

# Mathematics 3201

## June 2015 Public Exam Outcome Report

This examination follows the specifications, conventions and standards set out in the:  
**Mathematics Public Examination Standards**

- Chapters:**
- |                                      |  |
|--------------------------------------|--|
| 1 Set Theory                         | 6 Exponential Functions                  |
| 2 Counting Method                    | 7 Logarithmic Functions                  |
| 3 Probability                        | 8 Sinusoidal Functions                   |
| 4 Rational Expressions and Equations | 9 Financial Mathematics: Borrowing Money |
| 5 Polynomial Functions               |  |

**PART I: Selected Response**—Total Value: 50%

Item	Curriculum Guide Pages	Outcome	Cognitive Level	Outcome Description
1	22, 24, 26	LR2	2M	Given a Venn diagram with three sets, two of which are disjoint (non-intersecting), determine which mathematical statement about the elements in the union or intersection of two of the sets is always the empty set.
2	26	LR2	2M	Given a Venn diagram with two non-disjoint (intersecting) sets, determine the number of elements in the complement of the intersection of the sets.
3	22	LR2	2A	Given one finite set and one infinite set, each described using set notation, determine which statement about the two sets is true.
4	30	LR2	2A	Given $n(A), n(B), n(A \cap B)$ , and $n(A \cup B)$ determine $n(U)$ , where $U$ denotes the universal set.
5	54	P4	2M	Given multiple distinct elements in each of three categories, determine the total number of choices possible if one element is selected from each category.
6	58	P5	2M	Determine the number of ways in which $n$ distinct objects can be arranged in a line if order matters.

7	56 & 64	P4	2M	Determine the number of passwords that can be created using the digits 0 to 9 given the number of digits in the password, that repetition of digits is permitted, and that there is a restriction on the digits used for the first number of the password.
8	60	P5	2A	Simplify an algebraic fraction containing factorials in the numerator and in the denominator.
9	68	P5	2A	Determine the number of different arrangements of a word with $n$ letters, not all of which are distinct.
10	70, 74	P6	2A	Given the number of defensemen and the number of forwards on a hockey team, identify the notation that indicates the number of ways in which a starting lineup of 5 players (no goalie) can be selected if the lineup must include exactly 2 defensemen.
11	88	P3	2M	Identify the Venn diagram which illustrates mutually exclusive events.
12	82	P1	2M	Given the probability of an event as a percent, determine the odds in favour of the event as a ratio.
13	88	P2	2A	Given a Venn diagram containing $n(A \setminus B)$ , $n(B \setminus A)$ , $n(A \cap B)$ and $n(A \cup B)$ , determine the probability (as a percentage) that a randomly selected element is in $(A \setminus B) \cup (B \setminus A)$ .
14	94	P3	2A	Given a deck containing $n$ cards divided into 4 equal different coloured sets, determine the probability that the first card is a given colour and the second card is the same colour as the first or any two other colours, if the cards are drawn without replacement.
15	94	P3	L3	Given the probability that a person attends a music festival if a certain band is playing at the festival, the probability that the person attends if the band is not playing, and the probability that the band plays at the festival, determine the probability that the person will attend the festival.
16	100	RF1	2M	Identify the rational expression (in factored form) that has the given non-permissible values.
17	108	RF2	2M	Determine the LCD for an equation where one side of the equation consists of the sum of two rational expressions with denominators $x + k$ and $c(x + k)$ and the other side of the equation consists of a rational expression with denominator $x + b$ , where $k$ , $c$ , and $b$ are integers.

