Canadian Pension Plan (CPP), Employment Insurance (EI) and income tax deductions for a given gross pay.	pp. 88-93
	N2.9 Determine net pay when given deductions; e.g., health plans, uniforms, union dues, charitable donations, payroll tax.	pp. 88-93
	N2.10 Determine gross pay for earnings acquired by: • base wage, plus commission • single commission rate.	p. 94
	N2.11 Describe the advantages and disadvantages for a given method of earning income; e.g., hourly wage, tips, piecework, salary, commission, contract work.	p. 94

Topic: Algebra	General Outcome: Develop algebraic reasoning.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
A1 Solve problems that require the manipulation and application of formulas related to: • **perimeter** • area • the Pythagorean theorem • primary trigonometric ratios • income. [C, CN, ME, PS, R]	A1.1 Create and solve a contextual problem that involves a formula.	pp. 54, 66, 128, 148
	A1.2 Describe, using examples, how a given formula is used in a trade or occupation.	pp. 54, 126, 148
	A1.3 Solve a contextual problem that involves the application of a formula that does not require manipulation.	pp. 54, 66-69, 84, 88-93, 128, 146
	A1.4 Solve a contextual problem that involves the application of a formula that requires manipulation.	pp. 54, 66, 128, 146
	A1.5 Identify and correct errors in a solution to a problem that involves a formula.	pp. 66, 148
	A1.6 Explain and verify why different forms of the same formula are equivalent.	pp. 128, 146

Topic: Measurement	General Outcome: Develop spatial sense through direct and indirect measurement.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
M1 Demonstrate an understanding of the Système International (SI) by: • describing the relationships of the units for length, area, volume, capacity, mass and temperature • applying strategies to convert SI units to imperial units. [C, CN, ME, V]	M1.1 Explain how the SI system was developed, and explain its relationship to base ten.	p. 32
	M1.2 Identify contexts that involve the SI system.	p. 36
	M1.3 Match the prefixes used for SI units of measurement with the powers of ten.	p. 36
	M1.4 Explain, using examples, how and why decimals are used in the SI system.	p. 36
	M1.5 Provide an approximate measurement in SI units for a measurement given in imperial units; e.g., 1 inch is approximately 2.5 cm.	p. 38
	M1.6 Convert a given measurement from SI to imperial units by using proportional reasoning (including formulas); e.g., Celsius to Fahrenheit, centimetres to inches.	pp. 38, 52
	M1.7 Write a given linear measurement expressed in one SI unit in another SI unit.	p. 50

Topic: Measurement	General Outcome: Develop spatial sense through direct and indirect measurement.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
M2 Demonstrate an understanding of the imperial system by: • describing the relationships of the units for length, area, volume, capacity, mass and temperature • comparing the American and British imperial units for capacity • applying strategies to convert imperial units to SI units. [C, CN, ME, V]	M2.1 Explain how the imperial system was developed.	p. 34
	M2.2 Compare the American and British imperial measurement systems; e.g., gallons, bushels, tons.	p. 34
	M2.3 Identify contexts that involve the imperial system.	p. 36
	M2.4 Explain, using examples, how and why fractions are used in the imperial system.	pp. 38, 46
	M2.5 Provide an approximate measure in imperial units for a measurement in SI units; e.g., 1 litre is approximately $\frac{1}{4}$ US gallon.	p. 38
	M2.6 Convert a given measurement from imperial to SI units using proportional reasoning (including formulas); e.g., Fahrenheit to Celsius, inches to centimetres.	pp. 38, 52
	M2.7 Write a given linear measure expressed in one imperial unit in another imperial unit.	p. 46
M3 Solve and verify problems that involve SI and imperial linear measurements, including decimal and fractional measurements. [CN, ME, PS, V]	M3.1 Provide an example of a situation in which a fractional linear measurement would be divided by a fraction.	p. 46
	M3.2 Measure inside diameters, outside diameters, lengths, widths of various given objects, and distances, using various measuring instruments.	pp. 48-51
	M3.3 Identify a referent for a given common SI or imperial unit of linear measurement.	pp. 48-51
	M3.4 Esitmate a linear measurement, using a referent.	pp. 48-51
	M3.5 Estimate the dimensions of a given regular 3-D object or 2-D shape, using a referent; e.g., the height of the desk is about three rulers long, so the desk is approximately three feet height.	pp. 48-51
	M3.6 Solve a linear measurement problem including perimeter, circumference, and length + width + height (used in shipping and air travel).	p. 54
	M3.7 Determine the operation that should be used to solve a linear measurement problem.	p. 54
	M3.8 Determine, using a variety of strategies, the midpoint of a linear measurement such as length, width, height, depth, diagonal and diameter of a 3-D object, and explain the strategies.	p. 56
	M3.9 Determine if a solution to a problem that involves linear measure is reasonable.	p. 56