18	104	RF1	2A	Given a rational expression with non-factored quadratic binomials in the numerator and denominator, determine the simplified form of the rational expression.
19	106	RF2	2A	Simplify the quotient of two rational expressions with like numeric denominators and monomials with the same variable but different degrees in each numerator.
20	108	RF2	2A	Simplify the sum of a numerical fraction and an algebraic rational expression with a monomial in its numerator and a linear binomial in its denominator.
21	110	RF3	2A	Solve an equation where one side of the equation is a numerical fraction and the other side of the equation is in the form $\frac{ax+k}{cx}$ , where $k$ , $c$ , and $a$ are integers.
22	126	RF7	2M	Given the equation of a cubic polynomial function in standard form, determine the $y$ -intercept and the end behaviour of the graph of the function.
23	120	RF7	2M	Given four graphs, determine the graph of a polynomial function.
24	122	RF7	2A	Determine which characteristic of a parabola would change if the sign of the leading coefficient of the quadratic equation is changed.
25	128	RF7	2A	Identify the polynomial function which best corresponds to the given graph.
26	124	RF7	2A	Given the graph of a polynomial function, determine the degree and leading coefficient of the corresponding polynomial function.
27	130	RF7	2A	Given the graph of a polynomial function $y = f(x)$ , determine the value of $f(x)$ for a given $x$ value.
28	140	RF6	2M	Given a description of the $y$ -intercept and the end behaviour of the graph of an exponential function, identify the equation of the function.
29	142	RF6	2M	Given the graph of an exponential function showing the $y$ -intercept, identify the equation of the function.
30	146	RF5	2A	Solve an exponential equation, involving powers with variable exponents, in which the bases can be made equal.
31	150	RF5	2A	Given an exponential equation of the form $G(t) = a(b)^{ct+d}$ , where $a$ , $b$ , $c$ , and $d$ are constants, which models the growth of a population over time, determine the amount of time taken for the population to reach a given value.

32	150	RF1, RF5	2A	Identify the exponential function that models the value of an investment over time given the principal invested, the annual interest rate, and the interest compounding frequency.
33	152	RF6	2A	Given an exponential graph which models the growth of an investment over time, determine the approximate value of the investment at a given time.
34	170	RF4	2M	Rewrite a given exponential equation in logarithmic form.
35	164	RF6	2M	Given the exponential graph $y = f(x)$ , identify the graph which results when the $x$ and $y$ coordinates of the points on $y = f(x)$ are interchanged.
36	170	RF4	2A	Determine the numerical value of a logarithm in the form $\log_b\left(\frac{1}{N}\right)$ where $N = b^k$ .
37	174	RF4	2A	Rewrite $k \log_b N - \log_b M$ , where $k, b, N$ , and $M$ are constants, as a single logarithm.
38	178	RF5	2A	Find the exact solution to an exponential equation, equating a power (with a constant base and a linear binomial in the exponent) and a constant, in which the bases cannot be made equal.
39	182	RF5	L3	Given an exponential function of the form $A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$ and the half-life of a radioactive substance, determine the amount of time it would take for a given percentage of a radioactive substance to remain.
40	194	RF8	2M	Determine the period of a given sinusoidal graph.
41	196	RF8	2M	Determine the equation of the midline of the graph of a sinusoidal function with equation $f(x) = a \cos b(x - c^\circ) + d$ , where $a, b, c$ , and $d$ are constants.
42	190	RF8	2A	Convert an angle measure from degrees to radians.
43	196	RF8	2A	Given the graph of a sinusoidal function plus a description of the maximum and minimum value of the function, determine the amplitude of the graph.
44	198	RF8	2A	Given the equation $y = a \sin b(x - c) + d$ , determine the value of $b$ that will result in a graph with a given period.

45	196,200	RF8	2A	Determine the range of the sinusoidal function with equation $y = a \cos(x + c^\circ) + d$ , where $a$ , $c$ , and $d$ are constants.
46	210	F1	2M	Given the repayment frequency and the number of payments required to repay a loan, determine the term of the loan.
47	216	F1	2M	Given a partial amortization table showing the payment period, the loan payment amount for each payment period, the amount of the principal repaid for each payment period, and the loan balance after each payment period, determine the amount of the initial loan balance that has been repaid after a given number of payments.
48	226	F2	2M	Identify the factor that might lead a person to purchase a home rather than rent.
49	208	F1	2A	Given a graph containing curves displaying the value of an investment earning the same annual interest rate with different interest compounding frequencies, identify the curve which represents the value of the investment for a given interest compounding frequency.
50	212	F1	2A	Given an exponential equation in the form $A = P(1+i)^n$ that models a bank loan, the interest compounding period and the repayment time, determine the annual interest rate.

**PART II: Constructed Response—Total Value: 50%**

Item	Curriculum Guide Page	Outcome	Cognitive Level	Value	Outcome Description
51	36	LR2	L3	3	Given $n(U), n(A), n(B), n(C), n(A \cap C), n(B \cap C),$ and $n(A \cap B \cap C)$ , use the provided Venn diagram to algebraically find the percentage of the elements in $B \setminus (A \cup C)$ .
52a	60,66	P5	L2A	3	Given an equation in the form ${}_n P_r = k$ , where $r$ and $k$ are constants, algebraically solve the equation for $n$ .

52b(i)	56	P4	L2A	1	Determine the total number of ways in which two letters followed by three digits can be selected from the 26 letters of the alphabet and the digits 0 through 9, if repetition is allowed.
52b(ii)	56	P4	L2A	1	Determine the number of outcomes that cannot occur in 52b(i) if repetition of letters is not allowed and there is a restriction on the numbers used for the first digit.
52c	70,74	P6	L3	3	Determine the number of ways in which $r$ elements can be chosen from $n$ elements in two different scenarios, one in which order is important and one in which order is not important, and explain why the answer in each scenario is different.
53a	84	P6	L2A	1	Given the number of girls and the number of boys on a student council, determine the probability that a sub-committee of size $n$ has $n$ girls.
53b	82	P1	L2A	2	Given the probability of one event and the odds in favour of a second independent event, determine which event is more likely to occur.
53c	92	P3	L2A	1	Given two independent events, determine the probability of a given outcome.
54a	106	RF2	L2A	3	Given the quotient of two algebraic rational expressions, both with binomials in the numerator and the denominator, simplify the expression and state the non-permissible values.
54b(i)	108	RF3	L3	1	Specifically identify the error in the “simplification” of the difference of two rational expressions.
54b(ii)	108	RF3	L2A	1	Provide the correct simplification for the rational expression in 54b(i).
54c	110	RF3	L3	1	Given a description of a real-world situation, the rational equation that models the situation, and the solutions to the rational equation, determine if the solutions make sense in the context of the situation and justify the decision.
55a(i)	126	RF7	L2A	1	On the axes provided, sketch a possible polynomial graph given a description of the end behaviour of the graph and the y-intercept of the graph.
55a(ii)	126	RF7	L2A	1	Write a polynomial equation that could represent the polynomial described in 55a(i).

55b	124	RF7	L2A	2	Given a polynomial graph, determine the end behaviour, the number of turning points, and the y-intercept of the graph plus the degree of the corresponding polynomial function.
55c	130	RF7	L3	2	Given a description of projectile motion that can be modeled using a quadratic function $y = ax^2 + bx + c$ and the values of $a$ , $b$ , and $c$ , determine if the projectile will reach a given height after travelling a given horizontal distance.
56a	142	RF5	L2A	4	Algebraically solve an exponential equation, equating a numerical radical with a power that has a constant base and a linear binomial exponent, in which the bases can be made equal.
56b(i)	154	RF6	L3	1	Given a table that models the growth of an investment over time, determine the exponential function that could predict the future value of the investment.
56b(ii)	154	RF6	L3	1	Determine how the function in 56b(i) would change if the annual interest rate changed but the compounding frequency remained the same.
56b(iii)	154	RF6	L3	1	Explain how the value of the investment in 56b(i) would be affected over time if the investment grew according to the interest rate in 56b(ii).
57a	176	RF5	L3	3	Given the formula for pH, $p(x) = -\log x$ , and the pH of a solution, determine the new pH of the solution if the concentration of hydrogen ions in the solution is doubled.
57b(i)	150,182	RF5/6	L2A	1	Given an exponential function in the form $A(t) = A_0 \left(\frac{1}{2}\right)^{\frac{t}{h}}$ which models the radioactive decay of a substance, and the numerical values of $A_0$ and $h$ , algebraically determine the amount remaining at a given time.
57b(ii)	150,182	RF5/6	L2A	3	Using the function in 57b(i), algebraically determine the time taken for the sample to decay to a given amount.

58a(i)	196,198	RF8	L2A	3	Given a sinusoidal function of the form $y = a \sin b(x - c) + d$ , where $a$ , $b$ , $c$ , and $d$ are constants, that models the depth of the water in a harbour, determine the amplitude, period, and the equation of the midline of the corresponding graph and the maximum and minimum depth of the water.
58a(ii)	196,198	RF8	L2A	1	Determine the depth of the water in the harbour in 58a(i) at a given time.
58b	196	RF8	L3	2	On the grid provided, sketch the graph of a sinusoidal function given the domain and range of the function and the period and y-intercept of the graph.
59	210	F1	L2A	3	Given the loan principal, the annual interest rate, the interest compounding frequency, and the term of the loan, determine the interest charged on the loan if the loan is repaid with a single payment at the end of the term.