Topic: Measurement	General Outcome: Develop spatial sense and through direct and indirect measurement.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
M4 Solve problems that involve SI and imperial area measurements of regular, composite and irregular 2-D shapes and 3-D objects, including decimal and fractional measurements, and verify the solutions. [ME, PS, R, V]	M4.1 Identify and compare referents for area measurements in SI and imperial units.	p. 62
	M4.2 Estimate an area measurement, using a referent.	p. 62
	M4.3 Estimate the area of a given regular, composite or irregular 2-D shape, using an SI square grid and an imperial square grid.	p. 62
	M4.4 Determine if a solution to a problem that involves an area measurement is reasonable.	p. 62
	M4.5 Identify a situation where a given SI or imperial area unit would be used.	p. 64
	M4.6 Write a given area measurement expressed in one SI unit squared in another SI unit squared.	p. 64
	M4.7 Write a given area measurement expressed in one imperial unit squared in another imperial unit squared.	p. 64
	M4.8 Solve a contextual problem that involves the area of a regular, a composite or an irregular 2-D shape.	pp. 66-69
	M4.9 Solve a problem, using formulas for determining the areas of regular, composite and irregular 2-D shapes, including circles.	pp. 66-69
	M4.10 Explain, using examples, the effect of changing the measurement of one or more dimensions on area and perimeter of rectangles.	p. 68
	M4.11 Solve a problem that involves determining the surface area of 3-D objects, including right cylinders and cones.	pp. 70-73

Topic: Geometry	General Outcome: Develop spatial sense.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
G1 Analyze puzzles and games that involve spatial reasoning, using problem-solving strategies. [C, CN, PS, R]	G1.1 Determine, explain and verify a strategy to solve a puzzle or to win a game; e.g., • guess and check • look for a pattern • make a systematic list • draw or model • eliminate possibilities • simplify the original problem • work backward • develop alternative approaches.	pp. 40, 74, 114, 130, 150
	G1.2 Identify and correct errors in the solution to a puzzle or in a strategy for winning a game.	pp. 40, 74, 114, 130, 150
	G1.3 Create a variation on a puzzle or a game, and describe a strategy for solving the puzzle or winning the game.	pp. 40, 74, 114, 130, 150
G2 Demonstrate an understanding of the Pythagorean theorem by: • identifying situations that involve right triangles • verifying the formula • applying the formula • solving problems. [C, CN, PS, V]	G2.1 Describe historical and contemporary applications of the Pythagorean theorem.	p. 120
	G2.2 Explain, using illustrations, why the Pythagorean theorem only applies to right triangles.	p. 122
	G2.3 Verify the Pythagorean theorem, using examples and counterexamples, including drawings, concrete materials and technology.	p. 122
	G2.4 Determine if a given triangle is a right triangle, using the Pythagorean theorem.	p. 124
	G2.5 Explain why a triangle with the side length ratio of 3:4:5 is a right triangle.	p. 124
	G2.6 Explain how the ratio of 3:4:5 can be used to determine if a corner of a given 3-D object is square (90 degrees) or if a given parallelogram is a rectangle.	p. 124
	G2.7 Solve a problem, using the Pythagorean theorem.	p. 128

Topic: Geometry	General Outcome: Develop spatial sense through direct and indirect measurement.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
G3 Demonstrate an understanding of similarity of convex polygons, including regular and irregular polygons. [C, CN, PS, V]	G3.1 Determine if two or more regular or irregular polygons are similar.	p. 136
	G3.2 Explain why two or more right triangles with a shared acute angle are similar.	p. 136
	G3.3 Explain the relationships between the corresponding sides of two polygons that have corresponding angles of equal measure.	p. 138
	G3.4 Explain why two given polygons are not similar.	p. 138
	G3.5 Solve a contextual problem that involves the similarity of polygons.	p. 138
	G3.6 Draw a polygon that is similar to a given polygon.	p. 140
G4 Demonstrate an understanding of primary trigonometric ratios (sine, cosine, tangent) by: • applying similarity to right triangles • generalizing patterns from similar right triangles • applying the primary trigonometric ratios • solving problems. [CN, PS, R, T, V]	G4.1 Show, for a specified acute angle in a set of right triangles, that the ratios of length of the side opposite to the length of the side adjacent are equal, and generalize a formula for the tangent ratio.	p. 144
	G4.2 Show, for a specified acute angle in a set of right triangles, that the ratios of length of the side opposite to the length of the hypotenuse are equal, and generalize a formula for the sine ratio.	p. 144
	G4.3 Show, for a specified acute angle in a set of right triangles, that the ratios of length of the side adjacent to the length of the hypotenuse are equal, and generalize a formula for the cosine ratio.	p. 144
	G4.4 Identify situations where the trigonometric ratios are used for indirect measurement of angles and lengths.	p. 146
	G4.5 Solve a contextual problem that involves right angles, using the primary trigonometric ratios.	p. 146
	G4.6 Determine if a solution to a problem that involves primary trigonometric ratios is reasonable.	p. 148

Topic: Geometry	General Outcome: Develop spatial sense.	
Specific Outcomes It is expected that students will:	**Achievement Indicators** *The following sets of indicators help determine whether students have met the corresponding specific outcome*	**Page Reference**
G5 Solve problems that involve parallel, perpendicular and transversal lines, and pairs of angles formed between them. [C, CN, PS, V]	G5.1 Sort a set of lines as perpendicular, parallel or neither, and justify this sorting.	p. 106
	G5.2 Illustrate and describe complementary and supplementary angles.	p. 108
	G5.3 Identify, in a set of angles, adjacent angles that are not complementary or supplementary.	p. 108
	G5.4 Identify and name pairs of angles formed by parallel lines and a transversal, including corresponding angles, vertically opposite angles, alternate interior angles, alternate exterior angles, interior angles on the same side of transversal, and exterior angles on the same side of the transversal.	p. 110
	G5.5 Explain and illustrate the relationships of angles formed by parallel lines and a transversal.	p. 110
	G5.6 Determine the measures of angles involving parallel lines and a transversal.	p. 110
	G5.7 Explain, using examples, why the angle relationships do not apply when the lines are not parallel.	p. 112
	G5.8 Determine if lines or planes are perpendicular or parallel, e.g., wall perpendicular to the floor, and describe the strategy used.	p. 112
	G5.9 Solve a contextual problem that involves angles formed by parallel lines and a transversal (including perpendicular transversals).	p. 112
G6 Demonstrate an understanding of angles, including acute, right, obtuse, straight and reflex, by: • drawing • replicating and constructing • bisecting • solving problems. [C, CN, ME, PS, R]	G6.1 Measure, using a protractor, angles in various orientations.	p. 100
	G6.2 Draw and describe angles with various measures, including acute, right, straight, obtuse and reflex angles.	p. 100
	G6.3 Identify referents for angles.	p. 102
	G6.4 Sketch a given angle.	p. 102
	G6.5 Estimate the measure of a given angle, using 22.5°, 30°, 45°, 60°, 90° and 180° as referent angles.	p. 102
	G6.6 Solve a contextual problem that involves angles.	pp. 102-105
	G6.7 Explain and illustrate how angles can be replicated in a variety of ways; e.g., Mira, protractor, compass and straightedge, carpenter's square, dynamic geometry software.	p. 104
	G6.8 Replicate angles in a variety of ways, with and without technology.	p. 104
	G6.9 Bisect an angle, using a variety of methods.	p.104

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